A Multilevel Factor Model: Identification, Asymptotic Theory and Applications*

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Abstract

This paper studies a multilevel factor model with global and country factors. The global factors affect all individuals while the country factors affect only those within each specific country. A sequential procedure to identify the global and country factors separately is proposed. In the initial step, the global factors are estimated by canonical correlation analysis. Using this initial estimator, the principal component estimators (PCEs) of the global and country factors are constructed. It is shown that the PCEs estimate the spaces of the global and country factors consistently and are normally distributed in the limit. Several information criteria that can estimate the numbers of the country factors are proposed. The number of the global factors is assumed to

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be known. Extensive simulation results demonstrate that the sequential procedure and the information criteria work well in finite samples. The method of this paper is applied to 25 OECD countries to identify international business cycle. It is reported that the method extracts a global factor reasonably well.

Keywords: multilevel factor model, canonical correlation analysis, principal component estimator, information criteria, international business cycles.

1 Introduction

Factor models have often been used in economics and finance for various purposes. First, factor model have been used to construct economic indicators as in Altissimo *et al.* (2001), Cristadoro *et al.* (2005) and Kapetanios (2004). Second, factor models have been useful for economic forecasting. A partial list includes Artis, Banerjee and Marcellino (2005); Banerjee and Marcellino (2006); Banerjee, Marcellino and Masten (2005); Camba-Mendez and Kapetanios (2005); den Reijer (2005); Forni *et al.* (2000, 2003); Giacomini and White (2006); Huang, Lee and Li (2006); Ludvigson and Ng (2007); Marcellino, Stock and Watson (2003); Schumacher and Dreger (2004); Schumacher (2007); and Stock and Watson (1999, 2002). Third, factor models have been employed for policy analysis. Bernanke, Boivin and Eliasz (2005); Favero, Marcellino and Neglia (2005); Forni *et al.* (2009); Giannone *et al.* (2002, 2005); and Sala (2003) are prime examples. Fourth, factor models can be used for instrumental-variables estimation as in Bai and Ng (2010). Last, some researchers have used factor models to model cross-sectional correlation in panel data analysis. See Bai (2009); Bai and Ng (2004), Greenaway–McGrevy, Han and Sul (2012); Moon and Perron (2004); Phillips and Sul (2003); Pesaran (2006); and Chudik and Pesaran (2015) for related articles. The reader is referred to Breitung and Choi (2013) for further references and discussions related to factor models.

The purpose of this paper is to study a multilevel factor model with global and country factors and apply the method thereof to international macro data. The global factors refer to unobserved factors that affect all individuals in the world. The unobserved country factors influence only those in one specific country. For example, macrovariables in one country are affected by the domestic factors as well as the global ones common across nations. By estimating the global factors, we can understand how the macrovariables are connected to the global factors and thereby measure the sensitivity of those variables to international influence.

More specifically, this paper develops a sequential procedure that can identify the global and country factors separately. For the proper use of the multilevel factor model, it is essential to separately identify the global and country factors. No inferential procedure is available yet in the literature for the separate identification of the global and country factors. In the initial step of the sequential procedure, the global factors are estimated by canonical correlation analysis. The initial estimator, albeit using the data from only two countries, can consistently estimate the space of the global factors. Using this initial estimator, we construct the principal component estimators (PCEs; cf. Stock and Watson, 2002; Bai, 2003) of the global and country factors. The PCEs estimate the spaces of the global and country factors consistently and are normally distributed in the limit. This paper also proposes information criteria that can estimate the numbers of the country factors. The number of the global factors is assumed to be known. We study the finite sample properties of the sequential procedure and the information criteria by means of simulation and show them to work well in finite samples. The method of this paper is applied to 25 OECD countries to identify international business cycle. It is reported that our method extracts the global factor successfully for the major OECD countries in the sense that the extracted global factor matches reasonably well with the major economic booms and recessions of the past 30 years.

Models similar to ours have been used to study the international business cycle. Gregory, Head and Raynauld (1997) employ a dynamic factor model with Kalman filtering to estimate the world business cycle. In their work, there are just three pieces of cross-sectional data for each country, which make their model different from ours in spirit. Furthermore, they do not provide any sampling properties for the estimated world business cycle. Kose, Otrok and Whiteman (2003, 2008) and Kose, Otrok and Prasad (2012) use a Bayesian dynamic latent factor model to estimate the world business cycle. In their model, there are three kinds of factors – world, region and country – but, again, there are only three pieces of cross-sectional data for each country as in Gregory, Head and Raynauld. The related work by Aruoba *et al.* (2010) extracts a single latent factor using the Kalman filter for each of the G-7 countries and then derives a common factor following Stock and Watson (1989). Wang (2008) studies exactly the same model as in this paper, but his asymptotic theory is applicable only under the assumption of an infinite number of regions (M in our notation of Section 2), which is unrealistic in most applications. Even in this case, it is not clear how an initial estimate of the global factor can be obtained. Dias, Pinheiro and Rua (2013) provide an information criterion for the multilevel factor model of this paper without separately identifying the global and country factors as in this paper. Moench, Ng and Potter (2013) propose a dynamic factor model with a multilevel structure and estimate it using the MCMC algorithm. No inferential theory is given there.

This paper is structured as follows. Section 2 introduces the model and identification strategy. Section 3 develops asymptotic theory for the multilevel factor model. Section 4 discusses information criteria for the selection of the numbers of factors. Section 5 reports simulation results. Section 6 applies the multilevel factor model to G-25 countries. Section 7 provides a summary and further remarks. Appendix A collects proofs and Appendix B some technical assumptions. Note that $P_A = A(A'A)^{-1}A'$, $M_A = I - P_A$ and $\|A\| = \sqrt{\mathrm{tr}(AA')}$ in this paper.

2 The model and identification strategy

This section introduces the multilevel factor model having both global and country factors. Additionally, a sequential strategy that can identify the global and country factors is proposed in this section. Asymptotic justifications of the sequential strategy will be discussed in the next section.

2.1 The model and assumptions

We are concerned with the multilevel factor model

$$x_{mit} = \gamma'_{mi} G_t + \lambda'_{mi} F_{mt} + e_{mit}, \ (m = 1, \ldots, M; \ i = 1, \ldots, N_m; \ t = 1, \ldots, T), \quad (1)$$

where m is the index for a country, i for each individual in country m, t for time, G_t is an s by 1 vector of unobserved, global factors that affect individuals in all the countries, F_{mt} is an r_m by 1 vector of unobserved, country factors that affect individuals only in country m, γ_{mi} and λ_{mi} are unobserved factor loadings, and e_{mit} is an idiosyncratic error. Note that each country is allowed to have a different number of individuals N_m. We assume that the numbers of the factors s and $\{r_m\}$ are known in this section and Section 3 and will discuss methods of estimating them in Section 4.

In vector notation, Model (1) is written as

$$\begin{aligned} X_{mt} &= \Gamma_m G_t + \Lambda_m F_{mt} + e_{mt} \\ &= [\Gamma_m \ \Lambda_m] \begin{bmatrix} G_t \\ F_{mt} \end{bmatrix} + e_{mt} \\ &= \Theta_m K_{mt} + e_{mt}, \ \text{say}, \end{aligned} \quad (2)$$

where

$$X_{mt} = \begin{bmatrix} x_{m1t} \\ \vdots \\ x_{mN_mt} \end{bmatrix}, \; e_{mt} = \begin{bmatrix} e_{m1t} \\ \vdots \\ e_{mN_mt} \end{bmatrix}, \; \Gamma_m = \begin{bmatrix} \gamma'_{m1} \\ \vdots \\ \gamma'_{mN_m} \end{bmatrix} \text{ and } \Lambda_m = \begin{bmatrix} \lambda'_{m1} \\ \vdots \\ \lambda'_{mN_m} \end{bmatrix}.$$

Let

$$X_m = \begin{bmatrix} X'_{m1} \\ \vdots \\ X'_{mT} \end{bmatrix}, \; G = \begin{bmatrix} G'_1 \\ \vdots \\ G'_T \end{bmatrix}, \; F_m = \begin{bmatrix} F'_{m1} \\ \vdots \\ F'_{mT} \end{bmatrix} \text{ and } e_m = \begin{bmatrix} e'_{m1} \\ \vdots \\ e'_{mT} \end{bmatrix}.$$

Model (2), when further stacked across time t, is written as

$$\begin{aligned} X_m &= G\Gamma'_m + F_m\Lambda'_m + e_m \\ &= K_m\Theta'_m + e_m. \end{aligned}$$

Regarding the global and country factors, we make the following assumption.

Assumption 1 *(i)* $\{G_t\}$, $\{F_{1t}\}, \ldots, \{F_{Mt}\}$ *are zero-mean, stationary processes that satisfy the conditions for the law of large numbers and the central limit theorem.*
(ii) $\{G_t\}$, $\{F_{1t}\}, \ldots, \{F_{Mt}\}$ *are uncorrelated. That is,* $E(F_{mt}F'_{nt}) = 0$ *for all* t *and* $m \neq n$*; and* $E(G_tF'_{mt}) = 0$ *for all* m*, and* t*.*
(iii) $\{G_t\}$, $\{F_{1t}\}, \ldots, \{F_{Mt}\}$ *satisfy conditions for the law of large numbers and the central limit theorem applied to their self- and cross-products.*
(iv) $N_1, \ldots, N_M$ *and* N *are of the same order of magnitude.*

The first and third parts of this assumption are of standard nature and do not require further elaboration. The second part, that the global and country factors are uncorrelated, seems to be intuitively appealing and will be used for the asymptotic justifications of our identification strategy. Note that the presence of the global factors makes all of the observed data correlated unless the global factor loadings are equal to zero.

In addition, the following technical assumptions are required for the asymptotic analysis of Sections 3 and 4.

Assumption 2 *(i)* $E(e_{mit}) = 0$ *for every* m, i *and* t; $\sup_{m,i,t} E|e_{mit}|^8 \leq \mathcal{M}$.
(ii) Let $E(e'_{ms}e_{mt}/N_m) = \omega_{mN_m}(s,t)$. *Then,* $\sup_{m,N_m} \left|\gamma_{mN_m}(s,s)\right| \leq \mathcal{M}$ *and* $\sup_{m,N_m} \sum_{s=1}^{T} |\omega_{mN_m}(s,t)| \leq \mathcal{M}$ *for every* $t \leq T$.
(iii) Let $E(e_{mit}e_{mjt}) = \tau_{ij,mt}$ *with* $\sup_t |\tau_{ij,mt}| \leq \tau_{ijm}$ *for some* τ_{ijm}. *In addition,* $\sup_{m,N_m} \sum_{j=1}^{N_m} |\tau_{ijm}| \leq \mathcal{M}$ *for every* $i \leq N_m$.
(iv) Let $E(e_{mit}e_{mjs}) = \tau_{ij,mts}$. *Then,* $\sup_{m,N_m,T}(N_mT)^{-1} \sum_{i=1}^{N_m} \sum_{j=1}^{N_m} \sum_{t=1}^{T} \sum_{s=1}^{T} |\tau_{ij,mts}| \leq \mathcal{M}$.
(v) $\sup_{t,s,m,N_m} E\left|N_m^{-1/2} \sum_{i=1}^{N_m} e_{mis}e_{mit} - E(e_{mis}e_{mit})\right|^4 \leq \mathcal{M}$.

Assumption 3 *(i)* $\sup_{m,t} E\|K_{mt}\|^4 < \infty$.
(ii) $\left\|\frac{G'G}{T} - \Sigma_{GG}\right\| \xrightarrow{p} 0$ *and* $\sup_m \left\|\frac{F'_mF_m}{T} - \Sigma_{F_m}\right\| \xrightarrow{p} 0$, *where* Σ_{GG} *and* Σ_{F_m} *are positive-definite matrices for every* m.
(iii) $\sup_{m,i} \|\gamma_{mi}\| \leq \bar{\gamma} < \infty$ *and* $\sup_{m,i} \|\lambda_{mi}\| \leq \bar{\lambda} < \infty$, *where* $\bar{\gamma}$ *and* $\bar{\lambda}$ *are constants.*
(iv) $\sup_m \left\|\frac{\Gamma'_m\Gamma_m}{N_m} - \Sigma_{\Gamma_m}\right\| \to 0$ *and* $\sup_m \left\|\frac{\Lambda'_m\Lambda_m}{N_m} - \Sigma_{\Lambda_m}\right\| \to 0$, *where* Σ_{Γ_m} *and* Σ_{Λ_m} *are positive-definite matrices.*
(v) The eigenvalues of the matrix $\Sigma_{\Gamma_m}\Sigma_{GG}$ *are distinct for every* m, *and so are the eigenvalues of the matrix* $\Sigma_{\Lambda_m}\Sigma_{F_m}$.

Assumption 4 *(i)* $\sup_{m,N_m,T} E\left(\frac{1}{N_m} \sum_{i=1}^{N_m} \left\|\frac{1}{\sqrt{T}} \sum_{t=1}^{T} K_{mt}e_{mit}\right\|\right) \leq \mathcal{M}$.
(ii) $\sup_{t,m,N_m,T} E\left\|\frac{1}{\sqrt{N_mT}} \sum_{s=1}^{T} \sum_{k=1}^{N_m} K_{ms}\left[e_{mks}e_{mkt} - E\left(e_{mks}e_{mkt}\right)\right]\right\|^2 \leq \mathcal{M}$.
(iii) $\sup_{m,N_m,T} E\left\|\frac{1}{\sqrt{N_mT}} \sum_{k=1}^{N_m} \sum_{t=1}^{T} K_{mt}\lambda'_{mk}e_{mkt}\right\|^2 \leq \mathcal{M}$.
(iv) For any t *and* i, *as* $T, N_m \to \infty$,

$$\begin{pmatrix} \frac{1}{\sqrt{N_m}} \sum_{i=1}^{N_m} \lambda_{mi}e_{mit} \\ \frac{1}{\sqrt{T}} \sum_{s=1}^{T} K_{ms}e_{mis} \end{pmatrix} \xrightarrow{d} N\left(0, \begin{matrix} \Psi_{mt} & 0 \\ 0 & \Phi_{mi} \end{matrix}\right),$$

where $\Psi_{mt} = \lim_{N_m\to\infty} \frac{1}{N_m} \sum_{i=1}^{N_m} \sum_{j=1}^{N_m} \lambda_{mi}\lambda'_{mj}E(e_{mit}e_{mjt})$ *and* $\Phi_{mi} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{s=1}^{T} E\left(K_{mt}K'_{ms}e_{mit}e_{mis}\right)$.

These are essentially from Bai and Ng (2002) and Bai (2003). The reader is referred to these articles for their detailed explanations.

2.2 Identification strategy

Suppose that M is finite. Then the spaces of the factors and factor loadings can be identified via the following steps. Further details and asymptotic justifications of these steps will be given in Section 3. In Steps 2–4 below, Breitung and Tenhofen's (2011) and Choi's (2012) estimators can instead be employed to improve the efficiency of estimation. Because these estimators require more stringent model specifications than the PCE, the latter seems easier to use. So we will stick with the PCE throughout this paper. Extending the results of this paper using the more efficient estimators is beyond the scope of this paper.

Step 1: Select two countries 1 and 2 and obtain an estimator of G_t, denoted $\hat{G}_t^{(1)}$, from the canonical correlation analysis. The superscript (1) is employed to distinguish the current estimator from the estimator of G_t obtained in a later step. It will be shown later that $\hat{G}_{tj}^{(1)} \xrightarrow{p} G_{tj}$ or $-G_{tj}$ $(j = 1, \ldots, s)$ for every t as $N_1, N_2, T \to \infty$ under proper conditions, where G_{tj} denotes the j-th element of G_t. Equivalently, this can be written in vector notation as $\hat{G}_t^{(1)} \xrightarrow{p} Q_s G_t$, where Q_s is an $s \times s$ diagonal matrix with its diagonal elements consisting of 1 and -1.

Step 2: Rewrite Model (2) as

$$\begin{aligned} X_{mt} &= \Gamma_m Q_s^{-1} \hat{G}_t^{(1)} + \Lambda_m F_{mt} + e_{mt} - \Gamma_m Q_s^{-1} (\hat{G}_t^{(1)} - Q_s G_t) \\ &= \Gamma_m^* \hat{G}_t^{(1)} + \Lambda_m F_{mt} + e_{mt}^{(1)}, \text{ say,} \end{aligned} \tag{3}$$

and estimate Λ_m and F_{mt} by the principal component method. These estimators are denoted $\hat{\Lambda}_m^{(1)}$ and $\hat{F}_{mt}^{(1)}$, respectively.

Step 3: Using $\hat{\Lambda}_m^{(1)}\hat{F}_{mt}^{(1)}$, rewrite Model (2) as

$$\begin{aligned} X_{mt} - \hat{\Lambda}_m^{(1)}\hat{F}_{mt}^{(1)} &= \Gamma_m G_t + e_{mt} - (\hat{\Lambda}_m^{(1)}\hat{F}_{mt}^{(1)} - \Lambda_m F_{mt}) \\ &= \Gamma_m G_t + e_{mt}^{(2)}, \text{ say,} \end{aligned} \tag{4}$$

stack them as

$$\begin{bmatrix} X_{1t} - \hat{\Lambda}_1^{(1)}\hat{F}_{1t}^{(1)} \\ \vdots \\ X_{Mt} - \hat{\Lambda}_M^{(1)}\hat{F}_{Mt}^{(1)} \end{bmatrix} = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_M \end{bmatrix} G_t + \begin{bmatrix} e_{1t}^{(2)} \\ \vdots \\ e_{Mt}^{(2)} \end{bmatrix}, \tag{5}$$

and estimate $\{\Gamma_m\}$ and G_t by the principal component method. The estimators are written as $\left\{\hat{\Gamma}_m^{(1)}\right\}$ and $\hat{G}_t^{(2)}$. The estimator $\hat{G}_t^{(2)}$ uses all the information in the sample, whereas $\hat{G}_t^{(1)}$ is based on the sample information from only two countries.

