Introduction to Econometrics | In Choi
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Home Assignment 4

Due: June 8, 2017

1. Consider the panel data model

$$y_{it} = \mu + \beta x_{it} + u_{it}, (i = 1, ..., N; t = 1, ..., T) \tag{1}$$

where $x_{it} \sim iid(0, \sigma_x^2),\ u_{it} = \mu_i + v_{it},\ v_{it} \sim iid(0, \sigma_v^2)$, and x_{it} is independent of v_{js} for all i, t, j and s.

(a) Assuming that $N \to \infty$ and that T is fixed, calculate the variance of $\sqrt{N}(\hat{\beta}_{FE} - \beta)$, where $\hat{\beta}_{FE}$ is the fixed effect estimator of β.

(b) Consider the differenced model

$$\Delta y_{it} = \beta \Delta x_{it} + \Delta v_{it}.$$

Under the same assumptions as in part (a), calculate the variance $\sqrt{N}(\hat{\beta}_{OLS} - \beta)$, where $\hat{\beta}_{OLS}$ is the OLS estimator of β using the differenced model.

(c) Which estimator is more efficient?

2. Consider the simultaneous equation model

$$y_{1t} = \beta y_{2t} + \alpha z_{1t} + u_t. \tag{2}$$

The reduced form model for this model is written as

$$\begin{aligned} y_{1t} &= \pi_{11} z_{1t} + \pi_{12} z_{2t} + v_{1t}; \\ y_{2t} &= \pi_{21} z_{1t} + \pi_{22} z_{2t} + v_{2t}, \end{aligned}$$

where $(v_{1t}, v_{2t})' \sim iid\ (0, \Sigma)$.[1] Derive the probability limit of the OLS estimator of β using the structural equation (2). Is it consistent?

3. Let the true regression model be

$$y_i = \alpha + \beta x_i + \varepsilon_i.$$

But we observe

$$x_i^* = x_i + w_i \ \left(w_i \sim iid\left(0, \sigma_w^2\right)\right)$$

instead of x_i due to measurement error. When y_i is regressed on x_i^*, what is the probability limit of the OLS estimator? Assume that $\{\varepsilon_i\}$ and $\{w_i\}$ are independent.

[1] $\Sigma = \begin{pmatrix} Ev_{1t}^2 & Ev_{1t}v_{2t} \\ Ev_{1t}v_{2t} & Ev_{2t}^2 \end{pmatrix}$ for all t.