Homework 2

Mathematical Statistics (Fall, 2016)
Total points: 7
Due date: 9. 29 (Th)

1. Let $X \sim U(0,1)$. Let F be a cdf. Show that the cdf of $Y = F^{-1}(X)$ is F, where $F^{-1}(t) = \inf\{x \in R : F(x) \geq t\}$.

2. Let X be a random variable having the pdf $\frac{2x}{\pi^2} I_{(0,\pi)}(x)$. Derive the pdf of $Y = \sin X$.

3. Let X have the pdf
$$f(x) = \frac{1+x}{2}, \ -1 < x < 1.$$
Find the pdf of $Y = X^2$.

4. Let $X_1, ..., X_k$ be independent random variables and $Y = X_1 + ... + X_k$. Prove the following.
   (a) If $X_i \sim Binomial(n_i, p),\ Y \sim Binomial(n_1 + ... + n_k, p)$.
   (b) If X_i has the Cauchy distribution, Y/k has the same distribution as X_1.

5. Let X be a continuous, nonnegative random variable with pdf f. Show that
$$EX = \int_0^\infty (1 - F_X(x))dx.$$

6. Let X be a random variable such that $E(X-b)^2$ exists for all real b. Show that $E(X-b)^2$ is a minimum when $b = EX$.

7. Let X have the distribution function
$$\begin{aligned} F(x) &= 0, \ x < 0 \\ &= \frac{x+1}{4}, \ 0 \leq x < 1 \\ &= 1, \ 1 \leq x. \end{aligned}$$
Find EX and $VarX$.