Step 4: Using $\left\{\hat{\Gamma}_m^{(1)}\right\}$ and $\hat{G}_t^{(2)}$, rewrite Model (2) as

$$\begin{aligned} X_{mt} - \hat{\Gamma}_m^{(1)}\hat{G}_t^{(2)} &= \Lambda_m F_{mt} + e_{mt} - \left(\hat{\Gamma}_m^{(1)}\hat{G}_t^{(2)} - \Gamma_m G_t\right) \\ &= \Lambda_m F_{mt} + e_{mt}^{(3)}, \text{ say,} \end{aligned} \tag{6}$$

and estimate the spaces of Λ_m and F_{mt} by the principal component method. We denote these estimators as $\hat{\Lambda}_m^{(2)}$ and $\hat{F}_{mt}^{(2)}$, respectively. These estimators use $\hat{G}_t^{(2)}$ which is based on the whole sample. In contrast, those in Step 2 use $\hat{G}_t^{(1)}$, which are estimated by using the sample from only two countries.

3 Asymptotic theory for the multilevel factor model

This section details the estimation procedure for Model (1) and develops asymptotic theory that justifies the identification strategy of Subsection 2.2.

3.1 Step 1: Initial estimation of the global factor

Suppose that we chose two countries, 1 and 2. In practice, it is advisable to select two countries that yield the maximum sample mean of the eigenvalues from relation (7) below. Denote the PCE of K_{mt} in Model (2) as $\hat{K}_{mt}$ $(m = 1,\ 2)$. Under Assumptions 2∼4, $\hat{K}_{mt}$ estimates a rotation of K_{mt}, $L_m' K_{mt}$, consistently, where the matrix L_m is $O_p(1)$ and defined in Bai (2003). Equivalently, we may write $\hat{K}_{mt} = L_m' K_{mt} + o_p(1)$.

The global factor G_t is present at both K_{1t} and K_{2t}. Thus, the maximal correlation of arbitrary linear combinations of K_{1t} and K_{2t} should be equal to plus or minus one. In other words, the maximal correlation of arbitrary linear combinations of K_{1t} and K_{2t} carries information on the presence of the global factor at K_{1t} and K_{2t}. This prompts us to use the canonical correlation analysis to estimate the global factor.

For the canonical correlation analysis, let $\hat{S}_{ab} = T^{-1} \sum_{t=1}^{T} \hat{K}_{at} \hat{K}_{bt}'$ $(a, b = 1, 2)$. The generalized eigenvalues, $\hat{\mu}$, that satisfy the determinant equation

$$\left| \hat{S}_{12} \hat{S}_{22} \hat{S}_{21} - \hat{\mu} \hat{S}_{11} \right| = 0 \tag{7}$$

provide the squared sample correlation coefficients between arbitrary linear combinations of $\hat{K}_{1t}$ and $\hat{K}_{2t}$, or R^2s resulting from regressing arbitrary linear combinations of $\hat{K}_{1t}$ on $\hat{K}_{2t}$ (see Chapter 12 of Anderson, 2003). We order the eigenvalues such that $\hat{\mu}_1 \geq \cdots \geq \hat{\mu}_{s+r_1}$. The coefficients for the linear combination of $\hat{K}_{1t}$ that correspond to $\hat{\mu}_j$ are given by the eigenvector, $\hat{p}_j$, that satisfies the equation

$$\left(\hat{S}_{12} \hat{S}_{22}^{-1} \hat{S}_{21} - \hat{\mu}_j \hat{S}_{11} \right) \hat{p}_j = 0$$

with the restriction $\hat{p}_j' \hat{p}_j = 1$ $(j = 1, \ldots, s+r_1)$ and $\hat{p}_j' \hat{p}_k = 0$ $(k = 1, \ldots, s+r_1-1, k < j)$. In other words, the squared sample multiple correlation coefficient between $\hat{p}_j' \hat{K}_{1t}$ and $\hat{K}_{2t}$ is equal to $\hat{\lambda}_j$.

Since $\hat{S}_{ab} = L_a' T^{-1} \sum_{t=1}^{T} K_{at} K_{bt}' L_b + o_p(1) = L_a' S_{ab} L_b + o_p(1)$, where $S_{ab} =$

$T^{-1}\sum_{t=1}^{T} K_{at}K'_{bt}$ $(a, b = 1, 2)$, the determinant equation (7) is equivalent to

$$\left|S_{12}S_{22}^{-1}S_{21} - \hat{\mu}_j S_{11} + o_p(1)\right| = 0 \tag{8}$$

and the relevant equation for the eigenvector is

$$\left(S_{12}S_{22}^{-1}S_{21} - \hat{\mu}_j S_{11} + o_p(1)\right)\hat{q}_j = 0, \tag{9}$$

where $\hat{q}_j = L_1\hat{p}_j$. This eigenvector cannot be computed in practice since L_1 is not known, but will be used later to derive an estimator of G_t.

Under Assumption 1, the population equations corresponding to equations (8) and (9) are written as, respectively,

$$\left|\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \mu_j\Sigma_{11}\right| = 0$$

and

$$\left(\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \mu_j\Sigma_{11}\right)q_j = 0,$$

where $\Sigma_{ab} = E(K_{at}K'_{bt})$ $(a, b = 1, 2)$ for any arbitrary t. We order the population eigenvalues such that $\mu_1 \geq \cdots \geq \mu_{s+r_1}$. As shown in Lemma A.1 in Appendix A, $\mu_1 = \cdots = \mu_s = 1$, $\mu_{s+1} = \cdots = \mu_{s+r_1} = 0$ and $q_j = (0, \ldots, \overset{j\text{-th}}{1}, \ldots, 0)'$ $(j = 1, \ldots, s + r_1)$.

The sample eigenvalues and eigenvectors converge to their population counterparts (up to the sign in case of the sample eigenvectors) as shown in the following lemma. The proof of this lemma follows from the standard theory of multivariate analysis (e.g., Anderson, 2003, Chapter 13) and is omitted.

Lemma 1 *Under Assumptions 1∼4, the following results hold as $N_1, N_2, T \to \infty$.*
(i) $\hat{\mu}_j \xrightarrow{p} 1$, *for* $j = 1, \ldots, s$.
(ii) $\hat{\mu}_j \xrightarrow{p} 0$, *for* $j = s + 1, \ldots, s + r_1$.
(iii) $\hat{q}_j \xrightarrow{p} q_j$ *or* $-q_j$, *where* $q_j = (0, \ldots, \overset{j-th}{1}, \ldots, 0)'$ *for* $j = 1, \ldots, s + r_1$.

Let $\hat{G}_{tj}^{(1)} = \hat{p}_j' \hat{K}_{1t}$ $(j = 1, \ldots, s)$. Then, we obtain Proposition 2 from Lemma 1.

Proposition 2 *Under Assumptions 1∼4, the following results hold for every t and j, as $N_1, N_2, T \to \infty$,*

(i) $\hat{G}_{tj}^{(1)} \xrightarrow{p} G_{tj}$ *or* $-G_{tj}$, *for* $j = 1, \ldots, s$.

(ii) $\hat{G}_{tj}^{(1)} \xrightarrow{p} F_{1tj}$ *or* $-F_{1tj}$, *for* $j = s+1, \ldots, s+r_1$.

(iii) $\hat{G}_{tj}^{(1)} \pm G_{tj} = O_p\left(\frac{1}{\min\{\sqrt{T}, \sqrt{N_1}\}}\right)$, *for* $j = 1, \ldots, s$.

(iv) $\hat{G}_{tj}^{(1)} \pm F_{tj} = O_p\left(\frac{1}{\min\{\sqrt{T}, \sqrt{N_1}\}}\right)$, *for* $j = s+1, \ldots, s+r_1$.

Part (i) of this proposition shows that the global factor $\{G_t\}$ can be consistently estimated up to sign by $\hat{G}_t^{(1)}$. Notice that $\hat{G}_{tj}^{(1)}$ estimates either $+G_{tj}$ or $-G_{tj}$ where the sign is common for all t. In other words, the event where $\hat{G}_{tj}^{(1)}$ estimates G_{tj} while $\hat{G}_{sj}^{(1)}$ does $-G_{sj}$ $(s \neq t)$ would not happen. Part (ii) of this proposition shows the rate of consistency of $\hat{G}_t^{(1)}$. This will be used for Proposition 3 in the next subsection.

3.2 Step 2: Initial estimation of the country's common components

This subsection studies Model (3). The PCEs for Model (3) can be derived by the conditional maximum likelihood estimation method as shown in Choi (2012). Therefore, this subsection starts from the conditional maximum likelihood estimation of Model (3). To this end, assume $e_{mt} \mid \{K_{mt}\} \sim iid\ N(0, I_{N_m})$. Then, treating $\left\{\hat{G}_t^{(1)}\right\}$ as if they were $\{G_t\}$ and ignoring the $o_p(1)$ term $\Gamma_m Q_s^{-1}\left(\hat{G}_t^{(1)} - Q_s G_t\right)$, the conditional log-likelihood function (multiplied by -2) for Model (3) is

$$\begin{aligned} l(\{\Gamma_m^*\}, \{\Lambda_m\}, \{F_m\}) &= T\sum_{m=1}^{M} N_m \ln(2\pi) + T\sum_{m=1}^{M} \ln|I_{N_m}| \\ &\quad + \sum_{m=1}^{M} \mathrm{tr}\left\{\left(X_m - \hat{G}^{(1)}\Gamma_m^{*\prime} - F_m\Lambda_m'\right)'\left(X_m - \hat{G}^{(1)}\Gamma_m^{*\prime} - F_m\Lambda_m'\right)\right\}. \end{aligned}$$

For the conditional maximum likelihood estimation, we need to minimize

$$\operatorname{tr}\left\{\left(X_m-\hat{G}^{(1)}\Gamma_m^{*\prime}-F_m\Lambda_m'\right)'\left(X_m-\hat{G}^{(1)}\Gamma_m^{*\prime}-F_m\Lambda_m'\right)\right\} \tag{10}$$

with respect to Γ_m^*, Λ_m and F_m. Using the standard theory of multivariate linear regression, we obtain

$$\hat{\Lambda}_m = X_m' M_{\hat{G}^{(1)}} F_m (F_m' M_{\hat{G}^{(1)}} F_m)^{-1} = X_m^{G\prime} F_m^G \left(F_m^{G\prime} F_m^G\right)^{-1},$$

with $X_m^G = M_{\hat{G}^{(1)}} X_m$ and $F_m^G = M_{\hat{G}^{(1)}} F_m$, and

$$\hat{\Gamma}_m^* = X_m' M_{F_m} \hat{G}^{(1)} \left(\hat{G}^{(1)\prime} M_{F_m} \hat{G}^{(1)}\right)^{-1} = X_m^{F\prime} \hat{G}^F \left(\hat{G}^{F\prime} \hat{G}^F\right)^{-1},$$

where $X_m^F = M_{F_m} X_m$ and $\hat{G}^F = M_{F_m} \hat{G}^{(1)}$. Plugging $\hat{\Lambda}_m$ and $\hat{\Gamma}_m^*$ successively into objective function (10), we obtain $\operatorname{tr}\left\{X_m^{G\prime} M_{F_m} X_m^G\right\}$. With the standardization $F_m^{G\prime} F_m^G = T \times I_{r_m}$, the PCE of F_m^G, denoted $\hat{F}_m^{(1)}$, is obtained by maximizing $\operatorname{tr}\left\{F_m^{G\prime} X_m^G X_m^{G\prime} F_m^G\right\}$ with respect to F_m^G. It is $\sqrt{T}$ times the matrix consisting of the eigenvectors corresponding to the r_m largest eigenvalues of the matrix $X_m^G X_m^{G\prime}$. The PCE of Λ_m is given by $\hat{\Lambda}_m^{(1)} = \frac{1}{T} X_m^{G\prime} \hat{F}_m^{(1)}$.

Note that the PCE $\hat{F}_m^{(1)}$ estimates the space of $M_{\hat{G}^{(1)}} F_m$, not that of F_m. However, the PCE estimates a rotation of F_m as will be discussed in more detail later. The intuition is that G_t and F_{mt} are uncorrelated, so $\hat{G}^{(1)}$ and F_{mt} are asymptotically.

Proposition 3 reports asymptotic results for the PCEs. This follows from Bai's (2003) Theorems 1, 2 and 3 upon minor adaptations.

Proposition 3 *Under Assumptions 1∼4, the following results hold for any m as $N_m, T \to \infty$.*

(i) (a) If $\frac{\sqrt{N_m}}{T} \to 0$, $\hat{F}_{mt}^{(1)} - H_m' F_{mt}^G = O_p(N_m^{-1/2})$ for each t, where $\hat{F}_{mt}^{(1)}$ is the t-th column of $\hat{F}_m^{(1)\prime}$ and H_m is a square random matrix such that $||H_m|| = O_p(1)$.[1]

[1]See Bai's (2003) Theorem 1 for the definition of H_m.

(b) If $\liminf \frac{\sqrt{N_m}}{T} \geq \tau > 0$*,* $\hat{F}_{mt}^{(1)} - H_m' F_{mt}^G = O_p(T^{-1})$ *for each* t*.*

(ii) (a) If $\frac{\sqrt{T}}{N_m} \to 0$*,* $\hat{\lambda}_{mi}^{(1)} - H_m^{-1}\lambda_{mi} = O_p(T^{-1/2})$ *for each* i*.*

(b) If $\liminf \frac{\sqrt{T}}{N_m} \geq \tau > 0$*,* $\hat{\lambda}_{mi}^{(1)} - H_m^{-1}\lambda_{mi} = O_p\left(N_m^{-1}\right)$ *for each* i*.*

(iii) Let $\hat{C}_{mit}^{(1)} = \hat{\lambda}_{mi}^{(1)\prime}\hat{F}_{mt}^{(1)}$ *and* $C_{mit}^G = \lambda_{mi}' F_{mt}^G$*. Then,* $\hat{C}_{mit}^{(1)} - C_{mit}^G = O_p\left(\kappa_{N_mT}^{-1}\right)$*, where* $\kappa_{N_mT} = \min\left\{\sqrt{N_m}, \sqrt{T}\right\}$.

Part (i) of this proposition shows that $\hat{F}_{mt}^{(1)} - H_m' F_{mt}^G \xrightarrow{p} 0$. Since $H_m = O_p(1)$, $F_m'\hat{G}^{(1)} = F_m' G Q_s + F_m'(\hat{G}^{(1)} - GQ_s) = O_p(T^{1/2}) + O_p(T\kappa_{N_1T}^{-1})$ and $(T^{-1}\hat{G}^{(1)\prime}\hat{G}^{(1)})^{-1} = O_p(1)$, we have

$$
\begin{aligned}
\hat{F}_{mt}^{(1)} - H_m' F_{mt}^G &= \hat{F}_{mt}^{(1)} - H_m' F_{mt} - H_m' F_m' \hat{G}^{(1)} \left(\hat{G}^{(1)\prime}\hat{G}^{(1)}\right)^{-1} \hat{G}_t^{(1)} \\
&= \hat{F}_{mt}^{(1)} - H_m' F_{mt} + O_p(\kappa_{N_1T}^{-1}), \qquad (11)
\end{aligned}
$$

which implies that $\hat{F}_{mt}^{(1)} - H_m' F_{mt} \xrightarrow{p} 0$. That is, $\hat{F}_{mt}^{(1)}$ consistently estimates a rotation of F_{mt}.

Part (ii) of this proposition shows that $H_m^{-1}\lambda_{mi}$ can be estimated consistently by $\hat{\lambda}_{mi}^{(1)}$ at the rate of either $T^{1/2}$ or N_m.

Using the same arguments as for equation (11), we have

$$
\hat{C}_{mit}^{(1)} - C_{mit}^G = \hat{C}_{mit}^{(1)} - C_{mit} + O_p(\kappa_{N_1T}^{-1}). \qquad (12)
$$

Thus, $\hat{C}_{mit}^{(1)}$ estimates the common components C_{mit} consistently with the convergence rate κ_{N_mT} or κ_{N_1T} since $\hat{C}_{mit}^{(1)} - C_{mit}^G = O_p(\kappa_{N_mT}^{-1})$.

3.3 Step 3: Estimation of the spaces of the global factor and factor loadings

This subsection considers estimating the spaces of the global factor and factor loadings using the estimators of the country's common components from Step 2. For this, write

Model (5) as

$$Z_t = \Gamma G_t + e_t^{(2)}, \tag{13}$$

where, letting $N = \sum_{m=1}^{M} N_m$, Z_t is an N by 1 vector, Γ is an N by s matrix. Since $e_{mit}^{(2)} = e_{mit} + O_p\left(\kappa_{N_m T}^{-1}\right)$ for each m and i, Model (13) is the typical factor model except for the presence of the $O_p\left(\kappa_{N_m T}^{-1}\right)$ terms. The PCEs of γ_{mi} and G_t, denoted $\hat{\gamma}_{mi}^{(1)}$ and $\hat{G}_t^{(2)}$, respectively, follow the asymptotic distributions given in Bai (2003) and are stated concretely in Proposition 4.

Proposition 4 *Under Assumptions 1∼4, the following results hold as $N, T \to \infty$.*
(i) (a) If $\frac{\sqrt{N}}{T} \to 0$, $\hat{G}_t^{(2)} - J'G_t = O_p(N^{-1/2})$ for each t, where $\hat{G}_t^{(2)}$ is the t-th column of $\hat{G}^{(2)\prime}$ and J is a square random matrix such that $||J|| = O_p(1)$.
(b) If $\liminf \frac{\sqrt{N}}{T} \geq \tau > 0$, $\hat{G}_t^{(2)} - J'G_t = O_p(T^{-1})$ for each t.
(ii) (a) If $\frac{\sqrt{T}}{N} \to 0$, $\hat{\gamma}_{mi}^{(1)} - J^{-1}\gamma_{mi} = O_p(T^{-1/2})$ for each m and i.
(b) If $\liminf \frac{\sqrt{T}}{N} \geq \tau > 0$, $\hat{\gamma}_{mi}^{(1)} - J^{-1}\gamma_{mi} = O_p\left(N^{-1}\right)$ for each m and i.
(iii) $\hat{\gamma}_{mi}^{(1)\prime}\hat{G}_t^{(2)} - \gamma_{mi}'G_t = O_p\left(\delta_{NT}^{-1}\right)$, where $\delta_{NT} = \min\left\{\sqrt{N}, \sqrt{T}\right\}$.

