Homework 8

Mathematical Statistics (Fall, 2018)
Total points: 10
Due date: 12. 10 (M)

1. Let $X_1, ..., X_n$ be a random sample from a $N(0, \sigma_X^2)$ and let $Y_1, ..., Y_n$ be a random sample from a $N(0, \sigma_Y^2)$, independent of the Xs. Define $\lambda = \sigma_Y^2 / \sigma_X^2$.

   (a) Find the level α LRT of $H_0 : \lambda = \lambda_0$ versus $H_0 : \lambda \neq \lambda_0$.

   (b) Express the rejection region of the LRT of part (a) in terms of an F random variable.

   (c) Find a $1 - \alpha$ confidence interval for λ.

2. (a) Find the $1 - \alpha$ confidence set for a that is obtained by inverting the LRT of $H_0 : a = a_0$ versus $H_1 : a \neq a_0$ based on a sample $X_1, ..., X_n$ from a $N(\theta, a\theta)$ family, where θ is unknown.

   (b) A similar question can be asked about the related family, the $N(\theta, a\theta^2)$ family. If $X_1, ..., X_n$ are iid $N(\theta, a\theta^2)$, where θ is unknown, find the $1 - \alpha$ confidence set based on inverting the LRT of $H_0 : a = a_0$ versus $H_1 : a \neq a_0$.

3. Find a $1 - \alpha$ confidence interval for θ, given $X_1, ..., X_n$ iid with pdf

   (a) $f(x \mid \theta) = 1, \ \theta - \frac{1}{2} < x < \theta + \frac{1}{2}$.

   (b) $f(x \mid \theta) = 2x/\theta^2, 0 < x < \theta, \ \theta > 0$.

4. Let X be a single observation from the $beta(\theta, 1)$ pdf.

   (a) Let $Y = -(logX)^{-1}$. Evaluate the confidence coefficient of the set $[y/2, y]$.

   (b) Find a pivotal quantity and use it to set up a confidence interval having the same confidence coefficient as the interval in part (a).

   (c) Compare the two confidence intervals.

5. If $X_1, ..., X_n$ are iid with pdf $f(x \mid \mu) = e^{-(x-u)} \mathbf{1}_{[\mu,\infty]}(x)$, then $Y = \min_{1 \le i \le n} X_i$ is sufficient for μ with pdf

$$f_Y(y \mid \mu) = ne^{-n(y-\mu)} \mathbf{1}_{[\mu,\infty]}(x).$$

   A $1 - \alpha$ confidence interval was found using the method of using cdf. Compare that interval to $1 - \alpha$ intervals obtained by likelihood and pivotal methods.

6. Let $X_1, ..., X_n$ be a random sample from a $N(\mu, \sigma^2)$ population. Compare expected lengths of $1 - \alpha$ confidence intervals for it that are computed assuming

   (a) σ^2 is known.

(b) σ^2 is unknown.

7. Let $X_1, ..., X_n$ be a random sample from a $N(\mu, \sigma^2)$ population, where both μ and σ^2 are unknown. Consider confidence intervals for σ^2 of the form

$$\{\sigma^2 : \frac{(n-1)s^2}{a} \leq \sigma^2 \leq \frac{(n-1)s^2}{b}\},$$

where s^2 is the sample variance and a and b are constant.

(a) Find the shortest length $1 - \alpha$ confidence interval of this form.

(b) For $\alpha = 0.1$ and $n = -3$, find the numerical values of a and b.Compare the length of this interval to the one obtained by splitting α equally.