Econometrics for Financial Time Series

Chapter 8: Cointegration

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Spurious regression

- We often find in applied econometric work equations of high R^2, but with low value of the D-W statistic (positive autocorrelation).
- This phenomina are contradictory, because low DW implies that there is a specification error and that the model fitted is inadequate. According to a standard theory in econometrics, we expect that low DW accompanies low R^2.
- Granger and Newbold first reported this dubious regression results (high R^2, low DW) by simulations and showed that these spurious regression results can occur when we regress one random walk process on another independent random walk process.

Spurious regression

- Specifically, Granger and Newbold studied the regression model

$$y_t = \alpha + \beta x_t + u_t \tag{1}$$

where

$$y_t = y_{t-1} + \upsilon_t, x_t = x_{t-1} + w_t$$

and

$$\upsilon_t \sim iidN(0, \sigma_\upsilon^2), w_t \sim iidN(0, \sigma_w^2).$$

- Since υ_t is independent of w_t, x_t and y_t have no statistical relation, and hence we expect that $H_0 : \beta = 0$ will be rejected by the usual t-test. However, Granger and Newbold found that the null hypothesis is rejected at the 5% level about 3/4 of 100 simulations. Further, R^2 is moderately high and DW is low in their experimental results. Because the regression results in this case cannot be relied upon, we call these regressions "spurious".

Spurious regression

- Phillips (1986; JOE) developed asymptotic results for spurious regressions by using the invariance principle. Phillips analyzed model (1) and reported that

$$\hat{\beta} \xrightarrow{d} \text{ a random variable (hence } \hat{\beta} \overset{P}{\nrightarrow} \beta)$$

$$T^{-1/2}\hat{\alpha} = O_p(1) \ (\hat{\alpha} \xrightarrow{P} 0)$$

$$T^{-1/2}t_\beta = O_p(1) \ (|t_\beta| \xrightarrow{P} \infty)$$

$$T^{-1/2}t_\alpha = O_p(1) \ (|t_\alpha| \xrightarrow{P} \infty)$$

$$R^2 = O_p(1)$$

$$DW \xrightarrow{P} 0.$$

- These asymptotic results explain why we tend to reject the nulls $\alpha = 0$ and $\beta = 0$, and find low DW and moderately high R^2 in the regressions involving two random walks. Further, these results show that the OLS estimates for α and β are not consistent.

Definitions

- A series with no deterministic component which has a stationary, invertible, ARMA representation after differencing d times, is said to be integrated of order d, denoted $x_t \sim I(d)$.

Example If

$$y_t = y_{t-1} + u_t, u_t \sim WN(0, \sigma^2),$$

$y_t \sim I(1)$.

Definitions

- Properties of $I(1)$ series

1. Growing variance. (If $y_t \sim I(1)$, then $var(y_t) \rightarrow \infty$ as $t \rightarrow \infty$.)
2. An innovation has a permanent effect on the value of y_t. $(y_t = \sum_{-\infty}^{t} u_i.)$
3. $f_{yy}(0) = \infty$. There exiats a strong long-term component. ($f_{yy}(\cdot)$:spectral density function of $\{y_t\}$.
4. The expected time between crossings of $x = 0$ is infinite.
5. Theoretical autocorrelations converge to 1 at all lags as $t \rightarrow \infty$.

Definitions

- If $x_t \sim I(d)$ and $y_t \sim I(d)$, it is generally true that $z_t = x_t - ay_t \sim I(d)$. When $z_t \sim I(b)$, $b < d$, we say that x_t and y_t are cointegrated.
- More formally, the components of the vector x_t are said to be cointegrated of order (d, b), denoted by $x_t \sim CI(d, b)$, if

1. all components of x_t are $I(d)$.
2. there exists a vector $\alpha(\neq 0)$ so that $z_t = \alpha' x_t \sim I(d - b)$, $b > 0$. The vector α is called the cointegrating vector.

- The vector α represents the long-run equilibrium relationship among variables.

Definitions

- A vector time series has an error correction representation if it can be expressed as

$$A(B)(1-B)x_t = -\gamma z_{t-1} + u_t$$

where u_t is a stationary multivariate disturbance, with $A(0) = I$, $A(1)$ has all elements finite, $z_t = \alpha' x_t$ and $\gamma > 0$.

- In this representation, the change in one period is explained by the disequilibrium errors in previous periods.

$$\left[(1-B)x_t = -\gamma A^{-1}(B) z_{t-1} + A^{-1}(B) u_t\right].$$

- For example, if $A(B) = I$, $x_t = x_{t-1} - \gamma z_{t-1} + u_t$. Thus, when the equilibrium error of the previous period $t-1$ is positive, x_t will decrease, and vice versa.

Asymptotics for cointegrating regressions

- Let $\{y_t\}$ be generated by

$$y_t = \alpha' x_t + u_t \tag{2}$$

where α is an $m \times 1$ coefficient matrix and the $m-$vector process $\{x_t\}_0^\infty$ satisfies

$$x_t = x_{t-1} + v_t.$$

x_0 can be any random variable.

Example

y_t : consumption, x_t : income

Example

y_t : money, x_t : income, interest rate

Asymptotics for cointegrating regressions

The OLS estimator of α for model (2) is

$$\hat{\alpha} = \left(\sum_{t=1}^{n} x_t x_t' \right)^{-1} \left(\sum_{t=1}^{n} x_t y_t \right).$$

Main properties of $\hat{\alpha}$ is:

1. $\hat{\alpha}$ converges to α in probability.
2. $T(\hat{\alpha} - \alpha)$ converges in distribution to a nonnormal distribution.