This proposition implies that the global factor space is consistently estimated by the PCE $\hat{G}_t^{(2)}$ at the rate either $\sqrt{N}$ or T. $\hat{G}_t^{(2)}$ estimates the global factor space faster than the initial estimator $\hat{G}_t^{(1)}$ which has the consistency rate $\kappa_{N_1 T}$. The intuition behind this is that the PCE $\hat{G}_t^{(2)}$ uses all the information in the sample, while $\hat{G}_t^{(1)}$ is based on the sample from only countries 1 and 2. Moreover, the common components $\gamma_{mi}'G_t$ are consistently estimated at the rate δ_{NT}. This will be used for Step 4 in the next subsection.

3.4 Step 4: Estimation of the spaces of the country factor and factor loadings

In this subsection, we consider estimating the spaces of the country factor and factor loadings using Model (6) and the estimators of the common components from Step

3. Write Model (6) as

$$Y_{mt} = \Lambda_m F_{mt} + e_{mt}^{(3)}.$$

Since $e_{mit}^{(3)} = e_{mit} + O_p(\delta_{NT}^{-1})$, we obtain the following for the PCEs $\hat{F}_{mt}^{(2)}$ and $\hat{\lambda}_{mi}^{(2)}$.

Proposition 5 *The results in Proposition 3 hold as $N_m, T \to \infty$, once $\hat{F}_{mt}^{(1)}$, F_{mt}^G, $\hat{\lambda}_{mi}^{(1)}$, and $\hat{C}_{mit}^{(1)}$ are replaced by $\hat{F}_{mt}^{(2)}$, F_{mt}, $\hat{\lambda}_{mi}^{(2)}$, and $\hat{\lambda}_{mi}^{(2)\prime}\hat{F}_{mt}^{(2)}$, respectively; and H_m is appropriately redefined.*

This proposition implies that a rotation of F_{mt} can consistently be estimated at the rate of either $\sqrt{N_m}$ or T. In contrast, $\hat{F}_{mt}^{(1)} - H_m' F_{mt} = O_p(N_m^{-1/2}) + O_p(\kappa_{N_1 T}^{-1})$ or $O_p(T^{-1}) + O_p(\kappa_{N_1 T}^{-1})$ and the additional $O_p(\kappa_{N_1 T}^{-1})$ term can adversely affect the estimates. This will be studied further via simulation in Section 5. A similar argument applies to the estimators of the common components.

4 Estimation of the numbers of country factors

There are various procedures available for the selection of the number of static factors. Bai and Ng (2002) and Choi and Jeong (2012) propose several information criteria for the number of factors in approximate factor models. Dias, Pinheiro and Rua (2013) provide an information criterion for the multilevel factor model of this paper without separately identifying the global and country factors as in this paper. Onatski (2009, 2010) and Ahn and Horenstein (2013) suggest methods using ordered eigenvalues. See Breitung and Choi (2013) for detailed discussions of these methods. When the number of static factors is determined, we may use the methods of Amengual and Watson (2007), Bai and Ng (2007), and Breitung and Pigorsch (2012) to determine the number of dynamic (or primitive) factors.

Motivated from the empirical example in Section 6, we assume that the number of global factors, s, is known. Then, one only needs to estimate the total number of factors in each country, $s + r_m$, which can be consistently estimated by applying one of the aforementioned information criteria to each country. Having obtained an estimate for the total number of factors, the number of country factors is computed simply by subtracting s. Alternatively, one can apply an information criterion in Step 2. In other words, one eliminates the global factors by $\hat{G}^{(1)}$ and applies an information criterion to $X_m^G = M_{\hat{G}^{(1)}} X_m$ in each country. For the theoretical justification of the second method, we provide the following result.

Let $V_m(k_m, \tilde{F}_{k_m}^{(1)})$ be the sum of squared residuals for country m in Step 2 when k_m country factors are estimated by $\tilde{F}_{k_m}^{(1)}$, that is,

$$V_m(k_m, \tilde{F}_{k_m}^{(1)}) = \frac{1}{N_m T} tr \left[X_m^{G\prime} M_{\tilde{F}_{k_m}^{(1)}} X_m^G \right].$$

We prove the consistency of an information criterion given by

$$PC(k_m) = V_m(k_m, \tilde{F}_{k_m}^{(1)}) + k_m \cdot g_m.$$

Proposition 6 *Suppose that Assumptions 1∼4 hold. Let $\hat{r}_m$ be the minimizer of $PC(k_m)$ over $0 \leq k_m \leq k_m^{\max}$. Assume that each element of $\hat{G}^{(1)}$ satisfies the results in Proposition 2. Then,*

$$\lim_{N,T \to \infty} \Pr(\hat{r}_m = r_m) = 1$$

if $g_m \to 0$ and $\kappa_{N_m T}^2 g_m \to \infty$ for each m.

The above result can readily be extended to some of popular information criteria. The information criteria we consider in this paper are:

$$\begin{aligned} IC_{p2} &= \ln\left(V_m(k_m, \tilde{F}_{k_m}^{(1)}) \right) + \ln(\min\{N_m, T\}) \left(\frac{k_m(N_m + T)}{N_m T} \right); \\ BIC &= T \cdot tr \left(\ln \left(\frac{1}{T} X_m^{G\prime} M_{\tilde{F}_{k_m}^{(1)}} X_m^G \right) \right) + \ln(N_m T) \left[k_m(N_m + T) + N_m \right]; \\ HQ_c &= T \cdot tr \left(\ln \left(\frac{1}{T} X_m^{G\prime} M_{\tilde{F}_{k_m}^{(1)}} X_m^G \right) \right) + c \ln\ln(N_m T) \left[k_m(N_m + T) + N_m \right], \end{aligned}$$

where c is a constant of our choice. The first one is Bai and Ng's IC_{p2}, which shows good performance according to the simulation results of Choi and Jeong (2012). The second and third are BIC and Hannan and Quinn's (1979) criterion suggested in Choi and Jeong, respectively. Choi and Jeong show that BIC and HQ_c perform well in finite samples.

Although these criteria use some log transformation of the goodness of fit measure, their consistency continues to hold. See Corollary 1 of Bai and Ng (2002) for IC_{p2} and Appendix III in Choi and Jeong (2012) for BIC and HQ_c. For the penalty term g_m, let $g_m = (N_m + T)\ln(\min\{N_m, T\})/N_m T$ for IC_{p2}. Then, g_m satisfies the conditions in Proposition 6. For BIC and HQ_c, rescale them by $N_m T$ and let $g_m = \ln(N_m T)(N_m + T)/N_m T$ for BIC and $g_m = c \ln\ln(N_m T)(N_m + T)/N_m T$ for HQ_c. The conditions in Proposition 6 are satisfied for BIC if $\ln(N_m)/T \to 0$ and $\ln(T)/N_m \to 0$, and for HQ_c if $\ln\ln(N_m)/T \to 0$ and $\ln\ln(T)/N_m \to 0$.

Finite-sample properties of these information criteria are studied in Section 5. In our Monte Carlo simulation experiments, applying an information criterion in Step 2 shows better finite sample performance.

5 Simulation

This section evaluates finite-sample performance of the stepwise estimators of the global and country factors and the model-selection criteria. For our simulation, data were generated by the following data-generating process (DGP):

$$\begin{aligned} x_{mit} &= \gamma'_{mi}G_t + \sqrt{\theta_{m1}}\lambda'_{mi}F_{mt} + \sqrt{\theta_{m2}}e_{mit} \\ &= \sum_{k=1}^{s}\gamma_{mik}g_{tk} + \sqrt{\theta_{m1}}\sum_{k=1}^{r_m}\lambda_{mik}f_{mtk} + \sqrt{\theta_{m2}}e_{mit} \\ \gamma_{mik} &\sim iid\ N\left(0,\sigma_\gamma^2\right);\ \lambda_{ik} \sim iid\ N\left(0,\sigma_\lambda^2\right) \\ G_t &= \alpha G_{t-1} + v_t,\ v_t \sim iid\ N(0, I_s) \\ F_{mt} &= \phi F_{mt-1} + w_{mt},\ w_t \sim iid\ N(0, I_{r_m}) \\ e_{mit} &= \rho e_{mit-1} + \varepsilon_{mit} + \beta \sum_{1\le|j|\le 8} \varepsilon_{m,i-j,t},\ \varepsilon_{mit} \sim iid\ N(0,1). \end{aligned} \tag{14}$$

This DGP allows serially correlated factors, and serially and cross-sectionally correlated idiosyncratic errors. The parameters α and ϕ control the degree of serial correlations in the global and country factors, respectively. In addition, the parameters ρ and β set the degrees of serial and cross-sectional correlations of the idiosyncratic errors, respectively. Since

$$\text{Var}\left(\gamma'_{mi}G_t\right) = \sum_{k=1}^{s} E(\gamma_{mik}g_{tk})^2 = \sum_{k=1}^{s} E(\gamma_{mik}^2)E(g_{tk}^2) = \frac{s\sigma_\gamma^2}{1-\alpha^2};$$

$$\text{Var}\left(\lambda'_{mi}F_{mt}\right) = \sum_{k=1}^{r_m} E(\lambda_{mik}f_{mtk})^2 = \sum_{k=1}^{r_m} E\left(\lambda_{mik}^2\right) E\left(f_{mtk}^2\right) = \frac{r_m\sigma_\lambda^2}{1-\phi^2};$$

and

$$\text{Var}(e_{mit}) = \frac{1+16\beta^2}{1-\rho^2},$$

we set $\theta_{m1} = \left(\frac{s\sigma_\gamma^2}{1-\alpha^2}\right) / \left(\frac{r_m\sigma_\lambda^2}{1-\phi^2}\right)$ and $\theta_{m2} = \left(\frac{s\sigma_\gamma^2}{1-\alpha^2}\right) / \left(\frac{1+16\beta^2}{1-\rho^2}\right)$ to make the three components in (14) have the same variance, $\frac{s\sigma_\gamma^2}{1-\alpha^2}$.

We consider three cases for the true number of global factors $s \in \{1, 2, 3\}$. The true numbers of country factors are assumed to be 2 for all countries. We set the number of countries $M \in \{5, 20\}$. The cross-sectional sample size, N_m, is either 20 or 100, and the time-series sample size, T, is either 100 or 200. The parameters

σ_γ^2 and σ_λ^2 are assumed to be 1. For the autocorrelation parameters for the global and country factors, we consider $(\alpha, \phi) = (0.5, 0, 5)$, but the results for $(\alpha, \phi) = (0.5, 0, 85), (0.85, 0, 5)$ and $(0.85, 0, 85)$ are qualitatively similar to the case of $(\alpha, \phi) = (0.5, 0, 5)$ and are unreported here.[2] For the parameters ρ and β, we consider the combinations of $\rho = 0, 0.5, 0.85$ and $\beta = 0, 0.2$. The simulation results in Section 5.1 and 5.2 are based on $5,000$ Monte Carlo replications, and those in 5.3 are based on $1,000$ replications.

5.1 Efficiency in estimating the global and country factors

This subsection studies how well our procedure estimates the true G_t and F_{mt} by calculating the trace-ratio statistic. The trace-ratio statistic is a generalized squared correlation coefficient used as a goodness-of-fit measure in multivariate analysis. Denote the true global factor and its estimate by G and $\hat{G}$, respectively. Then, the trace ratio is defined as

$$\mathrm{tr}(\hat{G}) = \frac{\mathrm{tr}(G'\hat{G}(\hat{G}'\hat{G})^{-1}\hat{G}'G)}{\mathrm{tr}(G'G)}.$$

A value of the trace ratio closer to one implies better performance of the factor estimates. To study the efficiency gains obtained by going through the four estimation steps, we report the trace-ratio statistics of the factor estimates from each estimation step.

Table 1 reports averaged values of the trace ratios for the estimates of global and country factors for $s = 1$, 2 and 3. Each row shows the results for different sample sizes $(M,\ N_m,\ T)$. Each column shows the results for different structures of the idiosyncratic errors. The rows labeled $\hat{G}^{(1)}$ and $\hat{G}^{(2)}$ report, respectively, the averages of the trace ratios of the initial (Step 1) and the final (Step 3) estimates of the global factors across $5,000$ replications. The rows labeled $\hat{F}^{(1)}$ and $\hat{F}^{(2)}$ show the trace ratios of the initial (Step 2) and the final (Step 4) estimates of the country factors, averaged

[2] The results are available upon request.

across countries and across 5,000 replications, respectively.

Most notably, the final estimates perform better than the initial estimates: $\hat{G}^{(2)}$ outperforms $\hat{G}^{(1)}$, and $\hat{F}^{(2)}$ outperforms $\hat{F}^{(1)}$. The improvements are much more noticeable for the global than the country factor estimators. This is due to the fact that $\hat{G}^{(2)}$ uses more observations than $\hat{G}^{(1)}$ while $\hat{F}^{(2)}$ and $\hat{F}^{(1)}$ use the same number of observations. $\hat{F}^{(2)}$ does better than $\hat{F}^{(1)}$ because $\hat{F}^{(2)}$ is based on a better global factor estimator ($\hat{G}^{(2)}$) than $\hat{F}^{(1)}$.

Overall, the final estimates of the global factors perform very well – for example, the trace ratios in Part I of Table 1 range from 0.9866 to 0.9995 when there are no serial and cross-sectional correlations in the idiosyncratic errors. The degrees of serial and cross-sectional correlations of the idiosyncratic errors have little impact on the performance of the estimates of the global factors. The trace ratios of the final estimates of the global factors do not fall and sometimes even increase as the idiosyncratic errors become more serially and cross-sectionally correlated. In addition, the performance of the estimates of the global factors improves as T, N_m, or M increases.

According to Parts II and III of Table 1, the trace ratios of the initial estimates of the global factors fall as the number of global factors increases. However, those of the final estimates deteriorate only slightly. Moreover, the efficiency gains from the initial to the final estimates becomes much more noticeable as s increases, demonstrating the advantage of our step-wise estimation method.

The performance of the estimates of the country factors is affected more by the properties of the idiosyncratic errors and by the sample sizes than the properties of the global factors. The trace ratios for the final estimates of the country factors in Part I of Table 1 range from 0.8785 to 0.9716 when there are no serial and cross-sectional correlations in the idiosyncratic errors. But the performance of the estimates of the country factors deteriorates as the errors become more cross-sectionally correlated, while it is not significantly affected by the degree of serial correlation of the errors.

While the number of time series observations, T, is not important in determining the performance of the country factor estimates, the number of cross-sectional observations, N_m, significantly affects the performance of $\hat{F}$. As N_m increases, both the initial and final estimates of the country factors show improved performance. The improvement is particularly large when the idiosyncratic errors are serially and cross-sectionally correlated. A larger number of countries marginally improves the performance of the country factor estimates. When the number of cross-sectional observations is not large, however, adding more countries worsens the performance of $\hat{F}$.

Parts II and III of Table 1 show that the performance of the estimates of the country factors slightly worsens as the number of the global factors increases. However, the implications obtained from Part I do not essentially change as s becomes larger.

5.2 Efficiency in estimating the variance of the global common components

This subsection studies how well our procedures estimate the variance ratio

$$\frac{\mathrm{Var}\left(\gamma_{mi}' G_t\right)}{\mathrm{Var}(x_{mit})} = \frac{1}{3}.$$

This measure is important in studying the international business cycle as will be seen later. Table 2 presents the variance ratios of the initial and final estimates of the global factors. When there are no serial and cross-sectional correlations in the idiosyncratic errors, the estimated variance ratios are close to $\frac{1}{3}$. Moreover, when N_m is large, the variance ratios stabilize around $\frac{1}{3}$, unless the idiosyncratic errors become highly serially and cross-sectionally correlated. The estimated variance ratios tend to increase as the idiosyncratic errors become more serially and cross-sectionally correlated. As expected, $\hat{G}^{(2)}$ estimates the variance ratios more accurately than $\hat{G}^{(1)}$, showing again that going through an additional step of estimation improves the performance of the global factor estimators.

5.3 Estimation of the number of country factors

This subsection studies finite-sample performance of the model-selection criteria. The two approaches explained in Section 4 are compared. The first approach applies an information criterion to each country and estimates the total number of factors. The number of country factors is estimated simply by subtracting the number of global factors from the estimate of the total number. The second approach formulates an information criterion using the estimation results from Step 2. For each approach, we simulate three information criteria. The first one is Bai and Ng's (2002) IC_{p2}, which shows good performance according to the simulation results of Choi and Jeong (2012). The second and third are BIC and Hannan and Quinn's (1979) HQ_c criterion suggested in Choi and Jeong (2012), respectively. Choi and Jeong (2012) show that BIC and HQ_c perform well in finite samples.

We use the same simulation specifications as in the previous subsections. The true number of global factors is 1 and the numbers of country factors are set to be 2 for all countries.[3] The maximum number of country factors to be tested is set at 3. For HQ_c, we chose $c = 4$. The number of global factors is assumed to be known.

Table 3a contains the results for the first approach and Table 3b for the second approach. In each of them, Part I reports the sample means of the estimated numbers of country factors for each set of parameter values and sample sizes. The sample means are taken across countries and across $1,000$ iterations. Part II reports the root mean squared errors (RMSE) of the estimated numbers of factors.

First, see the case where there is no cross-sectional correlation ($\beta = 0$) in the idiosyncratic errors. The performance of IC_{p2} is similar across the first and second approaches. IC_{p2} estimates the true number in most cases, but it tends to overes-

[3]The results for the specifications with $s = 2$, $s = 3$, and $(\alpha, \phi) = (0.5, 0, 85)$, $(0.85, 0, 5)$, $(0.85, 0.85)$ are not reported, since thery are very similar to those of $s = 1$ and $(\alpha, \phi) = (0.5, 0.5)$.

timate when there is strong serial correlation ($\rho = 0.85$) in the idiosyncratic errors. BIC and HQ_4 are less affected by the serial correlation in the idiosyncratic errors. However, these two criteria tend to underestimate when N_m is small ($N_m = 20$), and the degree of underestimation is much more severe in the first approach.

When mild cross-sectional correlation ($\beta = 0.2$) is introduced in the idiosyncratic errors, IC_{p2} overestimates the numbers of factors in all cases. BIC and HQ_4 perform better than IC_{p2}. Their mean values of the estimated factor numbers are closer to the true value, 2. When there is strong cross-sectional correlation ($\beta = 0.5$) in the idiosyncratic errors, no criterion is working properly. All of them overestimate the numbers of factors. The second approach slightly outperforms the first approach with cross-sectionally correlated idiosyncratic errors.

