

Homework 4

Mathematical Statistics (Fall, 2016)

Total points: 7

Due date: 11. 01 (Tu)

1. Let X_1 and X_2 be independent standard normal variables. Find the joint pdf of (Y_1, Y_2) where $Y_1 = \sqrt{X_1^2 + X_2^2}$ and $Y_2 = X_1/X_2$. Show that Y_1 and Y_2 are independent.
2. Let X_1 and X_2 be independent standard normal variables. Find the joint pdf of (Y_1, Y_2) where $Y_1 = X_1^2 + X_2^2$ and $Y_2 = X_1/\sqrt{Y_1}$. Show that Y_1 and Y_2 are independent.
3. Let X_1, X_2, X_3 be iid $N(\mu, \sigma^2)$ and define

$$Y_1 = X_1 + \delta X_3$$

and

$$Y_2 = X_2 + \delta X_3.$$

- (a) Find the means and variances of Y_1 and Y_2 and their correlation coefficients.
 - (b) Find the joint mgf of Y_1 and Y_2 .
4. Let X and Y have a bivariate normal distribution. Show that $X + Y$ and $X - Y$ are independent if and only if $VarX = VarY$.
 5. Prove that $ES \leq \sigma$, and if $\sigma^2 > 0$, then $ES < \sigma$.
 6. Let $X_i \sim iid N(\mu, \sigma^2)$. Find the mean and variance of $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$. [Hint: use the distribution of $(n-1)S^2/\sigma^2$.]
 7. Let X be a random variable with a Student's t-distribution with p degrees of freedom. Derive the mean and variance of X .