Financial Econometrics
Sogang University

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Midterm Examination

Time: 03:00-4:15 PM
Total points: 80 points
Instructions: (i) Consulting an A4-sized sheet of paper on which you wrote down necessary information is allowed.
(ii) Write either in English or in Korean.

1. (10 points) A government bond pays 500,000 won each year forever. If the discount rate is 0.02 per annum, what should be its equilbrium price?

2. (10 points) The sample kurtosis is defined as

$$\hat{K}_X = \frac{1}{(T-1)\hat{\sigma}_X^4} \sum_{t=1}^{T} (X_t - \hat{\mu}_X)^4$$

   where $\{X_t\}$ denotes observations, $\hat{\mu}_X = \frac{1}{T}\sum_{t=1}^{T} X_t$ and $\hat{\sigma}_X^2 = \frac{1}{T-1}\sum_{t=1}^{T}(X_t - \hat{\mu}_X)^2$. If a test statististic

$$K = \frac{\hat{K}_X - 3}{\sqrt{\frac{24}{T}}}$$

   takes a large value, what does it tell about characteristics of the observations $\{X_t\}$?

3. (10 points) Consider the AR(1) model $r_t = \phi r_{t-1} + a_t, (t = 1, 2, ..., T)$, $|\phi| < 1$, $a_t \sim WN(0, \sigma^2)$. Assume that the coefficients ϕ is known to us. (In practice, this is estimated.)

   (a) How can we predict r_{T+1}? What is the variance of the forecast error?

   (b) How can we predict r_{T+2}? What is the variance of the forecast error?

4. (10 points)

   (a) Suppose that $r_t = 0.9r_{t-1} + a_t$, where $a_t \sim iid(0, 1)$. Is this process stationary?

   (b) Suppose that $r_t = a_t + a_{t-1}$, where $a_t \sim iid(0, 1)$. Is this process stationary?

5. (10 points) Consider the AR(2) model

$$\begin{aligned} r_t &= \phi_1 r_{t-1} + \phi_2 r_{t-2} + a_t, \\ a_t &\sim WN(0, \sigma^2). \end{aligned}$$

   (a) Explain how we can test the presence of a unit root in the AR(2) model.

   (b) The left-hand-side tails of the Dickey-Fuller are usually used for unit root tests. Why is it so?

6. (10 points) Suppose that a time series r_t can be decomposed as $r_t = a_0 + a_1 t + S_t + X_t$, where $a_0 + a_1 t$ denotes the trend component (a_0 and a_1 are unknown constant), S_t the seasonal component and X_t the random component. Suppose that the seasonal component has the property $S_t = S_{t-4}$ for all t. Show that a seasonal differencing can eliminate both the seasonal and trend components.

7. (10 points) Consider a stationary VAR model

$$\begin{pmatrix} z_t \\ x_{1t} \\ x_{2t} \end{pmatrix} = \sum_{i=1}^{p} \begin{bmatrix} \Phi_{11i} & \Phi_{12i} & \Phi_{13i} \\ \Phi_{21i} & \Phi_{22i} & \Phi_{23i} \\ \Phi_{31i} & \Phi_{32i} & \Phi_{33i} \end{bmatrix} \begin{bmatrix} z_{t-i} \\ x_{1(t-i)} \\ x_{2(t-i)} \end{bmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \\ a_{3t} \end{pmatrix}.$$

Write the null hypothesis that x_{2t} does not Granger-cause z_t at the horizon 1.

8. (10 points) Consider the ARCH(1) model

$$a_t = \sigma_t \epsilon_t, \;\; \sigma_t^2 = \alpha_o + \alpha_1 a_{t-1}^2,$$

where $\alpha_0 > 0$ and $1 > \alpha_1 \geq 0$. One of the empirical regularities for financial return series is that the returns are serially uncorrelated. Does the ARCH(1) model satisfy this property?