Overall, BIC and HQ_4 seem to perform better than IC_{p2} in most cases as long as N_m is not too small. HQ_4 tends to underestimate the factor numbers more than BIC does, although the differences are marginal. Between the two approaches, the second approach, the one applying a criterion in Step 2, appears to be better than the first approach.

6 An application to international business cycle

6.1 Data

This section applies the methodology developed in the previous sections to data from 25 OECD countries.[4] For the selection of variables, we start with the variables in the U.S. composite index of coincident indicators published by the Conference Board. They are the number of employees on nonagricultural payrolls, the index of

[4]The previous version of this paper used data from the G-7 countries to compare our application results with those of previous studies (e.g., Aruoba *et al.,* 2010). But one of the referees suggested to include more countries, because more data tend to bring better estimation results according to the simulation results of Section 5. To accommodate the referee comment, this version employs data from 25 OECD countries. However, the results of this version are not qualitatively different from those of the previous version.

industrial production, the level of manufacturing and trade sales, and the aggregate amount of personal income excluding transfer payments. The Philadelphia Fed has also published the U.S. coincident economic activity index constructed with a similar group of variables – nonfarm payroll employment, unemployment rate, average hours worked in manufacturing, and wages and salaries. Following the spirit of the above coincident indices, we include four kinds of variables in our application: GDP-related data, employment/unemployment-related data, sales and retail trade data, and international trade data.[5] All the variables listed in the indices above are used for our application. In addition, national income account variables (disposable personal income, compensation of employees, private consumption, private investment, construction of structures, and change in inventories), employment-related variables (initial unemployment claims) and international trade variables (exports and imports of goods and services) are used. The variables included in our application is listed in Table 4. The data frequency is quarterly and the data span the first quarter 1980 through the second quarter 2013. Some variables were seasonally adjusted with the X-12 ARIMA procedure if they were not originally seasonally adjusted. Quantity variables except rate and ratio variables were logarithmically transformed and all variables were detrended with the Hodrick–Prescott filtering method.[6]

6.2 Estimation results

We performed our factor analysis assuming that the number of global factors is known to be one. Because we include only real national account variables and labor-market variables in our application, it is natural to consider a single, real factor that drives business fluctuations in the 25 OECD countries. Figure 1 plots the estimated global

[5] Aruoba *et al.* (2010) use six variables for the G-7 countries: payroll employment, GDP, household disposable income, initial unemployment claims, industrial production index, and retail trade index.

[6] Other detrending methods (e.g., log first- differencing) may be considered. We only report the empirical results with HP filtering for comparison with such recent studies as Gregory, Head and Raynauld (1997) and Aruoba *et al.* (2010).

factor and the GDP fluctuations of 8 major countries – U.S., Canada, U.K., Japan, France, Germany, Korea, and Mexico – along with the official recession dates of each country illustrated as shaded areas.[7] We report plots only for the 6 countries in the G-7 and the 2 emerging economies – Korea and Mexico – to save the space. Aruoba *et al.* (2010) also extract similar common components in the G-7 countries, but our global factor is smoother and shows more persistence. The global factor seems to co-move well with the GDP fluctuations and can be interpreted as an international business cycle factor common in the 25 OECD countries' GDP fluctuations. In particular, the global factor shows clearly the depth and severity of the 2008 global financial crisis (the Great Recession). Comparing the extracted global factor with the U.S. GDP fluctuations, we see that the global factor shows a close relationship with the U.S. recessions particularly in recent periods. The recessions in the U.S. over the early 1980s and in the Great Recession period correspond to declines in the global factor, while there are slight lags in the early 1990s and the early 2000s' technology bubble bursting and terrorist attack. It is interesting to see that the Canadian GDP fluctuations are very similar to those of the global factor. This may be interpreted as follows: The U.S. business cycle may be most important for the fluctuations of the global factor, and Canada seems to be affected much by the U.S. business fluctuations. The GDP fluctuations in France and Germany are closely associated with the fluctuations of the global factor even though there are sometimes time lags in the 1980s and 1990s which is in accordance with the findings by Kose, Otrok and Whiteman (2008). Japan seems to show more synchronized fluctuations of GDP with the global factor after 2000. Korea and Mexico's GDP fluctuations show a tendency of co-movements with the global factor in more recent periods. Mexico's close relationship with the global factor may be related to the NAFTA that she signed in 1992.

Figure 2 draws the fluctuations of the global factor and the HP-filtered cyclical component of the industrial production index of the G-7 countries published by

[7] For the initial estimate of the global factor in Step 1, data from the U.S. and Canada are used since they yield the maximum sample mean of the eigenvalues from equation (7).

the OECD.[8] The G-7 industrial production index may be regarded as representing international business cycle, although it excludes many other countries. To our satisfaction, the extracted global factor closely mimics the HP-filtered G-7 industrial production index, which suggests that our method extracts a global business cycle factor quite well.

The next questions investigated in this section are: (i) the relative importance of the global and country factors in explaining GDP fluctuations and (ii) changes in their relative importance over time. Before these questions are tackled, the numbers of country factors should be estimated. Table 5 reports selection results for the number of country factors using the three information criteria – IC_{p2}, BIC, and HQ_4 – for the whole sample period and three subperiods (1981–1990, 1991–2000 and 2001–2013).[9] We find that IC_{p2} tends to choose the maximum number of country factors while BIC and HQ_4 tend to provide the same numbers for country factors. Using the selected numbers of the country factors, variance decompositions were calculated and reported in Figure 3. In Figure 3, the variance of the global factor refers to the sample variance of the common components involving the global factor (i.e., the sample variance of $\hat{\gamma}_{mi}^{(1)}\hat{G}_t^{(2)}$ from Step 3), while that of the country factors is the sample variance of the common components involving the country factors (i.e., the sample variance of $\hat{\lambda}_{mi}^{(2)\prime}\hat{F}_{mt}^{(2)}$ from Step 4). The residual variances were calculated by subtracting the sum of the variances of the global and country factors from the sample variance of the GDP data. The numbers of country factors were selected by BIC for Figure 3. Over the whole sample period, the global factor is most important in GDP fluctuations of France, Germany and Japan, which accords well with Gregory, Head and Raynauld (1997), Kose, Otrok and Whiteman (2003), Kose, Otrok and Prasad (2012) and Kose, Otrok and Whiteman (2008). The percentage shares of the global factors in explaining GDP fluctuations are 72, 75, and 43 for France, Germany, and

[8] The G-7 industrial production index was taken from the OECD website (http://stats.oecd.org) and detrended with the HP-filter.

[9] Among the two approaches proposed in Section 4, we use the second approach which applies information criteria to data from Step 2.

Japan, respectively, while the corresponding numbers are 30, 27, 29, 6, and 33 for the U.S., Canada, the U.K., Korea, and Mexico, respectively. The importance of global factor for Germany and Japan suggests that international trade plays an important role for the global factor. It is interesting to note that the U.S. GDP fluctuations are mostly explained by country factors. The low figure for Korea indicates significance of trade with China for the Korean economy. Recall that our data set does not include China.

To investigate the relative importance of the global and country factors over time, we divided the whole sample period into three subperiods (1981–1990, 1991–2000, and 2001–2013), estimated the global and country factors using the selected factor numbers reported in Table 5, and calculated variance decompositions as before.[10] Figure 3 reports relative importance of the global and country factors and the residual component in the 8 major countries' GDP fluctuations over the three subperiods. During the recent subperiod, 2001-2013, the global factor is most important and the percentage share of the global factor increased drastically from the previous decade in all of the 8 countries. In 1980s, country factors explain much of the GDP fluctuations in the U.S. and Canada, while the global factor is most important in the U.K. and France. The residual variance is most important in Japan, Germany, Korea, and Mexico. By contrast, in 1990s, country factors are important in most countries except Japan and Germany. Japan also shows a significant increase in the importance of the country factors during 1990s which may reflect a long recession of 1990s separating Japanese economy from the global economic fluctuations. In the years of 2001 through 2013, which includes the recent global financial crisis, the global factor regains importance for most of the countries except Korea. For the Korean economy,

[10] We may extract the global and country factors using the whole sample and calculate the fraction of each factor's contribution to GDP fluctuations over the subperiods. However, this approach may make sum of the variances of the factors exceed that of the GDP fluctuations, which is obviously unnatural. Thus, we estimated the global and country factors using the data from each subperiod and reported the variance decomposition results for the subperiods. Most previous studies (e.g., Kose, Otrok and Prasad, 2012; and Kose, Otrok and Whiteman, 2008) also allow model parameters to vary across subsamples.

trade with China has increasingly become important in recent years. The low figure for Korea reflects the fact that China is excluded in our data set. In all of the 8 countries, the global factor became more important for their GDP fluctuations than in the previous decade. Out of the 25 OECD countries, the number of the countries where the global factor is most important for GDP fluctuations increased from 5 during 1990s to 18 during 2001-2013, while that of the countries having country factors as most important factors declined from 13 to 3 over the same subperiods.

The empirical findings of this section are summarized as follows. First, our method extracts the global factor successfully for the major OECD countries in the sense that the extracted global factor matches reasonably well with the major economic booms and recessions of the past 30 years. Second, the global factor is most important in explaining GDP fluctuations for France, Germany, and Japan over the whole sample period and for most of the OECD countries in the period after 2000. Third, the relative importance of the global factor in explaining GDP fluctuations has changed over the three subperiods. Over the 1980s and the 1990s global factors are less important than now. In the period 2001 through 2013, which includes the recent global financial crisis, the global factor is dominant in GDP fluctuations of most of the OECD countries.

7 Summary and further remarks

We have studied a multilevel factor model with global and country factors, and proposed a sequential procedure that can identify the global and country factors separately. This procedure estimates the global factors by canonical correlation analysis in the initial step. This initial estimator is used to construct the PCEs of the global and country factors. The PCEs consistently estimate the spaces of the global and country factors and are normally distributed in the limit. We then establish consistency of information criteria that can estimate the number of country factors. Simulation results have shown that the sequential procedure and the information criteria work well in finite samples. The method of this paper is applied to 25 OECD countries in order to identify international business cycles. The method seems to extract global factors successfully for the 25 countries in the sense that the extracted global factors match reasonably well with the major economic recessions of the past 30 years. Moreover, the global factors are most important in explaining GDP fluctuations for most of the 25 countries.

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Table 1: Trace Ratios of the Estimated Factors

Note: (i) Entry numbers denote the averages of the trace-ratio statistics for the estimates of the global and country factors across 5,000 iterations.
(ii) Data were generated by equation (14).
(iii) The parameters ρ and β set the idiosyncratic errors' degrees of serial and cross-sectional correlation, respectively.
(iv) The number of countries is M, the number of cross-sectional samples in country m is N_m, and the number of time-series samples is T.

Part I: $s = 1$

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	20	100	$\hat{G}^{(1)}$	0.9549	0.9552	0.9500	0.9619	0.9619	0.9610	0.9750	0.9753	0.9745
			$\hat{G}^{(2)}$	0.9866	0.9861	0.9812	0.9886	0.9882	0.9863	0.9921	0.9920	0.9913
			$\hat{F}^{(1)}$	0.8746	0.8729	0.8422	0.6205	0.6334	0.6537	0.5690	0.5787	0.5995
			$\hat{F}^{(2)}$	0.8785	0.8770	0.8475	0.6222	0.6350	0.6557	0.5699	0.5796	0.6005
5	20	200	$\hat{G}^{(1)}$	0.9557	0.9558	0.9540	0.9626	0.9623	0.9619	0.9752	0.9755	0.9763
			$\hat{G}^{(2)}$	0.9873	0.9871	0.9856	0.9893	0.9890	0.9882	0.9925	0.9924	0.9924
			$\hat{F}^{(1)}$	0.8829	0.8826	0.8708	0.6097	0.6178	0.6321	0.5526	0.5565	0.5692
			$\hat{F}^{(2)}$	0.8866	0.8864	0.8753	0.6109	0.6192	0.6339	0.5535	0.5573	0.5700
5	100	100	$\hat{G}^{(1)}$	0.9906	0.9905	0.9890	0.9908	0.9907	0.9888	0.9908	0.9902	0.9881
			$\hat{G}^{(2)}$	0.9978	0.9976	0.9961	0.9976	0.9975	0.9959	0.9975	0.9972	0.9957
			$\hat{F}^{(1)}$	0.9630	0.9627	0.9539	0.9557	0.9537	0.9338	0.9418	0.9375	0.8960
			$\hat{F}^{(2)}$	0.9633	0.9629	0.9542	0.9560	0.9539	0.9342	0.9421	0.9378	0.8965
5	100	200	$\hat{G}^{(1)}$	0.9907	0.9907	0.9905	0.9913	0.9912	0.9906	0.9913	0.9911	0.9903
			$\hat{G}^{(2)}$	0.9979	0.9978	0.9974	0.9978	0.9977	0.9973	0.9977	0.9976	0.9971
			$\hat{F}^{(1)}$	0.9714	0.9712	0.9683	0.9654	0.9645	0.9586	0.9552	0.9532	0.9393
			$\hat{F}^{(2)}$	0.9716	0.9714	0.9685	0.9656	0.9647	0.9588	0.9554	0.9534	0.9396
20	20	100	$\hat{G}^{(1)}$	0.9644	0.9648	0.9624	0.9784	0.9783	0.9776	0.9902	0.9903	0.9901
			$\hat{G}^{(2)}$	0.9961	0.9958	0.9931	0.9973	0.9971	0.9961	0.9983	0.9983	0.9980
			$\hat{F}^{(1)}$	0.8756	0.8746	0.8442	0.6214	0.6353	0.6546	0.5690	0.5786	0.6004
			$\hat{F}^{(2)}$	0.8794	0.8785	0.8496	0.6226	0.6364	0.6561	0.5694	0.5790	0.6009
20	20	200	$\hat{G}^{(1)}$	0.9654	0.9654	0.9646	0.9788	0.9786	0.9785	0.9904	0.9905	0.9903
			$\hat{G}^{(2)}$	0.9965	0.9963	0.9955	0.9975	0.9974	0.9970	0.9984	0.9984	0.9983
			$\hat{F}^{(1)}$	0.8842	0.8839	0.8721	0.6108	0.6186	0.6335	0.5526	0.5579	0.5711
			$\hat{F}^{(2)}$	0.8877	0.8875	0.8763	0.6118	0.6196	0.6348	0.5530	0.5583	0.5715
20	100	100	$\hat{G}^{(1)}$	0.9917	0.9919	0.9908	0.9931	0.9929	0.9920	0.9939	0.9938	0.9929
			$\hat{G}^{(2)}$	0.9994	0.9993	0.9980	0.9994	0.9993	0.9981	0.9994	0.9993	0.9982
			$\hat{F}^{(1)}$	0.9632	0.9627	0.9538	0.9557	0.9537	0.9342	0.9422	0.9375	0.8965
			$\hat{F}^{(2)}$	0.9634	0.9630	0.9542	0.9559	0.9540	0.9346	0.9424	0.9377	0.8970
20	100	200	$\hat{G}^{(1)}$	0.9917	0.9918	0.9918	0.9933	0.9932	0.9930	0.9944	0.9943	0.9939
			$\hat{G}^{(2)}$	0.9995	0.9994	0.9991	0.9995	0.9994	0.9991	0.9995	0.9994	0.9991
			$\hat{F}^{(1)}$	0.9713	0.9712	0.9683	0.9655	0.9646	0.9586	0.9553	0.9533	0.9393
			$\hat{F}^{(2)}$	0.9716	0.9714	0.9685	0.9656	0.9648	0.9588	0.9554	0.9534	0.9395

Part II: $s = 2$

M	N_m	T	ρ / β	0 / 0	0.5 / 0	0.85 / 0	0 / 0.2	0.5 / 0.2	0.85 / 0.2	0 / 0.5	0.5 / 0.5	0.85 / 0.5
5	20	100	$\hat{G}^{(1)}$	0.8977	0.8954	0.8624	0.7840	0.7838	0.7813	0.7799	0.7851	0.7832
			$\hat{G}^{(2)}$	0.9720	0.9697	0.9441	0.9360	0.9298	0.9063	0.9388	0.9338	0.9127
			$\hat{F}^{(1)}$	0.8510	0.8481	0.8037	0.5813	0.5971	0.6163	0.5325	0.5443	0.5660
			$\hat{F}^{(2)}$	0.8621	0.8598	0.8208	0.5952	0.6113	0.6308	0.5441	0.5552	0.5776
5	20	200	$\hat{G}^{(1)}$	0.8988	0.8979	0.8853	0.7762	0.7723	0.7796	0.7785	0.7743	0.7729
			$\hat{G}^{(2)}$	0.9738	0.9730	0.9661	0.9441	0.9406	0.9324	0.9503	0.9460	0.9344
			$\hat{F}^{(1)}$	0.8675	0.8667	0.8493	0.5780	0.5851	0.6008	0.5236	0.5282	0.5417
			$\hat{F}^{(2)}$	0.8774	0.8769	0.8628	0.5915	0.5994	0.6151	0.5340	0.5389	0.5525
5	100	100	$\hat{G}^{(1)}$	0.9802	0.9798	0.9714	0.9747	0.9732	0.9559	0.9638	0.9597	0.9282
			$\hat{G}^{(2)}$	0.9955	0.9950	0.9889	0.9949	0.9941	0.9862	0.9938	0.9925	0.9808
			$\hat{F}^{(1)}$	0.9470	0.9460	0.9360	0.9391	0.9365	0.9146	0.9247	0.9193	0.8739
			$\hat{F}^{(2)}$	0.9478	0.9468	0.9370	0.9402	0.9376	0.9172	0.9266	0.9215	0.8796
5	100	200	$\hat{G}^{(1)}$	0.9804	0.9804	0.9778	0.9767	0.9759	0.9705	0.9685	0.9668	0.9561
			$\hat{G}^{(2)}$	0.9957	0.9956	0.9939	0.9954	0.9951	0.9930	0.9948	0.9943	0.9917
			$\hat{F}^{(1)}$	0.9629	0.9627	0.9594	0.9568	0.9558	0.9494	0.9465	0.9440	0.9286
			$\hat{F}^{(2)}$	0.9635	0.9633	0.9600	0.9575	0.9566	0.9504	0.9476	0.9452	0.9308
20	20	100	$\hat{G}^{(1)}$	0.9073	0.9056	0.8778	0.7957	0.7959	0.7976	0.7924	0.7955	0.7945
			$\hat{G}^{(2)}$	0.9908	0.9895	0.9724	0.9810	0.9777	0.9615	0.9850	0.9824	0.9711
			$\hat{F}^{(1)}$	0.8528	0.8500	0.8076	0.5826	0.5950	0.6140	0.5281	0.5388	0.5606
			$\hat{F}^{(2)}$	0.8651	0.8631	0.8284	0.6012	0.6145	0.6338	0.5461	0.5567	0.5785
20	20	200	$\hat{G}^{(1)}$	0.9090	0.9082	0.8985	0.7911	0.7933	0.7896	0.7893	0.7888	0.7900
			$\hat{G}^{(2)}$	0.9921	0.9916	0.9877	0.9845	0.9837	0.9784	0.9884	0.9874	0.9843
			$\hat{F}^{(1)}$	0.8693	0.8683	0.8519	0.5765	0.5846	0.5970	0.5167	0.5214	0.5349
			$\hat{F}^{(2)}$	0.8801	0.8793	0.8667	0.5940	0.6022	0.6161	0.5330	0.5376	0.5513
20	100	100	$\hat{G}^{(1)}$	0.9813	0.9811	0.9732	0.9771	0.9758	0.9606	0.9680	0.9649	0.9362
			$\hat{G}^{(2)}$	0.9988	0.9984	0.9930	0.9987	0.9982	0.9924	0.9984	0.9978	0.9912
			$\hat{F}^{(1)}$	0.9473	0.9461	0.9360	0.9395	0.9367	0.9159	0.9255	0.9203	0.8768
			$\hat{F}^{(2)}$	0.9481	0.9470	0.9372	0.9407	0.9379	0.9184	0.9271	0.9222	0.8818
20	100	200	$\hat{G}^{(1)}$	0.9815	0.9815	0.9793	0.9787	0.9783	0.9739	0.9725	0.9711	0.9615
			$\hat{G}^{(2)}$	0.9989	0.9988	0.9975	0.9989	0.9987	0.9973	0.9988	0.9986	0.9971
			$\hat{F}^{(1)}$	0.9631	0.9629	0.9596	0.9571	0.9562	0.9495	0.9467	0.9445	0.9297
			$\hat{F}^{(2)}$	0.9637	0.9635	0.9603	0.9578	0.9569	0.9505	0.9477	0.9456	0.9316

