Introduction to Econometrics
Sogang University

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Home Assignment 2

Due: April 11, 2017

1. Using the data caschool.xls, regress log(testscr) on log(str) and log(avginc).

   (a) Test the significance of the coefficients of log(str) and log(avginc) at the 5% level.

   (b) Construct the 95% confidence intervals for the coefficients of log(str) and log(avginc).

   (c) Test the joint significance of the coefficients of log(str) and log(avginc) at the 5% level.

   (d) When the student-teacher ratio increases by 1%, how much does test score increase?

2. Consider the linear regression model

$$y_t = \alpha + \beta t + u_t,\ \ u_t \sim iid(0, \sigma^2), (t = 1, ..., T).$$

An estimator of β is

$$b = \frac{y_T - y_1}{T - 1}.$$

   (a) Is estimator b consistent? (Hint: use Chebyshev's inequality)
   We may write

$$b - \beta = \frac{u_T - u_1}{T - 1}.$$

   Since for $\epsilon > 0$

$$\begin{aligned} P\left[\left|\frac{u_T - u_1}{T - 1}\right| > \epsilon\right] &\leq \frac{1}{\epsilon^2 (T-1)^2} E(u_T - u_1)^2 \\ &= \frac{2\sigma^2}{\epsilon^2 (T-1)^2} \to 0 \text{ as } T \to \infty, \end{aligned}$$

   $b \xrightarrow{p} \beta$.

   (b) If $u_t \sim i.i.d.\ N(0, 1)$, what is the asymptotic distribution of b?
   We have

$$(T - 1)(b - \beta) = u_T - u_1.$$

   But $u_T - u_1 \sim N(0, 2\sigma^2)$ for any T. Thus,

$$(T - 1)(b - \beta) \xrightarrow{d} N(0, 2\sigma^2).$$

3. For the linear regression model

$$y_t = \beta x_t + \varepsilon_t,\ \ \varepsilon_t \sim iid\left(0, \sigma^2\right),\ \ t = 1, \cdots, n,$$

an estimator

$$\bar{b} = \frac{\sum_{t=1}^n y_t}{\sum_{t=1}^n x_t}$$

is considered. Assume $\{X_t\}$ is a sequence of constants.

(a) What assumptions are required for the consistency of $\bar{b}$?

Write

$$\bar{b} - \beta = \frac{\sum_{t=1}^n u_t/n}{\sum_{t=1}^n x_t/n}.$$

Since $\sum_{t=1}^n u_t/n \xrightarrow{p} 0$ by the WLLN, $\bar{b}$ is consistent for β if $\sum_{t=1}^n x_t/n \xrightarrow{p} M\ (\neq 0)$.

(b) Derive the asymptotic distribution of $\bar{b}$.

$$\sqrt{n}(\bar{b} - \beta) = \frac{\sum_{t=1}^n u_t/\sqrt{n}}{\sum_{t=1}^n x_t/n} \xrightarrow{d} N(0, \sigma^2/M^2).$$

4. Let $\varepsilon_t \sim iid\,(0, \sigma^2)$ and $u_t = \varepsilon_t + \varepsilon_{t-1}$. What is the probability limit of $\frac{1}{n}\sum_{t=1}^n u_t$?
Since

$$\frac{1}{n}\sum_{t=1}^n u_t = \frac{1}{n}\sum_{t=1}^n \epsilon_t + \frac{1}{n}\sum_{t=1}^n \epsilon_{t-1}$$

and $\frac{1}{n}\sum_{t=1}^n \epsilon_t,\ \frac{1}{n}\sum_{t=1}^n \epsilon_{t-1} \xrightarrow{p} 0$ by the WLLN, $\frac{1}{n}\sum_{t=1}^n u_t \xrightarrow{p} 0$ as $n \to \infty$.

5. Let $X_1, X_2, \ldots$be independent random variables with $P\{X_n = 1\} = p_n$ and $P\{X_n = 0\} = 1 - p_n$. Show that $X_n \xrightarrow{p} 0$ if $p_n \to 0$ as $n \to \infty$.
For $\epsilon > 0$

$$P\left[|X_n| > \epsilon\right] = P\{X_n = 1\} = p_n.$$

Thus, $X_n \xrightarrow{p} 0$ if $p_n \to 0$ as $n \to \infty$.