Homework 3

Mathematical Statistics (Fall, 2018)
Due date: 10. 17 (W)

1. Let X be a random variable with moment-generating function $M_X(t)$, $-h < t < h$. Prove that $P(X > a) \leq e^{-at}M_X(t)$, $0 < t < h$.

2. Let X be a random variable such that $P(X \leq 0) = 0$ and let $\mu = EX$ exist. Show that $P(X \geq 2\mu) \leq \frac{1}{2}$.

3. If X is a random variable such that $EX = 3$ and $EX^2 = 13$, find a lower bound for the probability $P(-2 < X < 8)$.

4. Show that the Poisson family of distributions is an exponential family.

5. Let Z be a random variable with pdf $f(z)$. Define z_α to be a number that satisfies this relationship:
$$\alpha = P(Z > z_\alpha).$$
Show that if X is a random variable with pdf $(1/\sigma)f((x-\mu)/\sigma)$ and $x_\alpha = \sigma z_\alpha + \mu$, then $P(X > x_\alpha) = \alpha$.

6. Let X and Y have the joint pdf $f(x, y) = 15x^2y,\ 0 < x < y < 1,$ zero elsewhere. Find each marginal pdf and compute $P(X + Y \leq 1)$.

7. Let $f(x, y) = 21x^2y^3,\ 0 < x < y < 1,$ zero elsewhere, be the joint pdf of X and Y.

   (a) Find the conditional mean and variance of X, given $Y = y,\ 0 < y < 1$.

   (b) Find the distribution of $Z = E(X \mid Y)$.

   (c) Determine EZ and compare this to EX.

8. Let X and Y have the joint pdf $f(x, y) = \frac{1}{4}(1 + xy),\ |x| < 1,\ |y| < 1,$ zero elsewhere.

   (a) Show that X and Y are dependent.

   (b) Show that X^2 and Y^2 are independent.

9. Let X and Y have the joint pdf $f(x, y) = \pi^{-1},\ x^2 + y^2 \leq 1,$ zero elsewhere. Show that X and Y are uncorrelated and dependent.

10. X_1 and X_2 are independent $N(0, \sigma^2)$ random variables.

    (a) Find the joint distribution of $Y_1 = X_1^2 + X_2^2$ and $Y_2 = \frac{X_1}{\sqrt{Y_1}}$.

    (b) Show that Y_1 and Y_2 are independent.