Part III: $s = 3$

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	20	100	$\hat{G}^{(1)}$	0.8420	0.8366	0.7617	0.7258	0.7233	0.7161	0.7446	0.7413	0.7361
			$\hat{G}^{(2)}$	0.9560	0.9511	0.8858	0.9059	0.8943	0.8552	0.9095	0.8961	0.8657
			$\hat{F}^{(1)}$	0.8264	0.8198	0.7531	0.5640	0.5748	0.5987	0.5217	0.5321	0.5578
			$\hat{F}^{(2)}$	0.8451	0.8397	0.7803	0.5772	0.5893	0.6156	0.5304	0.5411	0.5680
5	20	200	$\hat{G}^{(1)}$	0.8439	0.8428	0.8103	0.7142	0.7137	0.7068	0.7305	0.7282	0.7206
			$\hat{G}^{(2)}$	0.9601	0.9586	0.9413	0.9241	0.9167	0.8965	0.9275	0.9198	0.8958
			$\hat{F}^{(1)}$	0.8524	0.8499	0.8221	0.5652	0.5742	0.5906	0.5176	0.5244	0.5404
			$\hat{F}^{(2)}$	0.8684	0.8665	0.8456	0.5771	0.5863	0.6047	0.5225	0.5296	0.5458
5	100	100	$\hat{G}^{(1)}$	0.9700	0.9690	0.9465	0.9508	0.9459	0.8881	0.9026	0.8944	0.8280
			$\hat{G}^{(2)}$	0.9933	0.9922	0.9769	0.9913	0.9892	0.9637	0.9867	0.9827	0.9446
			$\hat{F}^{(1)}$	0.9312	0.9292	0.9153	0.9216	0.9175	0.8846	0.9005	0.8917	0.8383
			$\hat{F}^{(2)}$	0.9326	0.9307	0.9176	0.9245	0.9208	0.8946	0.9098	0.9019	0.8550
5	100	200	$\hat{G}^{(1)}$	0.9702	0.9699	0.9635	0.9564	0.9544	0.9357	0.9242	0.9179	0.8697
			$\hat{G}^{(2)}$	0.9936	0.9933	0.9896	0.9926	0.9919	0.9867	0.9909	0.9895	0.9803
			$\hat{F}^{(1)}$	0.9547	0.9541	0.9502	0.9483	0.9467	0.9380	0.9359	0.9320	0.9066
			$\hat{F}^{(2)}$	0.9557	0.9551	0.9514	0.9499	0.9485	0.9411	0.9398	0.9366	0.9185
20	20	100	$\hat{G}^{(1)}$	0.8523	0.8467	0.7756	0.7326	0.7339	0.7243	0.7496	0.7464	0.7397
			$\hat{G}^{(2)}$	0.9853	0.9825	0.9389	0.9808	0.9769	0.9511	0.9849	0.9816	0.9631
			$\hat{F}^{(1)}$	0.8302	0.8243	0.7636	0.5667	0.5797	0.5967	0.5190	0.5283	0.5495
			$\hat{F}^{(2)}$	0.8511	0.8471	0.7996	0.5872	0.6014	0.6207	0.5354	0.5455	0.5669
20	20	200	$\hat{G}^{(1)}$	0.8535	0.8522	0.8228	0.7211	0.7184	0.7137	0.7356	0.7334	0.7267
			$\hat{G}^{(2)}$	0.9877	0.9869	0.9781	0.9851	0.9836	0.9778	0.9887	0.9877	0.9836
			$\hat{F}^{(1)}$	0.8543	0.8522	0.8266	0.5657	0.5724	0.5865	0.5104	0.5156	0.5279
			$\hat{F}^{(2)}$	0.8721	0.8707	0.8532	0.5826	0.5903	0.6051	0.5237	0.5289	0.5409
20	100	100	$\hat{G}^{(1)}$	0.9709	0.9703	0.9479	0.9530	0.9490	0.8936	0.9077	0.8978	0.8345
			$\hat{G}^{(2)}$	0.9981	0.9972	0.9837	0.9976	0.9964	0.9801	0.9966	0.9948	0.9766
			$\hat{F}^{(1)}$	0.9314	0.9291	0.9168	0.9233	0.9192	0.8916	0.9065	0.8981	0.8500
			$\hat{F}^{(2)}$	0.9331	0.9309	0.9195	0.9263	0.9225	0.9007	0.9138	0.9069	0.8660
20	100	200	$\hat{G}^{(1)}$	0.9712	0.9710	0.9650	0.9590	0.9570	0.9387	0.9283	0.9239	0.8764
			$\hat{G}^{(2)}$	0.9983	0.9981	0.9951	0.9981	0.9978	0.9944	0.9978	0.9973	0.9931
			$\hat{F}^{(1)}$	0.9549	0.9542	0.9505	0.9487	0.9474	0.9392	0.9374	0.9342	0.9136
			$\hat{F}^{(2)}$	0.9561	0.9554	0.9519	0.9504	0.9492	0.9422	0.9408	0.9379	0.9219

Table 2: Variance Ratios of the Estimated Global Factors

Note: (i) Entry numbers denote the averages of the variance-ratio statistics for the estimates of the global factors across 5,000 iterations.
(ii) Data were generated by equation (14).
(iii) The parameters ρ and β set the idiosyncratic errors' degrees of serial and cross-sectional correlation, respectively.
(iv) The number of countries is M, the number of cross-sectional samples in country m is N_m, and the number of time-series samples is T.

Part I: $s = 1$

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	20	100	$\hat{G}^{(1)}$	0.2632	0.2680	0.2786	0.2629	0.2672	0.2792	0.2631	0.2663	0.2799
			$\hat{G}^{(2)}$	0.3456	0.3507	0.3689	0.4423	0.4433	0.4483	0.5515	0.5492	0.5396
5	20	200	$\hat{G}^{(1)}$	0.2613	0.2637	0.2693	0.2609	0.2622	0.2690	0.2607	0.2634	0.2698
			$\hat{G}^{(2)}$	0.3446	0.3478	0.3581	0.4397	0.4408	0.4432	0.5500	0.5483	0.5434
5	100	100	$\hat{G}^{(1)}$	0.2634	0.2669	0.2776	0.2623	0.2670	0.2792	0.2637	0.2671	0.2798
			$\hat{G}^{(2)}$	0.3176	0.3245	0.3435	0.3199	0.3269	0.3453	0.3219	0.3301	0.3495
5	100	200	$\hat{G}^{(1)}$	0.2611	0.2627	0.2700	0.2612	0.2628	0.2698	0.2610	0.2631	0.2697
			$\hat{G}^{(2)}$	0.3178	0.3213	0.3307	0.3190	0.3226	0.3326	0.3212	0.3251	0.3361
20	20	100	$\hat{G}^{(1)}$	0.2609	0.2653	0.2762	0.2622	0.2656	0.2778	0.2621	0.2665	0.2794
			$\hat{G}^{(2)}$	0.3448	0.3510	0.3685	0.4416	0.4431	0.4482	0.5513	0.5483	0.5392
20	20	200	$\hat{G}^{(1)}$	0.2595	0.2614	0.2674	0.2597	0.2619	0.2689	0.2603	0.2623	0.2693
			$\hat{G}^{(2)}$	0.3441	0.3475	0.3574	0.4396	0.4400	0.4422	0.5497	0.5482	0.5427
20	100	100	$\hat{G}^{(1)}$	0.2627	0.2666	0.2783	0.2620	0.2668	0.2792	0.2627	0.2667	0.2790
			$\hat{G}^{(2)}$	0.3177	0.3244	0.3429	0.3197	0.3265	0.3450	0.3224	0.3300	0.3499
20	100	200	$\hat{G}^{(1)}$	0.2605	0.2628	0.2690	0.2603	0.2624	0.2692	0.2606	0.2626	0.2694
			$\hat{G}^{(2)}$	0.3177	0.3209	0.3311	0.3191	0.3226	0.3328	0.3211	0.3251	0.3358

Part II: $s = 2$

M	N_m	T	ρ / β	0 / 0	0.5 / 0	0.85 / 0	0 / 0.2	0.5 / 0.2	0.85 / 0.2	0 / 0.5	0.5 / 0.5	0.85 / 0.5
5	20	100	$\hat{G}^{(1)}$	0.3100	0.3160	0.3270	0.2949	0.3006	0.3186	0.2937	0.3013	0.3203
			$\hat{G}^{(2)}$	0.3278	0.3327	0.3491	0.4213	0.4210	0.4215	0.5225	0.5169	0.5027
5	20	200	$\hat{G}^{(1)}$	0.3046	0.3081	0.3151	0.2898	0.2917	0.3020	0.2892	0.2913	0.3011
			$\hat{G}^{(2)}$	0.3287	0.3319	0.3406	0.4207	0.4210	0.4213	0.5234	0.5214	0.5131
5	100	100	$\hat{G}^{(1)}$	0.3097	0.3163	0.3326	0.3095	0.3168	0.3324	0.3097	0.3167	0.3316
			$\hat{G}^{(2)}$	0.3007	0.3062	0.3226	0.3026	0.3084	0.3248	0.3051	0.3118	0.3295
5	100	200	$\hat{G}^{(1)}$	0.3044	0.3081	0.3170	0.3047	0.3079	0.3166	0.3040	0.3081	0.3169
			$\hat{G}^{(2)}$	0.3023	0.3053	0.3141	0.3034	0.3068	0.3159	0.3057	0.3091	0.3186
20	20	100	$\hat{G}^{(1)}$	0.3038	0.3091	0.3205	0.2840	0.2896	0.3062	0.2845	0.2909	0.3068
			$\hat{G}^{(2)}$	0.3267	0.3327	0.3487	0.4255	0.4254	0.4267	0.5285	0.5237	0.5123
20	20	200	$\hat{G}^{(1)}$	0.2997	0.3024	0.3096	0.2798	0.2834	0.2903	0.2809	0.2835	0.2921
			$\hat{G}^{(2)}$	0.3279	0.3312	0.3397	0.4243	0.4243	0.4260	0.5282	0.5263	0.5196
20	100	100	$\hat{G}^{(1)}$	0.3085	0.3152	0.3319	0.3080	0.3156	0.3315	0.3088	0.3150	0.3300
			$\hat{G}^{(2)}$	0.3006	0.3067	0.3223	0.3025	0.3083	0.3249	0.3048	0.3119	0.3298
20	100	200	$\hat{G}^{(1)}$	0.3034	0.3071	0.3158	0.3034	0.3068	0.3155	0.3032	0.3070	0.3154
			$\hat{G}^{(2)}$	0.3021	0.3052	0.3140	0.3034	0.3068	0.3158	0.3057	0.3090	0.3189

Part III: $s = 3$

M	N_m	T	ρ / β	0 / 0	0.5 / 0	0.85 / 0	0 / 0.2	0.5 / 0.2	0.85 / 0.2	0 / 0.5	0.5 / 0.5	0.85 / 0.5
5	20	100	$\hat{G}^{(1)}$	0.3351	0.3422	0.3517	0.3213	0.3293	0.3503	0.3222	0.3306	0.3559
			$\hat{G}^{(2)}$	0.3179	0.3235	0.3392	0.4084	0.4060	0.4034	0.5034	0.4955	0.4761
5	20	200	$\hat{G}^{(1)}$	0.3267	0.3312	0.3390	0.3127	0.3165	0.3270	0.3129	0.3176	0.3294
			$\hat{G}^{(2)}$	0.3214	0.3237	0.3325	0.4114	0.4102	0.4090	0.5086	0.5041	0.4928
5	100	100	$\hat{G}^{(1)}$	0.3357	0.3444	0.3638	0.3350	0.3437	0.3613	0.3337	0.3430	0.3586
			$\hat{G}^{(2)}$	0.2906	0.2960	0.3109	0.2926	0.2987	0.3143	0.2959	0.3021	0.3193
5	100	200	$\hat{G}^{(1)}$	0.3264	0.3308	0.3421	0.3262	0.3307	0.3414	0.3258	0.3302	0.3398
			$\hat{G}^{(2)}$	0.2943	0.2971	0.3050	0.2956	0.2989	0.3075	0.2977	0.3016	0.3110
20	20	100	$\hat{G}^{(1)}$	0.3239	0.3314	0.3375	0.3051	0.3128	0.3301	0.3078	0.3150	0.3338
			$\hat{G}^{(2)}$	0.3176	0.3231	0.3413	0.4184	0.4168	0.4164	0.5161	0.5106	0.4965
20	20	200	$\hat{G}^{(1)}$	0.3169	0.3212	0.3269	0.2980	0.3013	0.3103	0.3007	0.3044	0.3133
			$\hat{G}^{(2)}$	0.3206	0.3235	0.3332	0.4194	0.4193	0.4203	0.5186	0.5159	0.5094
20	100	100	$\hat{G}^{(1)}$	0.3331	0.3432	0.3615	0.3328	0.3421	0.3582	0.3303	0.3393	0.3538
			$\hat{G}^{(2)}$	0.2908	0.2958	0.3111	0.2926	0.2984	0.3153	0.2963	0.3030	0.3219
20	100	200	$\hat{G}^{(1)}$	0.3246	0.3292	0.3400	0.3246	0.3286	0.3395	0.3239	0.3280	0.3363
			$\hat{G}^{(2)}$	0.2942	0.2971	0.3054	0.2952	0.2988	0.3073	0.2976	0.3014	0.3120

Table 3a: Estimation of the Numbers of Country Factors Using Raw Data

Note: (i) Entry numbers in Part I denote the averages of the estimated numbers of country factors across 1,000 iterations.
(ii) Entry numbers in Part II denote the root mean squared errors of the estimated numbers of country factors.
(iii) Data were generated by equation (14). The true number of global factor is set to be 1 and the numbers of country factors are assumed to be 2 for all countries.
(iv) The parameters ρ and β set the idiosyncratic errors' degrees of serial and cross-sectional correlation, respectively.
(v) The number of countries is M, the number of cross-sectional samples in country m is N_m, and the number of time-series observations is T.

