Final Examination

Advanced Econometrics (Spring, 2011)
Total: 140 points
Instructions: Consulting an A4-sized sheet of paper on which you wrote down necessary information is allowed.

1. Discuss the validity of the following statements. Simple "Yes" or "No" answer will not be accepted for credit.
   (a) (10 points) We can obtain more efficient estimates of slope coefficients when regressors are highly correlated than when they are not.
   (b) (10 points) When the dependent variable is subject to measurement error, the OLS estimator of the slope coefficient is consistent.
   (c) (10 points) The feasible GLS estimator is more efficient than the OLS estimator whatever the sample size is.
   (d) (10 points) If the Durbin-Wu-Hausman test rejects the null hypothesis, using OLS is appropriate.
   (e) (10 points) To calculate Breush and Pagan's test for heteroskedasticity, we have to estimate the variances of the error terms.
   (f) (10 points) When the unobservable individual effect and the regressor are correlated, the random effect estimator is consistent.

2. Consider the panel data model

$$y_{it} = \mu + \beta x_{it} + u_{it}, (i = 1, ..., N; t = 1, ..., T) \tag{1}$$

   where $x_{it} \sim iid(0, \sigma_x^2)$, $u_{it} = \mu_i + v_{it}$, $v_{it} \sim iid(0, \sigma_v^2)$, and x_{it} is independent of v_{js} for all i, t, j and s.
   (a) (10 points) Assuming that $N \to \infty$ and that T is fixed, calculate the variance of $\sqrt{N}(\hat{\beta}_{FE} - \beta)$, where $\hat{\beta}_{FE}$ is the fixed effect estimator of β.
   (b) (10 points) Consider the differenced model

$$\Delta y_{it} = \beta \Delta x_{it} + \Delta v_{it}.$$

   Under the same assumptions as in part (a), calculate the variance $\sqrt{N}(\hat{\beta}_{OLS} - \beta)$, where $\hat{\beta}_{OLS}$ is the OLS estimator of β using the differenced model.
   (c) (10 points) Which estimator is more efficient?

3. Consider the following two-period fixed effects model with a single regressor, x_{it}

$$y_{it} = \lambda_i + \alpha x_{it} + u_{it}, \ (i = 1, \ldots, n; \ t = 1, 2), \tag{2}$$

where

$$x_{it} = z_i + a_{it} \text{ and } u_{it} = v_i + b_{it} \tag{3}$$

and z_i and v_i are random variables. As usual, λ_i is an individual effects variable correlated with x_{it}. Observed data are $\{y_{it}\}$ and $\{x_{it}\}$. But $\{z_i\}$ and $\{a_{it}\}$ are not separately observed. Random variables $\{a_{it}\}$ and $\{b_{it}\}$ bring time series variations to the observed data. For $\{a_{it}\}$ and $\{b_{it}\}$, assume
$\begin{pmatrix} n^\beta a_{it} \\ n^\gamma b_{it} \end{pmatrix} \sim iid \left(0, \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}\right)$ for every n, i and t.

(a) (10 points) Show that the Within-OLS and first-differenced estimators are identical for this two-period fixed effects model.

(b) (20 points) The first-differenced estimator is written as

$$\hat{\alpha}_d = \frac{\sum_{i=1}^n \Delta x_i \Delta y_i}{\sum_{i=1}^n (\Delta x_i)^2} = \alpha + \frac{\sum_{i=1}^n \Delta a_i \Delta b_i}{\sum_{i=1}^n (\Delta a_i)^2}$$

where $\Delta w_i = w_{i2} - w_{i1}$. Find the limiting distribution of $\frac{n^{\beta+\gamma}}{\sqrt{n}} \sum_{i=1}^n \Delta a_i \Delta b_i$ and the probability limit of $\frac{n^{2\beta}}{n} \sum_{i=1}^n (\Delta a_i)^2$ when $n \to \infty$. Using these results, find the limiting distribution of $n^{\frac{1}{2}-\beta+\gamma}(\hat{\alpha}_d - \alpha)$.

(c) (10 points) When is the estimator $\hat{\alpha}_d$ consistent?