

Home Assignment 3

Due: May 23, 2017

1. Consider the linear regression model

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad (1)$$

where x_i is a sequence of constants, $E(u_i) = 0$, $E(u_i u_j) = 0$ for $i \neq j$ and $E(u_i^2) = \sigma^2(x_i - \bar{x})^2$.

- (a) Is the OLS estimator of β_1 from equation (1) BLUE?
 - (b) Is the OLS estimator of β_1 from equation (1) consistent?
 - (c) Devise a GLS estimator of the regression coefficients β_0 and β_1 .
2. Using the Korean GDP data in the file `gini_fifth_gdp.xlsx`, estimate economic growth rate of the GDP during the sampling period. What happens to the estimate if the errors are assumed to follow an AR(1) process.
 3. Let $Z_t \sim iid N(0, \sigma^2)$. Which of the following processes are stationary? For each stationary process specify the mean and autocovariance function.
 - a. $X_t = 1 + Z_t + Z_{t-2}$
 - b. $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$ (c is a constant.)
 - c. $X_t = Z_t Z_{t-1}$
 4. If $x_t = a + bt$, where a and $b(\neq 0)$ are constant, show that for each fixed $h \geq 1$

$$\hat{\rho}(h) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

5. Consider the linear process

$$X_t = \sum_{j=0}^{\infty} 0.2^j u_{t-j}, \quad u_t \sim WN(0, \sigma^2).$$

- (a) Calculate the autocovariance function of $\{X_t\}$.
- (b) Is $\{X_t\}$ weakly stationary?