Part I: Sample mean of the estimated numbers of country factors

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	100	100	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.999	2.000	2.037	2.010	2.162	2.987	2.997	2.999	3.000
			HQ_4	1.999	2.000	2.073	2.029	2.264	2.994	3.000	3.000	3.000
5	100	200	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	2.000	2.000	2.000	2.125	2.439	2.991	3.000	3.000	3.000
			HQ_4	2.000	2.000	2.000	2.486	2.794	3.000	3.000	3.000	3.000
5	20	100	IC_{p2}	1.993	1.996	2.997	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	0.147	0.201	0.365	2.073	2.141	2.418	2.992	2.993	2.997
			HQ_4	0.072	0.098	0.203	1.564	1.649	2.047	2.985	2.986	2.990
5	20	200	IC_{p2}	1.998	1.998	2.770	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	0.122	0.141	0.202	2.079	2.109	2.274	2.993	2.997	2.995
			HQ_4	0.097	0.116	0.160	1.917	1.948	2.158	2.991	2.995	2.994
20	100	100	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.998	1.999	2.035	2.010	2.168	2.989	2.998	2.999	3.000
			HQ_4	2.000	1.999	2.076	2.027	2.268	2.995	3.000	3.000	3.000
20	100	200	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	2.000	2.000	2.000	2.126	2.448	2.994	3.000	3.000	3.000
			HQ_4	2.000	2.000	2.000	2.497	2.799	2.999	3.000	3.000	3.000
20	20	100	IC_{p2}	1.994	1.998	2.996	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	0.152	0.194	0.364	2.050	2.131	2.411	2.992	2.992	2.995
			HQ_4	0.074	0.098	0.205	1.530	1.644	2.039	2.985	2.985	2.991
20	20	200	IC_{p2}	1.998	1.998	2.772	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	0.119	0.141	0.209	2.053	2.105	2.258	2.993	2.994	2.996
			HQ_4	0.096	0.111	0.172	1.900	1.959	2.129	2.992	2.992	2.995

Part II: Root mean squared errors of the estimated numbers of country factors

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	100	100	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.033	0.013	0.192	0.106	0.404	0.993	0.998	1.000	1.000
			HQ_4	0.025	0.006	0.269	0.170	0.515	0.997	1.000	1.000	1.000
5	100	200	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.000	0.000	0.000	0.353	0.662	0.995	1.000	1.000	1.000
			HQ_4	0.000	0.000	0.000	0.697	0.891	1.000	1.000	1.000	1.000
5	20	100	IC_{p2}	0.082	0.094	0.998	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	1.891	1.853	1.742	0.998	0.980	0.950	0.996	0.996	0.998
			HQ_4	1.946	1.928	1.855	1.203	1.179	1.062	0.993	0.993	0.995
5	20	200	IC_{p2}	0.035	0.048	0.878	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	1.911	1.898	1.855	1.006	1.003	0.986	0.997	0.998	0.997
			HQ_4	1.929	1.915	1.884	1.068	1.064	1.025	0.996	0.998	0.997
20	100	100	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.035	0.027	0.187	0.101	0.410	0.994	0.999	1.000	1.000
			HQ_4	0.014	0.016	0.275	0.166	0.518	0.998	1.000	1.000	1.000
20	100	200	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.000	0.000	0.000	0.354	0.669	0.997	1.000	1.000	1.000
			HQ_4	0.000	0.000	0.000	0.705	0.894	1.000	1.000	1.000	1.000
20	20	100	IC_{p2}	0.078	0.078	0.998	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	1.888	1.859	1.743	1.006	0.985	0.958	0.996	0.996	0.998
			HQ_4	1.945	1.928	1.854	1.217	1.179	1.073	0.993	0.993	0.996
20	20	200	IC_{p2}	0.044	0.039	0.879	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	1.913	1.897	1.850	1.024	1.004	0.988	0.997	0.997	0.998
			HQ_4	1.929	1.919	1.877	1.080	1.057	1.031	0.996	0.996	0.998

Table 3b: Estimation of the Numbers of Country Factors Using Data from Step 2

Note: (i) Entry numbers in Part I denote the averages of the estimated numbers of country factors across 1,000 iterations.
(ii) Entry numbers in Part II denote the root mean squared errors of the estimated numbers of country factors.
(iii) Data were generated by equation (14). The true number of global factor is set to be 1 and the numbers of country factors are assumed to be 2 for all countries.
(iv) The parameters ρ and β set the idiosyncratic errors' degrees of serial and cross-sectional correlation, respectively.
(v) The number of countries is M, the number of cross-sectional samples in country m is $N_{m,}$ and the number of time-series observations is T.

Part I: Mean of the estimated numbers of country factors

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	100	100	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.999	2.000	2.038	2.008	2.144	2.986	2.996	3.000	3.000
			HQ_4	1.999	2.000	2.073	2.023	2.244	2.993	2.999	3.000	3.000
5	100	200	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	2.000	2.000	2.000	2.100	2.397	2.990	3.000	3.000	3.000
			HQ_4	2.000	2.000	2.000	2.439	2.764	2.999	3.000	3.000	3.000
5	20	100	IC_{p2}	2.040	2.087	2.995	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.031	1.051	1.122	2.173	2.203	2.428	2.986	2.987	2.988
			HQ_4	1.013	1.025	1.066	1.874	1.930	2.170	2.970	2.973	2.981
5	20	200	IC_{p2}	2.057	2.075	2.795	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.033	1.037	1.066	2.141	2.174	2.314	2.983	2.990	2.990
			HQ_4	1.024	1.026	1.050	2.048	2.084	2.230	2.979	2.987	2.987
20	100	100	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.999	1.999	2.035	2.007	2.150	2.988	2.995	2.999	3.000
			HQ_4	1.999	2.000	2.077	2.022	2.241	2.995	2.999	3.000	3.000
20	100	200	IC_{p2}	2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	2.000	2.000	2.000	2.095	2.403	2.993	3.000	3.000	3.000
			HQ_4	2.000	2.000	2.000	2.432	2.761	2.999	3.000	3.000	3.000
20	20	100	IC_{p2}	2.054	2.117	2.995	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.032	1.047	1.115	2.116	2.175	2.398	2.982	2.983	2.988
			HQ_4	1.013	1.021	1.062	1.816	1.888	2.138	2.964	2.965	2.977
20	20	200	IC_{p2}	2.058	2.091	2.805	3.000	3.000	3.000	3.000	3.000	3.000
			BIC	1.024	1.032	1.059	2.108	2.137	2.267	2.986	2.987	2.989
			HQ_4	1.018	1.024	1.046	2.015	2.043	2.178	2.982	2.984	2.986

Part II: Root mean squared errors of the estimated numbers of country factors

			ρ	0	0.5	0.85	0	0.5	0.85	0	0.5	0.85
M	N_m	T	β	0	0	0	0.2	0.2	0.2	0.5	0.5	0.5
5	100	100	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.024	0.006	0.194	0.090	0.381	0.993	0.998	1.000	1.000
			HQ_4	0.022	0.006	0.270	0.148	0.495	0.996	0.999	1.000	1.000
5	100	200	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.000	0.000	0.000	0.315	0.630	0.995	1.000	1.000	1.000
			HQ_4	0.000	0.000	0.000	0.662	0.874	1.000	1.000	1.000	1.000
5	20	100	IC_{p2}	0.204	0.289	0.997	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.984	0.974	0.937	0.808	0.816	0.850	0.994	0.994	0.995
			HQ_4	0.994	0.987	0.966	0.805	0.816	0.831	0.987	0.988	0.992
5	20	200	IC_{p2}	0.219	0.256	0.892	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.984	0.981	0.966	0.812	0.822	0.839	0.992	0.995	0.995
			HQ_4	0.988	0.987	0.975	0.811	0.820	0.836	0.991	0.994	0.994
20	100	100	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.033	0.024	0.188	0.090	0.387	0.994	0.998	0.999	1.000
			HQ_4	0.018	0.015	0.277	0.148	0.491	0.998	1.000	1.000	1.000
20	100	200	IC_{p2}	0.000	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.000	0.000	0.000	0.308	0.635	0.996	1.000	1.000	1.000
			HQ_4	0.000	0.000	0.000	0.657	0.872	1.000	1.000	1.000	1.000
20	20	100	IC_{p2}	0.250	0.346	0.997	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.984	0.976	0.941	0.813	0.820	0.850	0.991	0.992	0.995
			HQ_4	0.993	0.989	0.968	0.822	0.820	0.836	0.984	0.985	0.990
20	20	200	IC_{p2}	0.240	0.300	0.897	1.000	1.000	1.000	1.000	1.000	1.000
			BIC	0.988	0.984	0.970	0.822	0.825	0.838	0.994	0.994	0.995
			HQ_4	0.991	0.988	0.976	0.820	0.824	0.834	0.992	0.993	0.994

Table 4: Data Sources

Variable	Sources	Series Name, Modification
U.S. Series		
Real GDP	BEA	A191RX1
Real Personal Income excluding Transfer Payments	BEA	(A065RC1-W211RC1)*RPI/A065RC1
Real Disposable Personal Income	BEA	A067RX1
Real Compensation of Employees	BEA	A576RC1Q027SBEA/CPIAUCSL
Real Private Final Consumption	BEA	BEA Account Code: DPCERX1
Real Private Investment	BEA	GPDIC96
Real Change in Inventories/Real GDP	BEA	CBIC96/GDPC1
Real GDP: Structures	BEA	BEA Account Code: A755RX1
Industrial Production Index	Fed Board	INDPRO
Nonfarm Payroll Employment	BLS	PAYEMS
Initial Unemployment Claims	BLS	ICSA
Civilian Unemployment Rate	BLS	UNRATE
Average Weekly Hours of Production and Nonsupervisory Employees: Mfg.	BLS	AWHMAN
Real Manufacturing and Trade Industries Sales	St. Louis Fed	CQRMTSPL
Total Retail Trade, Index	OECD	USASARTMISMEI
Real Exports of Goods and Services	BEA	EXPGSC1
Real Imports of Goods and Services	BEA	IMPGSC96
Canada Series		
Real GDP	DATA Stream	CNXGDPR.D
Real Disposable Personal Income	DATA Stream	CNXPEDY.D
Real Compensation of Employees	DATA Stream, OECD	CND14805/CANCPIALLMINMEI
Real Private Final Consumption	DATA Stream	CNCNNPI.D
Real Private Investment	DATA Stream	CNXIPNR.D
Real GDP: Nonresidential Structures	DATA Stream	CN100114
Real Change in Inventories/Real GDP	DATA Stream	CNINVCH.D/CNXGDPR.D
Industrial Production Index	DATA Stream	CNXIPI.AQ
All Employees	DATA Stream	CNQLFT12O
Initial & Renewal Claims Received	DATA Stream	CNUNIRCLP
Harmonized Unemployment Rate	OECD	CANURHARMQDSMEI
Real Sales of Total Mfg. Goods	DATA Stream, OECD	CNQSLI09B/CANCPIALLMINMEI
Real Total Retail Trade	DATA Stream, OECD	CNQSLI07B/CANCPIALLMINMEI
Real Exports of Goods and Services	DATA Stream	CNEXNGS.D
Real Imports of Goods and Services	DATA Stream	CNIMNGS.D
U.K. Series		
Real GDP	DATA Stream	UKXGDPR.D
Real Disposable Personal Income	DATA Stream	UKXPEDY.D
Real Wages and Salaries	DATA Stream	UKROYJ..B/CPI
Real Private Final Consumption	DATA Stream	UKOCFPCND
Real Investment, Private Sector Business	DATA Stream	UKXIPNR.D
Real Fixed Investment: Buildings and Structures	DATA Stream	UKESFISXD
Real Changes in Inventories/Real GDP	DATA Stream	UKCAFU..D/UKXGDPR.D
Industrial Production Index	DATA Stream	UKXIPI.%Q
Total Employment	DATA Stream	UKXEMPT%Q
Unemployment, Claimant Count	DATA Stream	UKUNPTOTO
Unemployment Rate	DATA Stream	UKUN%O16Q
Hours Worked per Employee	DATA Stream	UKOCFHRBO
Retail Trade, Volume Index	DATA Stream	UKQSLI15G
Real Exports of Goods and Services	DATA Stream	UKIKBK..D
Real Imports of Goods and Services	DATA Stream	UKIMNGS.D
Japan Series		
Real GDP	DATA Stream	JPOCFGDPD
Real Disposable Personal Income	DATA Stream	JPXPEDY.C
Real Wages and Salaries	DATA Stream	JPCOMEM0D
Private Final Consumption	DATA Stream	JPXCPR..D

Real Private Investment	DATA Stream	JPXIPNR.D
Real Fixed Investment:		
Buildings and Structures	DATA Stream	JPGCFOBSB
Real Changes in Inventories/Real GDP	DATA Stream	JPINVCH.B/JPGDP...B
Industrial Production Index	DATA Stream	JPXIPI.%Q
Total Employment	DATA Stream	JPQLFT12Q
Harmonized Unemployment Rate	DATA Stream	JPQLRT16Q
Hours Worked per Employee	DATA Stream	JPOCFHRBO
Retail Trade, Volume Index	DATA Stream	JPQSLI15Q
Real Exports of Goods and Services	DATA Stream	JPOCFEGSD
Real Imports of Goods and Services	DATA Stream	JPOCFIGSD
France Series		
Real GDP	DATA Stream	FRGDP...D
Real Disposable Personal Income	DATA Stream	FRXPEDYDC
Real Wages and Salaries	DATA Stream	FRESNCMYQ*FRGDP...D
Real Private Final Consumption	DATA Stream	FROCFPCND
Real Private Investment	DATA Stream	FRXIPNR.D
Real Fixed Investment:		
Other Buildings and Structures	DATA Stream	FRES9KOZD
Real Changes in Inventories/Real GDP	DATA Stream	FRESNINYD/FRXPEDYDC
Industrial Production Index	DATA Stream	FRQ66..CE
Total Employment	DATA Stream	FREMPTOTO
Harmonized Unemployment Rate	DATA Stream	FRQUN014Q
Hours Worked per Employee	DATA Stream	FRESOVPWP/FREMPTOTO
Retail Trade, Volume Index	DATA Stream	FRQSLI15G
Real Exports of Goods and Services	DATA Stream	FREXNGS.D
Real Imports of Goods and Services	DATA Stream	FRIMNGS.D
Germany Series		
Real GDP	DATA Stream	BDXGDPR.D
Real Disposable Personal Income	DATA Stream	BDXPEDY.C
Private Final Consumption Expenditure	DATA Stream	BDQNA035Q
Real Investment, Private Sector Business	DATA Stream	BDXIPNR.C
Industrial Production Index	DATA Stream	BDXIPI.%Q
Total Employment	DATA Stream	BDQLFT12Q
Harmonized Unemployment Rate	DATA Stream	BDQLRT28Q
Hours Worked per Employee in Manufacturing	DATA Stream	BDOCFHRBO
Retail Trade, Volume Index	DATA Stream	BDQSLI15Q
Real Exports of Goods and Services	DATA Stream	BDXXTR..D
Real Imports of Goods and Services	DATA Stream	BDXMTR..D
Italy Series		
Real GDP	DATA Stream	ITXGDPR.D
Real Disposable Personal Income	DATA Stream	ITXPEDY.C
Private Final Consumption Expenditure	DATA Stream	ITXCPR..D
Real Investment, Private Sector Business	DATA Stream	ITXIPNR.D
Real Changes in Inventories/Real GDP	DATA Stream	ITINVCH.B/ITGDP...B
Industrial Production Index	DATA Stream	ITQ66..CE
Total Employment	DATA Stream	ITXEMPT.O
Harmonized Unemployment Rate	DATA Stream	ITOCFUNRQ
Hours Worked per Employee	DATA Stream	ITOCFHRBO
Retail Sales, Volume Index	DATA Stream	ITXRSVOAR
Real Exports of Goods and Services	DATA Stream	ITOCFEGSD
Real Imports of Goods and Services	DATA Stream	ITOCFIGSD
Australia Series		
Real GDP	DATA Stream	AUGDP...D
Real Disposable Personal Income	DATA Stream	AUGRDPINA/AUCONPRCF
Compensation of Employee	DATA Stream	AUCMPEMPB/AUCONPRCF
Private Final Consumption	DATA Stream	AUCNPER.D
Real Private Investment	DATA Stream	AUPRFXCPD
Real Fixed Investment:		
Buildings and Structures	DATA Stream	AUCPOTHBD

Real Changes in Inventories/Real GDP	DATA Stream	AUINVCH.D/AUGDP...D
Industrial Production Index	DATA Stream	AUIPTOT.G
Total Employment	DATA Stream	AUEMPEMPQ
Harmonized Unemployment Rate	DATA Stream	AUQLRT16Q
Hours Worked per Employee	DATA Stream	AUOCFHRBO
Retail Trade, Volume Index	DATA Stream	AUGPRETTD
Real Exports of Goods and Services	DATA Stream	AUOEXP01D
Real Imports of Goods and Services	DATA Stream	AUOEXP05D
Austria Series		
Real GDP	DATA Stream	OEXGDPR.D
Real Disposable Personal Income	DATA Stream	OEXPEDY.C
Real Private Final Consumption	DATA Stream	OEOCFPCND
Real Private Investment	DATA Stream	OEOCFITID
Real Changes in Inventories/Real GDP	DATA Stream	OEI93I..A/OEI99B..A
Industrial Production Index	DATA Stream	OEXIPI.%Q
Harmonized Unemployment Rate	DATA Stream	OEQLRT28Q
Retail Trade, Volume Index	DATA Stream	OEQSLI15G
Real Exports of Goods and Services	DATA Stream	OEOCFEGSD
Real Imports of Goods and Services	DATA Stream	OEOCFIGSD
Belgium Series		
Real GDP	DATA Stream	BGES0IFLD
Private Final Consumption Expenditure	DATA Stream	BGESNHHLD
Gross Capital Formation	DATA Stream	BGOCFITID
Gross Fixed Capital Formation	DATA Stream	BGESNGCLD
Private Non-residential Fixed Investment	DATA Stream	BGOCFIBSD
Real Changes in Inventories/Real GDP	DATA Stream	BGINVCH.B/BGGDP...B
Industrial Production Index	DATA Stream	BGXIPI..E
Total Employment	DATA Stream	BGEMPTOTO
Unemployment Rate	DATA Stream	BGXUPNA.Q
Hours Worked per Employee	DATA Stream	BGOCFHRBO
Retail Trade, Volume	DATA Stream	BGQSLI15Q
Real Exports of Goods and Services	DATA Stream	BGESNEXLD
Real Imports of Goods and Services	DATA Stream	BGESNIMLD
Denmark Series		
Real GDP	DATA Stream	DKXGDPR.D
Real Disposable Personal Income	DATA Stream	DKXPEDY.C
Compensation of Employee	DATA Stream	DKESENTCA/DKQCP009F
Private Final Consumption Expenditure	DATA Stream	DKXCPR..D
Real Fixed Investment	DATA Stream	DKXIFR..D
Real Changes in Inventories/Real GDP	DATA Stream	DKI93I..A/DKESNGDLA
Industrial Production Index	DATA Stream	DKQ66..BH
Total Employment	DATA Stream	DKXEMPTAQ
Unemployment Rate	DATA Stream	DKOCFUNRQ
Hours Worked per Employee	DATA Stream	DKOCFHRBO
Total Retail Trade, Volume	DATA Stream	DKQSLI15G
Real Exports of Goods and Services	DATA Stream	DKXXTR..D
Real Imports of Goods and Services	DATA Stream	DKXMTR..D
Finland Series		
Real GDP	DATA Stream	FNGDP...D
Net National Disposable Income	DATA Stream	FNOREL10D
Private Final Consumption Expenditure	DATA Stream	FNCNPER.D
Real Investment, Private Sector Business	DATA Stream	FNXIPNR.D
Private Non-residential Fixed Investment	DATA Stream	FNOCFIBSD
Machinery and Equipment Investment	DATA Stream	FNESX4BKD
Real Fixed Investment:		
Buildings and Structures	DATA Stream	FNESLI3ED
Real Changes in Inventories/Real GDP	OECD	FNINVCH.B/FINGDPNQDSMEI
Industrial Production Index	DATA Stream	FNQ66..CE
Total Employment	DATA Stream	FNESENN%Q

