Final Examination

Total points: 100

Instructions: (i) Consulting an A4-sized sheet of paper on which you wrote down necessary information is allowed.

(ii) Using a calculator is allowed.

1. (10 points) Why do you think β does or does not measure risk of an financial asset?

2. (10 points) The capital market line is written as

$$\mu_p = R_f + \left(\frac{\mu_q - R_f}{\sigma_q} \right) \sigma_p,$$

where μ_q and σ_q^2 are the mean and variance of the market portfolio, respectively, and R_f is the risk-free return. A prudent investor will remain on the capital market line. What is the expected return of the investor when he takes no risk? What is it when the investor takes the same level of risk as the market portfolio?.

3. (20 points) Discuss validity of the following statements.

   (a) Denote the holding period of an asset by k. If the Sharpe ratio of the asset return divided by $\sqrt{k}$ grows with k, we may say that it is safe to hold the asset for a long period of time.

   (b) If the variance ratio of an asset return is less than one, it implies that the asset is safer for "long-run investors" who can tolerate ups and downs of the market.

4. (10 points) Suppose that $\{r_t\}$ is an independent sequence of returns. Does it mean that $\{\frac{1}{r_t}\}$ is an uncorrelated sequence?

5. (10 points) Consider the ARCH(1) model

$$a_t = \mu + \sigma_t \epsilon_t, \ \sigma_t^2 = \alpha_o + \alpha_1 a_{t-1}^2, \ \epsilon_t \sim iid(0, 2),$$

where μ is a constant. Assume that $Var(a_t)$ does not change over time. What is $Var(a_t)$?

6. (10 points) Express the GARCH model

$$\begin{aligned} a_t &= \sigma_t \epsilon_t, \\ \sigma_t^2 &= \alpha_o + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned}$$

as an ARMA model.

7. (10 points) Some argue that the return of the present period is positively correlated with the conditional variance of the next period. Evaluate this statement by assuming that the return has zero-mean and follows the ARCH(1) model. (Hint: calculate $Cov(r_t, \sigma_{t+1}^2)$).

8. (20 points) Consider the following VAR process

$$r_t = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} r_{t-1} + a_t,$$

where $a_t \sim WN(0, \Sigma)$.

(a) Is the VAR process stationary?[1]

(b) Does the second element of r_t Granger-cause the first element?

[1]Hint: 1. The determinant of a 2×2 matrix is computed by the formula: $\left|\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right| = ad - bc$. 2. The roots of the equation $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. 3. The modulus of $a + bi$ is $\sqrt{a^2 + b^2}$.