Financial Econometrics
Sogang University

In Choi
Spring 2015

Midterm Examination

Time: 10:30-11:45 AM
Total points: 100
Instructions: (i) Consulting an A4-sized sheet of paper on which you wrote down necessary information is allowed.
(ii) Answers without explanations are not accepted for credit.

1. (10 points) An asset's value will be \$10 million 5 years later. If the discount rate is 0.04 per annum, what is its continuously compounded present value?

2. (10 points) Derive a two-step ahead forecast for an AR(1) process $\{r_t\}$

$$r_t = \phi_1 r_{t-1} + a_t, (t = 1, ..., T)$$

and compute its forecast error. Assume that the value of parameter ϕ_1 is known.

3. (20 points) Consider the AR(2) model

$$\begin{aligned} r_t &= \phi_1 r_{t-1} + \phi_2 r_{t-2} + a_t, (t = 1, 2, ..., T), \\ a_t &\sim WN(0, \sigma^2). \end{aligned}$$

(a) Suppose that $\phi_1 = 0.9$ and $\phi_1 = -0.14$. Is the process stationary?
(b) Explain how we can test the null hypothesis $H_0 : \phi_1 + \phi_2 = 1$ by using the ADF test.

4. (20 points) Derive the autocovariance function for the AR(1) process

$$\begin{aligned} r_t &= r_{t-1} + a_t, \ a_t \sim iid(0, 1) \\ r_0 &= 0. \end{aligned}$$

Using the derived autocovariance function, show that the autocorrelations of the AR(1) process are nearly 1 for large t.

5. (20 points) Consider the bivariate VAR model

$$\begin{aligned} r_{1t} &= \phi_{10} + \Phi_{11} r_{1,t-1} + \Phi_{12} r_{2,t-1} + a_{1t} \\ r_{2t} &= \phi_{20} + \Phi_{21} r_{1,t-1} + \Phi_{22} r_{2,t-1} + a_{2t} \end{aligned}$$

where $\begin{pmatrix} a_{1t} \\ a_{2t} \end{pmatrix} \sim iid(0, \Sigma)$.
(a) Discuss validity of the following statement.
Time series r_{1t} and r_{2t} are not concurrently correlated because the VAR model is concerned with only dynamic relationships between the two time series.
(b) What are the parameters that determine the feedback relationship between the time series r_{1t} and r_{2t}?

6. (20 points) Suppose that a time series r_t can be decomposed as $r_t = a_0 + a_1 t + S_t + X_t$, where $a_0 + a_1 t$ denotes the trend component (a_0 and a_1 are unknown constant) and S_t the seasonal component. Show that seasonal differencing eliminates both the trend and seasonal components.