Unemployment Rate	DATA Stream	FNOCFUNRQ
Hours Worked per Employee	DATA Stream	FNOCFHRBO
Retail Trade, Volume Index	DATA Stream	FNQSLI15G
Real Exports of Goods and Services	DATA Stream	FNEXNGS.D
Real Imports of Goods and Services	DATA Stream	FNIMNGS.D
Greece Series		
Real GDP	DATA Stream	GRXGDPR.D
Private Final Consumption Expenditure	DATA Stream	GRXCPR..D
Fixed Investment	DATA Stream	GRXIFR..D
Industrial Production Index	DATA Stream	GRQPRI35Q
Unemployment Rate	DATA Stream	GRXUPIL.Q
Hours Worked per Employee	DATA Stream	GROCFHRBO
Retail Trade, Volume Index	DATA Stream	GRQSLI15G
Real Exports of Goods and Services	DATA Stream	GRXXTR..D
Real Imports of Goods and Services	DATA Stream	GRXMTR..D
Iceland Series		
Real GDP	DATA Stream	ICOCFGDPD
Private Final Consumption Expenditure	DATA Stream	ICOCFPCND
Gross Capital Formation	DATA Stream	ICOCFITID
Fixed Investment	DATA Stream	ICOCFINVD
Total Employment	DATA Stream	ICOCFEMPO
Hours Worked per Employee	DATA Stream	ICOCFHRBO
Real Exports of Goods and Services	DATA Stream	ICOCFEGSD
Real Imports of Goods and Services	DATA Stream	ICOCFIGSD
Ireland Series		
Real GDP	DATA Stream	IRXGDPR.D
Private Final Consumption Expenditure	DATA Stream	IRXCPR..D
Fixed Investment	DATA Stream	IRXIFR..D
Stockbuilding/GDP	DATA Stream	IRXISR%.R
Industrial Production Index	DATA Stream	IRQPRI35G
Total Employment	DATA Stream	IRXEMPT.O
Unemployment Rate	DATA Stream	IRXUPNA.Q
Hours Worked per Employee	OECD	IRMNHOURP
Retail Trade, Volume Index	DATA Stream	IRQSLI15G
Real Exports of Goods and Services	DATA Stream	IRXX$LY.C
Real Imports of Goods and Services	DATA Stream	IRXM$LY.C
Korea Series		
Real GDP	DATA Stream	KOGDP...D
Real Disposable Personal Income	DATA Stream	KOXPEDY.C
Real Private Final Consumption	DATA Stream	KOCNPER.D
Real Private Investment	DATA Stream	KOXIPNR.D
Fixed Investment	DATA Stream	KOGFCF..D
Real Change in Inventories/Real GDP	DATA Stream	KOINVCH.D
Industrial Production Index	DATA Stream	KOQ66..CE
Total Employment	DATA Stream	KOOCFEMPO
Unemployment Rate	DATA Stream	KOXUPNA.R
Hours Worked per Employee	DATA Stream	KOOCFHRBO
Real Wholesale and Retail Trade	DATA Stream	KOGDPWHLD
Real Exports of Goods and Services	DATA Stream	KOEXNGS.D
Real Imports of Goods and Services	DATA Stream	KOIMNGS.D
Mexico Series		
Real GDP	DATA Stream	MXXGDPR.D
Real Disposable Personal Income	DATA Stream	MXXPEDY.C
Real Private Final Consumption	DATA Stream	MXOCFPCND
Total Investment	DATA Stream	MXOCFITID
Fixed Investment	DATA Stream	MXOCFINVD
Real Change in Inventories/Real GDP	DATA Stream	MXXSTCK.C/MXXGDPR.D
Industrial Production Index	DATA Stream	MXXIPI..E
Total Employment	DATA Stream	MXXEMPT.P
Unemployment Rate	DATA Stream	MXXUPNA.Q
Real Exports of Goods and Services	DATA Stream	MXOCFEGSD
Real Imports of Goods and Services	DATA Stream	MXOCFIGSD

Netherlands Series		
Real GDP	DATA Stream	NLXGDPR.D
Real Disposable Personal Income	DATA Stream	NLXPEDY.C
Compensation of Employee	DATA Stream	NLOCFCEMB/NLXCPI..F
Real Private Final Consumption	DATA Stream	NLOCFPCND
Total Fixed Investment	DATA Stream	NLXIFR..D
Real Fixed Investment:		
Buildings and Structures	DATA Stream	NLESK3SSC
Real Change in Inventories/Real GDP	DATA Stream	NLINVC95C/NLXGDPE.C
Industrial Production Index	DATA Stream	NLQ66..CE
Total Employment	DATA Stream	NLXEMPT%Q
Unemployment Rate	DATA Stream	NLXUPNA.Q
Hours Worked per Employee	DATA Stream	NLOCFHRBO
Total Retail Trade, Volume	DATA Stream	NLQSLI15Q
Real Exports of Goods and Services	DATA Stream	NLOCFEGSD
Real Imports of Goods and Services	DATA Stream	NLOCFIGSD
New Zealand Series		
Real GDP	DATA Stream	NZOCFGDPD
Real Private Final Consumption	DATA Stream	NZOCFPCND
Gross Capital Formation	DATA Stream	NZOCFITID
Real Gross Fixed Capital Formation	DATA Stream	NZOCFINVD
Private Non-residential Gross Fixed Capital Formation	DATA Stream	NZOCFIBSD
Industrial Production Index exclu. Construction	DATA Stream	NZQPRI35Q
Total Employment	DATA Stream	NZOCFEMPO
Unemployment Rate	DATA Stream	NZOCFUNRQ
Hours Worked per Employee	DATA Stream	NZOCFHRBO
Total Manufacturing Sales	DATA Stream	NZQSLI09A/NZOCFDGDE
Total Retail Trade, Volume	DATA Stream	NZOSLI15B
Real Exports of Goods and Services	DATA Stream	NZOCFEGSD
Real Imports of Goods and Services	DATA Stream	NZOCFIGSD
Norway Series		
Real GDP	DATA Stream	NWGDP...D
Real Disposable Personal Income	DATA Stream	NWXPEDY.C
Real Private Final Consumption	DATA Stream	NWCNPER.D
Gross Capital Formation	DATA Stream	NWGFCF..D
Gross Fixed Capital Formation	DATA Stream	NWXIFR..D
Private Non-residential Gross Fixed Capital Formation	DATA Stream	NWOCFIOBB
Real Change in Inventories/Real GDP	DATA Stream	NWINVCH.D/NWGDP...D
Industrial Production Index	DATA Stream	NWXIPI..E
Total Employment	DATA Stream	NWXEMPT%Q
Unemployment Rate	DATA Stream	NWOCFUNRQ
Hours Worked per Employee	DATA Stream	NWOCFHRBO
Total Retail Trade, Volume	DATA Stream	NWQSLI15G
Real Exports of Goods and Services	DATA Stream	NWEXNGS.D
Real Imports of Goods and Services	DATA Stream	NWIMNGS.D
Portugal Series		
Real GDP	DATA Stream	PTOCFGDPD
Real Disposable Personal Income	DATA Stream	PTXPED.%Q-ln(PTOCFGDPD)
Real Private Final Consumption	DATA Stream	PTOCFPCND
Gross Capital Formation	DATA Stream	PTOCFITID
Gross Fixed Capital Formation	DATA Stream	PTOCFINVD
Real Change in Inventories/Real GDP	DATA Stream	PTESENMTB/PTQ99BWCB
Industrial Production Index	DATA Stream	PTXIPI.%Q
Unemployment Rate	DATA Stream	PTOCFUNRQ
Real Exports of Goods and Services	DATA Stream	PTOCFEGSD
Real Imports of Goods and Services	DATA Stream	PTOCFIGSD

Spain Series		
Real GDP	DATA Stream	ESOCFGDPD
Real Disposable Personal Income	DATA Stream	ESXPEDY.C
Compensation of Employees	DATA Stream	ESESENTCB/ESQCP009F
Real Private Final Consumption	DATA Stream	ESOCFPCND
Real investment, Private Sector Business	DATA Stream	ESXIPNR.C
Gross Capital Formation	DATA Stream	ESOCFITID
Gross Fixed Capital Formation	DATA Stream	ESOCFINVD
Real Change in Inventories/Real GDP	DATA Stream	ESESNINYQ
Industrial Production Index	DATA Stream	ESQ66..CE
Total Employment	DATA Stream	ESEMPTOTO
Unemployment Rate	DATA Stream	ESOCFUNRQ
Hours Worked per Employee	DATA Stream	ESOCFHRBO
Retail Sales, Volume	DATA Stream	ESXRSVO%R
Real Exports of Goods and Services	DATA Stream	ESOCFEGSD
Real Imports of Goods and Services	DATA Stream	ESOCFIGSD
Sweden Series		
Real GDP	DATA Stream	SDOCFGDPD
Real Disposable Personal Income	DATA Stream	SDXPEDY.D
Compensation of Employees	DATA Stream	SDOCFCEMB/SDQCP009F
Real Private Final Consumption	DATA Stream	SDOCFPCND
Real investment, Private Sector Business	DATA Stream	SDXIPNR.C
Gross Capital Formation	DATA Stream	SDOCFITID
Gross Fixed Capital Formation	DATA Stream	SDOCFINVD
Real Change in Inventories/Real GDP	DATA Stream	SDESNINYR
Industrial Production Index	DATA Stream	SDXIPI.%Q
Total Employment	DATA Stream	SDOCFEMPO
Unemployment Rate	DATA Stream	SDOCFUNRQ
Hours Worked per Employee	DATA Stream	SDOCFHRBO
Total Retail Trade, Volume	DATA Stream	SDQSLI15G
Real Exports of Goods and Services	DATA Stream	SDOCFEGSD
Real Imports of Goods and Services	DATA Stream	SDOCFIGSD
Switzerland Series		
Real GDP	DATA Stream	SWGDP...D
Real Disposable Personal Income	DATA Stream	SWXPEDY.D
Real Private Final Consumption	DATA Stream	SWCNPER.D
Real investment, Private Sector Business	DATA Stream	SWXIPNRAQ
Gross Capital Formation	DATA Stream	SWOCFITID
Gross Fixed Capital Formation	DATA Stream	SWGFCF..D
Industrial Production Index	DATA Stream	SWQ66..BH
Total Employment	DATA Stream	SWKY3939F
Unemployment Rate	DATA Stream	SWOCFUNRQ
Hours Worked per Employee	DATA Stream	SWOCFHRBO
Total Retail Trade, Volume	DATA Stream	SWQSLI15G
Real Exports of Goods and Services	DATA Stream	SWEXNGS.D
Real Imports of Goods and Services	DATA Stream	SWIMNGS.D
Turkey Series		
Real GDP	DATA Stream	TKXGDSA.D
Real Disposable Personal Income	DATA Stream	TKXPED..A/TKQCP009F
Industrial Production Index	DATA Stream	TKQ66..BH
Total Retail Trade, Volume	DATA Stream	TKXRSVO.H
Real Exports of Goods and Services	DATA Stream	TKXXGSA.D
Real Imports of Goods and Services	DATA Stream	TKXMGSA.D

Table 5: Selection of Country Factors

Country	IC_{p2}	BIC	HQ_4
U.S.			
1981-1990	3	3	3
1991-2000	3	3	3
2001-2013	3	2	2
1981-2013	3	2	2
Canada			
1981-1990	3	2	2
1991-2000	3	3	1
2001-2013	3	3	2
1981-2013	3	2	2
U.K.			
1981-1990	3	1	1
1991-2000	3	3	2
2001-2013	3	1	1
1981-2013	3	1	1
Japan			
1981-1990	3	3	3
1991-2000	3	3	3
2001-2013	3	2	2
1981-2013	3	3	3
France			
1981-1990	3	3	3
1991-2000	3	2	2
2001-2013	3	2	2
1981-2013	3	2	2
Germany			
1981-1990	3	2	1
1991-2000	3	1	1
2001-2013	3	1	1
1981-2013	3	2	2
Korea			
1981-1990	3	3	3
1991-2000	3	3	3
2001-2013	3	3	3
1981-2013	3	2	2
Mexico			
1981-1990	3	3	3
1991-2000	3	3	3
2001-2013	3	3	3
1981-2013	3	3	3

Note: The maximum number of country factors is set to be three.

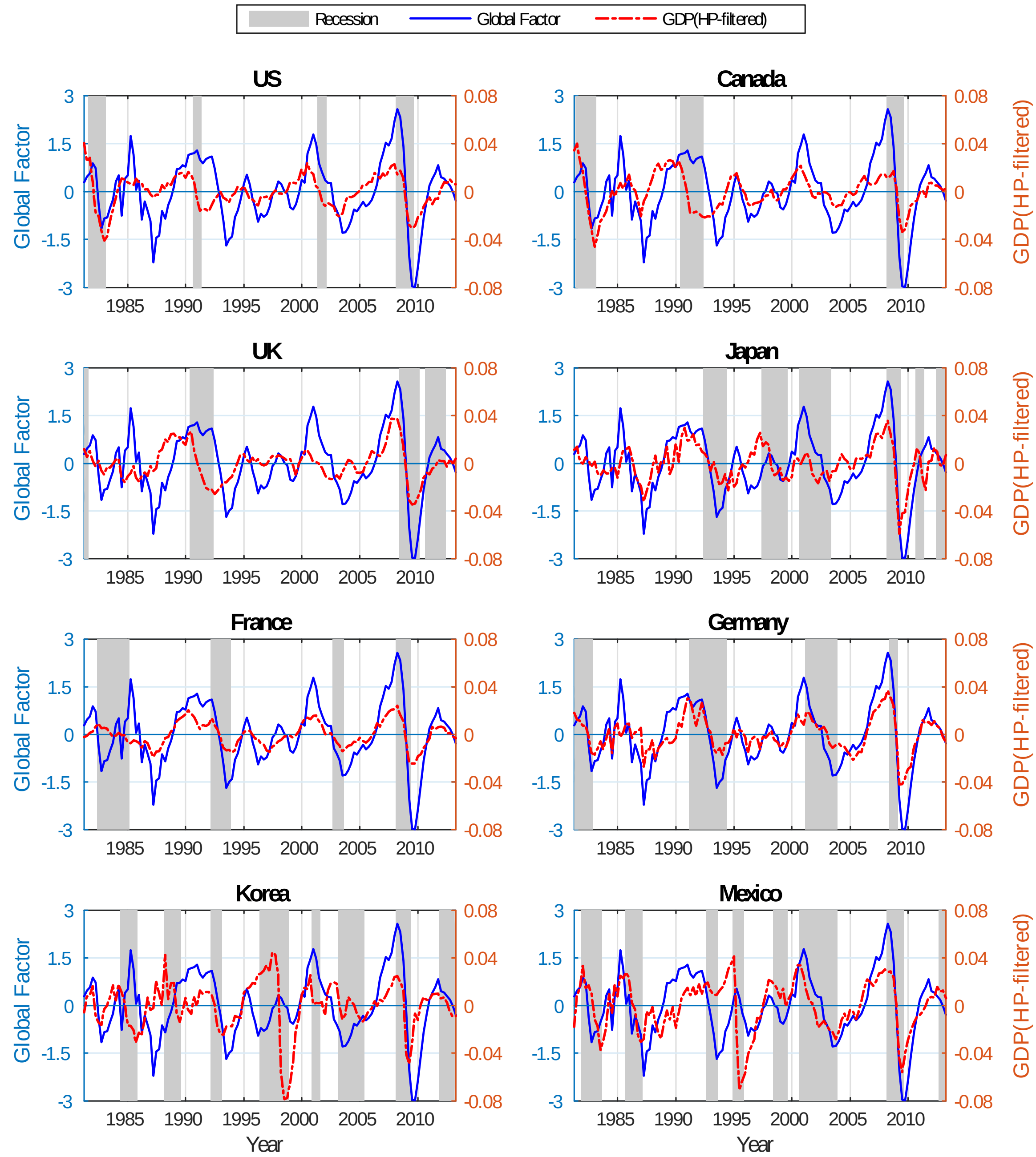

Figure 1. Global Factor and GDP Fluctuations in G-7 Countries
Note: Blue line is the extracted global factor and corresponds to the left axis while red lines are the HP-filtered GDP fluctuations in each of the G-7 countries and correspond to the right vertical axis. The shaded areas indicate recession periods for each country.

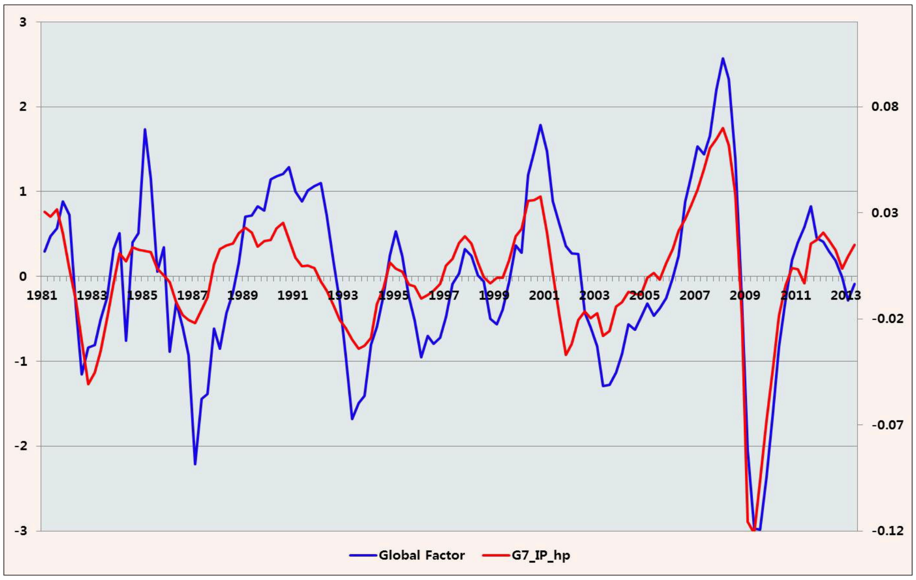

Figure 2. Global Factor and Industrial Production Index of G-7 Countries

Note: Blue line is the extracted global factor and corresponds to the left axis while red line is the HP-filtered Industrial Production Index of the G-7 Countries (G7_IP_hp) and corresponds to the right vertical axis. The G-7 Industrial Production Index was taken from the OECD website (http://stats.oecd.org) and detrended with the HP-filter.

