Introduction to Econometrics — In Choi
Sogang University — Fall 2017

Home Assignment 3

Due: November 7, 2017

1. Consider the linear regression model

$$y_t = \alpha + \beta t + u_t,\ \ u_t \sim iid(0, \sigma^2), (t = 1, ..., T).$$

Show that the OLS estimators of α and β are consistent.

2. For the linear regression model

$$y_t = \beta x_t + \varepsilon_t,\ \ \varepsilon_t \sim iid\left(0, \sigma^2\right),\ \ t = 1, \cdots, n,$$

an estimator

$$\bar{b} = \frac{\sum_{t=1}^{n} y_t}{\sum_{t=1}^{n} x_t}$$

is considered. Assume $\{x_t\}$ is a sequence of constants with $\sum_{t=1}^{n} x_t \to c(\neq 0)$ as $n \to \theta$.

(a) Is $\bar{b}$ consistent?

(b) Construct a t-ratio[1] for $H_0 : \beta = 0$ using $\bar{b}$ and derive its asymptotic distribution.

3. Let ε_t be i.i.d. following $B(m, p)$[2] and $u_t = \varepsilon_t + \varepsilon_{t-1}$. What is the probability limit of $\frac{1}{n}\sum_{t=1}^{n} u_t$?

4. Let $X_1, X_2, ...$be independent random variables with $P\{X_n = 1\} = p_n$ and $P\{X_n = 0\} = 1 - p_n$. Show that $X_n \xrightarrow{p} 0$ if $p_n \to 0$ as $n \to \infty$.

5. Let X_t be i.i.d. following $B(1, p)$. What is the limiting distribution of $\sqrt{n}(\bar{X} - p)$?

---

[1] The t-raito has the functional form

$$\frac{\text{estimator- point under } H_0}{\text{standard error}}.$$

[2] $B(m, p)$ denotes the binomial distribution with the number of trials m and success probability p.