Homework 5

Mathematical Statistics (Fall, 2018)
Total points: 9
Due date: 11. 14 (W)

1. Let $X_1, ..., X_n$ be a random sample from the pdf

$$f(x \mid \mu, \sigma) = \frac{1}{\sigma} e^{-(x-\mu)}, \ \mu < x < \infty, 0 < \sigma < \infty.$$

   Find a two dimensional sufficient statistic for (μ, σ).

2. Let $X_1, ..., X_n$ be a random sample from the pdf

$$f_{X_i}(x \mid \theta) = \{ \begin{array}{c} e^{i\theta - x} \ , x \geq i\theta \\ 0, \ o.w. \end{array}$$

   Prove that $T = \min(X_i/i)$ is a sufficient statistic for θ.

3. Let $X_i \sim iid \ U(0, \theta), \ \theta > 0, \ i = 1, 2, ..., n$. Show that $\max X_i$ is a complete and sufficient sufficient statistic for θ. [Hint: Let f be a function satisfying $Ef(\max X_i) = 0$ for all $\theta > 0$. Use the derivative of this integral to show the completeness.]

4. Suppose that X_1 and X_2 are iid observations from the pdf $f(x \mid \alpha) = \alpha x^{\alpha-1} e^{-x^{\alpha}}, \ x > 0, \alpha > 0$. Show that $\log X_1 / \log X_2$ is an ancillary statistic.

5. Let $X_i \sim iid \ N(\mu, 1), \ i = 1, 2, ..., n.$

   (a) Show that $\bar{X}$ is complete and sufficient for μ.

   (b) Show that $S^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/(n-1)$ is an ancillary statistic.

   (c) Using Basu' theorem, prove that $\bar{X}$ is independent of S^2.

6. Let $X_1, ..., X_n$ be a random sample from the pdf

$$f(x \mid \mu) = e^{-(x-\mu)}, \ -\infty < \mu < x < \infty.$$

   (a) Show that $X_{(1)} = \min X_i$ is a complete sufficient statistic.

   (b) Using Basu' theorem, prove that $X_{(1)}$ and S^2 are independent.