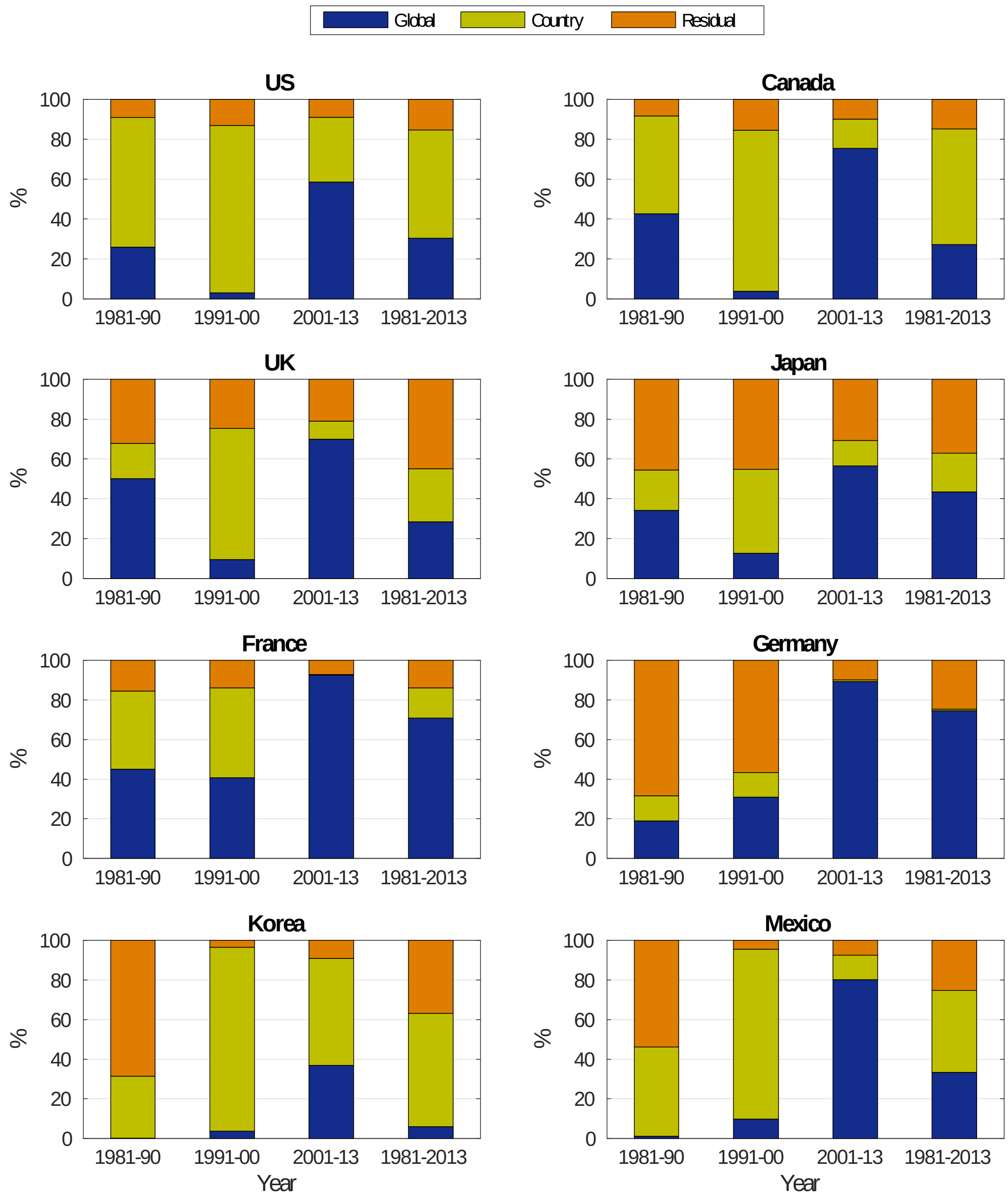

Figure 3. Relative Importance of Global Factor, Country Factors, and Residual Components in GDP Fluctuations over Time

Appendix: Proofs

Lemma A.1 $\mu_1 = \cdots = \mu_s = 1$, $\mu_{s+1} = \cdots = \mu_{s+r_1} = 0$ *and* $q_{1j} = (0, \ldots, \overset{j-th}{1}, \ldots, 0)'$ $(j = 1, \ldots, s + r_1)$.

Proof. Suppose $r_1 \leq r_2$. Since $\{G_t\}$, $\{F_{1t}\}$ and $\{F_{2t}\}$ are uncorrelated, we have

$$\begin{aligned} \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \lambda\Sigma_{11} &= \begin{bmatrix} \Sigma_{GG} & 0 \\ 0 & 0 \end{bmatrix} - \mu \begin{bmatrix} \Sigma_{GG} & 0 \\ 0 & \Sigma_{F_1} \end{bmatrix} \\ &= \begin{bmatrix} (1-\mu)\,\Sigma_{GG} & 0 \\ 0 & -\mu\Sigma_{F_1} \end{bmatrix}, \end{aligned}$$

where $\Sigma_{GG} = E(G_t G_t')$ for any arbitrary t. Thus, the solutions of the determinant equation $\left|\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \mu\Sigma_{11}\right| = 0$ are $\mu = 1$ (of multiplicity s) and $\mu = 0$ (of multiplicity r_1). If $r_1 > r_2$, we have $\mu_{s+1} = \cdots = \mu_{s+r_2} = 0$ for the same reason as above and $\mu_{s+r_2+1} = \cdots = \mu_{s+r_1}$ by construction (cf. Anderson, 2003, Subsection 12.2). The eigenvectors are trivially $q_{1j} = (0, \ldots, \overset{j\text{-th}}{1}, \ldots, 0)'$. ■

Proof of Proposition 2: (i) Since $\hat{p}_j = L_1^{-1}\hat{q}_{1j}$ and $\hat{K}_{1t} - L_1' K_{1t} \overset{p}{\longrightarrow} 0$ for each t, we obtain $\hat{G}_{tj}^{(1)} = \hat{p}_j'\hat{K}_{1t} \overset{p}{\longrightarrow} q_{1j}L_{1\infty}'^{-1}L_{1\infty}'K_{1t} = \pm G_{tj}$ as required, where $L_{1\infty} = p\lim_{N,T\to\infty} L_1$ and G_{tj} is the j-th element of G_t.
(ii) This follows in the same manner as in part (i).
(iii) Assume without loss of generality that $\hat{q}_{1j} \overset{p}{\longrightarrow} q_j$. Write

$$\begin{aligned} \hat{G}_{tj}^{(1)} - G_{tj} &= \hat{q}_{1j}' L_1^{-1\prime}\hat{K}_{1t} - q_j' K_{1t} \\ &= (\hat{q}_{1j} - q_j)'\, L_1^{-1\prime}\hat{K}_{1t} + q_j' L_1^{-1\prime}(\hat{K}_{1t} - L_1' K_{1t}). \end{aligned} \tag{A.1}$$

Since $S_{11}^{-1}S_{12}S_{22}^{-1}S_{21} - \Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} = O_p(\frac{1}{\sqrt{T}})$ under Assumption 1, $\hat{q}_{1j} - q_j = O_p\left(\frac{1}{\sqrt{T}}\right)$ (see Anderson, 2003, Chapter 13, for related techniques). The second term in equation (A.1) is either $O_p\left(\frac{1}{\sqrt{N_1}}\right)$ or $O_p\left(\frac{1}{T}\right)$ (cf. Bai, 2003). Thus, $\hat{G}_{tj}^{(1)} - G_{tj} = O_p\left(\frac{1}{\min\{\sqrt{N_1}, \sqrt{T}\}}\right)$ as required.
(iv) Use the same arguments as in part (iii). ■

Proof of Proposition 3: (i) (a) Multiplying $M_{\hat{G}^{(1)}}$ on both sides of model (3), we obtain

$$\begin{aligned} X_{mt}^G &= \Lambda_m F_{mt}^G + e_{mt} - e_m'\hat{G}^{(1)}\left(\hat{G}^{(1)\prime}\hat{G}^{(1)}\right)^{-1}\hat{G}_t^{(1)} - \Gamma_m Q_s^{-1}(\hat{G}_t^{(1)} - Q_s G_t) \\ &\quad + \Gamma_m Q_s^{-1}\left(\hat{G}^{(1)} - GQ_s\right)'\hat{G}^{(1)}\left(\hat{G}^{(1)\prime}\hat{G}^{(1)}\right)^{-1}\hat{G}_t^{(1)} \\ &= \Lambda_m F_{mt}^G + e_{mt} + a_{mtT}, \text{ say.} \end{aligned}$$

The only differences between this model and the one in Bai (2003) are the presence of the term a_{mtT}, which is $O_p\left(\frac{1}{\min\{\sqrt{N_1},\sqrt{T}\}}\right)$, and the use of F^G_{mt} instead of F_{mt}. Since $\frac{1}{N_m}e'_{ms}a_{mtT} = O_p\left(\frac{1}{\min\{\sqrt{N_1},\sqrt{T}\}}\right)$ for any m, s and t, a_{mtT} has no effects on Lemmas A.1, A.2 and A.3 of Bai under Assumption 4 (iv) except part (b) of Lemma A.2. The term $\frac{1}{N_m}e'_{ms}a_{mtT}$ affects part (b) of Lemma A.2 such that the term there becomes $O_p\left(\frac{1}{\min\{N_m,T\}}\right)$. However, because this does not affect the proof of Theorem 1 of Bai, it holds without any changes.

(b) The second terms of Lemmas B.1 and B.2 of Bai (2003) are affected by the presence of a_{mtT}, but the main results of Lemmas B.1 and B.2 remain intact. Thus, Theorem 2 of Bai holds without any changes.

(c) This follows from parts (a) and (b) as in Bai (2003). ■

Proof of Proposition 4: This follows from Bai (2003) with some minor changes as in the proof of Proposition 3. The details are omitted. ■

Proof of Proposition 5: This is omitted since it is similar to those of Propositions 3 and 4. ■

Proof of Proposition 6: Since $\hat{G}^{(1)}_t$ is assumed to satisfy Proposition 2, it can also be shown that

$$\kappa^2_{N_mT}\left(\frac{1}{T}\sum_{t=1}^{T}\left\|\hat{G}^{(1)}_t - Q'_sG_t\right\|^2\right) = O_p(1) \tag{A.2}$$

for some matrix Q_s.

Let $\hat{F}^{(1)}_{k_m}$ be the eigenvectors of $(N_mT)^{-1}X^G_mX^{G\prime}_m$ corresponding to the k_m largest eigenvalues with the usual normalization $T^{-1}\hat{F}^{(1)\prime}_{k_m}\hat{F}^{(1)}_{k_m} = I_{k_m}$. To use the results in Bai and Ng (2002), re-normalize $\hat{F}^{(1)}_{k_m}$ to get $\tilde{F}^{(1)}_{k_m} = N^{-1}_mX^G_m\hat{\Lambda}^{(1)}_m$ where $\hat{\Lambda}^{(1)\prime}_{k_m} = T^{-1}\hat{F}^{(1)\prime}_{k_m}X^G_m$. Then, $\tilde{F}^{(1)}_{k_m} = \hat{F}^{(1)}_{k_m}V_m$ where V_m is a diagonal matrix of the eigenvalues. While each column of $\tilde{F}^{(1)}_{k_m}$ is an eigenvector of $(N_mT)^{-1}X^G_mX^{G\prime}_m$ corresponding to a strictly positive eigenvalue, each column of $\hat{G}^{(1)}$ is an eigenvector of the same matrix corresponding to zero eigenvalue. Hence, we have $\hat{G}^{(1)\prime}\tilde{F}^{(1)}_{k_m} = 0$.

Lemma A.2 *Suppose that $\hat{G}^{(1)}$ satisfies (A.2). Then, there exists an $r_m \times k_m$ matrix D_m such that*

$$\frac{1}{T}\sum_{t=1}^{T}\left\|\tilde{F}^{(1)}_{k_mt} - D'_mF_{mt}\right\|^2 = O_p(\kappa^{-2}_{N_mT}),$$

where $||D_m|| = O_p(1)$.

The proof of Lemma A.2 is provided later. Based on the above lemma, the following results hold.

Lemma A.3 *For any k_m, $1 \leq k_m \leq r_m$, and D_m as in Lemma A.2,*

$$V_m(k_m, \tilde{F}^{(1)}_{k_m}) - V_m(k_m, F_m D_m) = o_p(1).$$

Lemma A.4 *For any matrix D_m defined in Lemma A.2, and for each k_m with $k_m < r_m$, there exists a $\tau_{k_m} > 0$ such that*

$$p \lim_{N_m, T} \inf V_m(k_m, F_m D_m) - V_m(r_m, F_m) = \tau_{k_m}.$$

Lemma A.5 *For any fixed k_m with $k_m \geq r_m$,*

$$V_m(k_m, \tilde{F}^{(1)}_{k_m}) - V_m(r_m, \tilde{F}^{(1)}_{r_m}) = O_p(\kappa^{-2}_{N_m T}).$$

Lemmas A.3$\sim$A.5 correspond to Lemmas 2, 3, and 4 in Bai and Ng (2002) respectively. Hence, the consistency of the information criteria follows from the same argument in Bai and Ng (2002). The proofs for Lemmas A.3$\sim$A.5 are available from the authors upon request. ■

Proof of Lemma A.2: Write

$$\begin{aligned} \tilde{F}^{(1)}_{k_m} &= \frac{1}{N_m T} M_{\hat{G}} X_m X'_m M_{\hat{G}^{(1)}} \hat{F}^{(1)}_{k_m} \\ &= M_{\hat{G}^{(1)}} K \left(\frac{1}{N_m} \Theta'_m \Theta_m \right) \left(\frac{1}{T} K'_m \hat{F}^{(1)}_{k_m} \right) + s_m \end{aligned}$$

with

$$s_m = \frac{1}{N_m T} M_{\hat{G}} \left(e_m \Theta_m K'_m + K_m \Theta'_m e'_m + e_m e'_m \right) M_{\hat{G}^{(1)}} \hat{F}^{(1)}_{k_m}.$$

Let

$$\left(\frac{1}{N_m} \Theta'_m \Theta_m \right) \left(\frac{1}{T} K'_m \hat{F}^{(1)}_{k_m} \right) = \begin{pmatrix} L_m \\ D_m \end{pmatrix},$$

where $||D_m|| = O_p(1)$ and $||L_m|| = O_p(1)$. Using this partition,

$$\begin{aligned} \tilde{F}^{(1)}_{k_m} &= M_{\hat{G}^{(1)}} (G, F_m) \begin{pmatrix} L_m \\ D_m \end{pmatrix} + s_m \\ &= M_{\hat{G}^{(1)}} G L_m + M_{\hat{G}^{(1)}} F_m D_m + s_m \\ &= F_m D_m + M_{\hat{G}^{(1)}} G L_m - P_{\hat{G}^{(1)}} F_m D_m + s_m. \end{aligned} \qquad \text{(A.3)}$$

The second term in (A.3) satisfied the relation

$$\begin{aligned}
\frac{1}{T}tr\left[L_m'G'\hat{M}_G G L_m\right] &= tr\left[L_m'\left(\frac{1}{T}(G-\hat{G}^{(1)}Q_s^{-1})'M_{\hat{G}}(G-\hat{G}^{(1)}Q_s^{-1})\right)L_m\right] \\
&\leq tr\left[L_m'Q_s^{-1\prime}\left(\frac{1}{T}(GQ_s-\hat{G}^{(1)})'(GQ_s-\hat{G}^{(1)})\right)Q_s^{-1}L_m\right] \\
&\leq tr\left[\frac{1}{T}(GQ_s-\hat{G}^{(1)})'(GQ_s-\hat{G}^{(1)})\right]tr\left[Q_s^{-1}L_mL_m'Q_s^{-1\prime}\right] \\
&= O_p(\kappa_{N_mT}^{-2}). \qquad (A.4)
\end{aligned}$$

For the third term in (A.3), we have

$$\begin{aligned}
\frac{1}{T}tr\left[D_m'F'P_{\hat{G}^{(1)}}FD_m\right] &= tr\left[D_m'\left(\frac{1}{T}F'\hat{G}\right)\left(\frac{1}{T}\hat{G}'\hat{G}\right)^{-1}\left(\frac{1}{T}\hat{G}'F\right)D_m\right] \\
&= O_p(\kappa_{N_mT}^{-2}), \qquad (A.5)
\end{aligned}$$

which holds because $||D_m|| = O_p(1)$, $(T^{-1}\hat{G}^{(1)\prime}\hat{G}^{(1)})^{-1} = O_p(1)$, and

$$\begin{aligned}
\frac{1}{T}\hat{G}'F_m &= \frac{1}{T}(\hat{G}^{(1)}-GQ_s)'F_m + \frac{1}{T}Q_s'G'F_m \\
&= O_p(\kappa_{N_mT}^{-2}) + O_p(T^{-1/2}).
\end{aligned}$$

For the last term in (A.3), Bai and Ng (2002) provide

$$\frac{1}{T}tr\left[s_m's_m\right] = O_p(\kappa_{N_mT}^{-2}). \qquad (A.6)$$

Combining (A.4)∼(A.6), we obtain

$$\begin{aligned}
&\frac{1}{T}\sum_{t=1}^{T}\left\|\tilde{F}_{k_mt}^{(1)} - D_m'F_{mt}\right\|^2 \\
=\ & \frac{1}{T}tr\left[(\hat{F}-FD_m)'(\hat{F}-FD_m)\right] \\
\leq\ & \frac{4}{T}tr\left[L_m'G'M_{\hat{G}^{(1)}}GL_m\right] + \frac{4}{T}tr\left[D_m'F_m'P_{\hat{G}^{(1)}}F_mD_m\right] + \frac{4}{T}tr\left[s_m's_m\right] = O_p(\kappa_{N_mT}^{-2}). \blacksquare
\end{aligned}$$