Introduction to Econometrics — In Choi
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Home Assignment 4

Due: November 28, 2017

1. Using the Korean GDP data in the file gini_fifth_gdp.xlsx, estimate economic growth rate of the GDP during the sampling period. What happens to the estimate if the errors are assumed to follow an AR(1) process.

2. Let $Z_t \sim iid\ N(0, \sigma^2)$. Which of the following processes are stationary?

   (a) $X_t = 1 + Z_t + Z_{t-2}$

   (b) $X_t = Z_t Z_{t-1}$

   (c) $X_t = 0.7X_{t-1} - 0.1X_{t-2} + Z_t$

3. Consider the linear process

$$X_t = \sum_{j=0}^{\infty} 0.2^j u_{t-j},\ u_t \sim WN(0, \sigma^2).$$

   (a) Calculate the autocovariance function of $\{X_t\}$.

   (b) Is $\{X_t\}$ weakly stationary?

4. Consider the panel data model

$$y_{it} = \mu + \beta x_{it} + u_{it}, (i = 1, ..., N; t = 1, ..., T) \tag{1}$$

   where $x_{it} \sim iid(0, \sigma_x^2),\ u_{it} = \mu_i + v_{it},\ v_{it} \sim iid(0, \sigma_v^2)$, and x_{it} is independent of v_{js} for all i, t, j and s.

   (a) Suppose that $E(x_{it}\mu_i) = b$ for all i and t. What is the probability limit of the pooled OLS estimator of β when T is fixed and $N \to \infty$?

   (b) Assuming that $N \to \infty$ and that T is fixed, derive the limiting distribution of $\sqrt{N}(\hat{\beta}_{FE} - \beta)$, where $\hat{\beta}_{FE}$ is the fixed effect estimator of β.

   (c) Consider the differenced model

$$\Delta y_{it} = \beta \Delta x_{it} + \Delta v_{it}.$$

   Under the same assumptions as in part (b), derive the limiting distribution of $\sqrt{N}(\hat{\beta}_{OLS} - \beta)$, where $\hat{\beta}_{OLS}$ is the OLS estimator of β using the differenced model.

(d) Suppose that $E(x_{it}\mu_i) = 0$ for all i and t. What is the limiting distribution of the pooled OLS estimator of β when T is fixed and $N \to \infty$?

(e) Suppose that $E(x_{it}\mu_i) = 0$ for all i and t. Which estimator among the three studied above is most efficient?

5. Let the true regression model be

$$y_i = \alpha + \beta x_i + \varepsilon_i,$$

where $x_i \sim iid\,(0, \sigma_x^2)$, $\varepsilon_i \sim iid\,(0, \sigma_\varepsilon^2)$ and $\{x_i\}$ and $\{\varepsilon_i\}$ are independent. But we observe

$$x_i^* = x_i + w_i \;\; \left(w_i \sim iid\left(0, \sigma_w^2\right)\right)$$

instead of x_i due to measurement error. Assume that $\{\varepsilon_i\}$ and $\{w_i\}$ are independent.

(a) When y_i is regressed on x_i^*, what is the probability limit of the OLS estimator?

(b) There is another variable with measurement errors

$$z_i = x_i + v_i,$$

where $\{v_i\}$ and $\{\varepsilon_i\}$ are independent. Is the IV estimator using $\{z_i\}$ as an instrument consistent?