# Model Selection for Factor Analysis: Some New Criteria and Performance Comparisons

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**Abstract**

This paper derives Akaike's (1973) Akaike information criterion (AIC), Hurvich and Tsai's (1989) corrected AIC, the Bayesian information criterion (BIC) of Akaike (1978) and Schwarz (1978), and Hannan and Quinn's (1979) information criterion for factor models and studies the consistency properties of these information criteria. It also reports extensive simulation results comparing the performance of the extant and new procedures for the selection of the number of factors. The data generating process for the simulation consists of serially correlated factors and serially and cross-sectionally correlated idiosyncratic errors. The idiosyncratic errors are either homoskedastic or heteroskedastic. Idiosyncratic errors with fat tails and those with outliers having a much larger variance than the rest of the errors are also considered. The simulation results show the difficulty of determining which criterion performs best. In practice, it is advisable to consider several criteria at the same time, especially BIC, Hannan and Quinn's information criterion, Bai and Ng's (2002) $IC_{p2}$ and $BIC_3$, and Onatski's (2010) and Ahn and Horenstein's (2009) eigenvalue-based criteria. The model-selection criteria considered in this paper are also applied to Stock and Watson's (2002, 2005) data sets. The results differ considerably depending on the model-selection

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criterion in use, but evidence suggesting four factors for Stock and Watson's (2002) data and six or seven factors for Stock and Watson's (2005) is obtainable.

**Keywords**: Factor model, Akaike information criterion, corrected Akaike information criterion, Bayesian information criterion, Hannan and Quinn's (1979) information criterion

# 1 Introduction

Factor models have become popular in economics and finance because they can effectively extract information from data sets. They have been used for various purposes. First, economic indicators have been constructed from factor models. Examples are Altissimo *et al.* (2001), Cristadoro *et al.* (2005) and Kapetanios (2004). Second, factor models have been widely used for the purpose of forecasting economic variables. Papers belonging to this category include Artis *et al.* (2005), Banerjee and Marcellino (2006), Banerjee *et al.* (2005), Camba-Mendez and Kapetanios (2005), den Reijer (2005), Forni *et al.* (2000, 2003), Giacomini and White (2006), Huang *et al.* (2006), Ludvigson and Ng (2007), Marcellino *et al.* (2003), Schumacher and Dreger (2004), Schumacher (2007) and Stock and Watson (1999, 2002). According to these works, factor models may provide more accurate forecasts than autoregressive and vector-autoregressive models. Third, factor models can facilitate policy analysis. Bernanke *et al.* (2005), Favero *et al.* (2005), Forni *et al.* (2009), Giannone *et al.* (2002, 2005) and Sala (2003) demonstrate how factor models are used effectively to study monetary policy. Fourth, factor models are used for instrumental-variables estimation. Bai and Ng (2010) assume endogenous regressors to be driven by a small number of unobserved, exogenous factors and the estimated factors are used as instruments. Last, cross-sectional correlation has been modelled by factor models in panel data analysis. Examples of this approach include Bai and Ng (2004), Moon and Perron (2004), Phillips and Sul (2003), and Pesaran (2006). Breitung and Choi (2012) review the literature on factor models and provide additional references.

An important issue when using factor models is the selection of the number of static factors. A few methods have already been proposed and used in applications. The simplest method is to select the number of factors from the "scree plot" – the plot of the ordered eigenvalues – as in Cattell (1966). Related procedures are suggested by Onatski (2009) and Onatski (2010) using the slope of the scree plot and the difference of ordered eigenvalues, respectively. In addition to these, Ahn and Horenstein (2009) consider maximizing the ratio of successive eigenvalues or their growth ratio. Information criteria have also been used to select the number of factors. Bai and Ng (2002) propose several information criteria for the number of factors in approximate factor models and show them to be consistent. The relationship between the information criteria and those based on eigenvalues is explored in Ahn and Horenstein (2009). Once the number of static factors is determined, the number of dynamic (or primitive) factors can be determined using methods proposed by Amengual and Watson (2007), Bai and Ng (2007), and Breitung and Pigorsch (2012).

The first purpose of this paper is to propose some additional information criteria for the selection of the number of static factors. Akaike's (1973) Akaike information criterion (AIC), Hurvich and Tsai's (1989) corrected AIC (CAIC), the Bayesian information criterion (BIC) of Akaike (1978) and Schwarz (1978), and Hannan and Quinn's (1979) information criterion will be derived. These have conventionally been used in the literature on model selection, but have not been considered in the literature on factor models, so it is worthwhile to consider them. Note that the BIC of this paper is different from Bai and Ng's (2008) BIC. This paper also studies the consistency properties of the new information criteria and shows that the BIC and Hannan and Quinn's information criterion are consistent, unlike the AIC and CAIC.

The second purpose of this paper is to provide extensive simulation results that compare performance of the extant and new procedures for the selection of the number of factors. These results will provide a practical guideline for empirical researchers employing factor models. The data-generating process (DGP) for the simulation consists of serially correlated factors, and serially and cross-sectionally correlated idiosyncratic errors. The idiosyncratic errors are either homoskedastic or heteroskedastic. Idiosyncratic errors with fat tails and those with outliers having a much larger variance than the others are also considered. The simulation results show that it is hard to determine which criterion performs best. In practice, it is advised to consider several criteria at the same time, especially the BIC of this paper, Hannan and Quinn's (1979) information criterion, Bai and Ng's $IC_{p2}$ and $BIC_3$, Onatski's (2010) and Ahn and Horenstein's (2009) eigenvalue-based criteria. These perform relatively well and have their own advantages depending on the DGP.

The third purpose of this paper is to apply the model selection criteria considered in this paper to Stock and Watson's (2002, 2005) data sets. The results vary considerably depending on the model selection criterion in use. However, the evidence suggests four factors for Stock and Watson's (2002) data and six or seven factors for Stock and Watson's (2005).

This paper proceeds as follows. Section 2 introduces the model, notation and conditional maximum likelihood estimation. Section 3 introduces model selection criteria for the static factor model. Section 4 studies the consistency of the new model selection criteria. Section 5 reports extensive simulation results for the model selection criteria. Section 6 employs the model selection criteria considered in this paper to estimate the number of factors in Stock and Watson's (2002, 2005) panel data sets. Section 7 summarizes and provides further remarks. The proofs are relegated to appendices.

All limits are taken as $N, T \to \infty$ unless otherwise specified. The elements of a matrix $A$ are denoted as $a_{it}$. For matrices $X$ and $Y$, $X \oplus Y = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}$. Convergence in probability is denoted as $\xrightarrow{p}$.

# 2 The model, notation and conditional maximum likelihood estimation

Consider that the true factor model is given by

$$
\begin{aligned}
x_{it} &= \sum_{k=1}^{r^o} \lambda_{ik} f_{tk} + e_{it}, \qquad (1) \\
&= \xi_{it} + e_{it} \ (t = 1, \ldots, T; i = 1, \ldots, N),
\end{aligned}
$$

where $x_{it}$ are observed data, $\lambda_{ik}$ factor loadings, $f_{tk}$ latent factors, $e_{it}$ idiosyncratic errors and $r^o$ the unknown true number of factors. We assume that $e_t = [e_{1t}, \ldots, e_{Nt}]' \sim$ i. i. d. $N(0, \Omega_e)$ with $\Omega_e = \text{diag}\left[\sigma_{e1}^2, \ldots, \sigma_{eN}^2\right]$ to derive some model selection criteria in the next section.

The candidate model we use for model selection is written as

$$x_{it} = \sum_{k=1}^{r} \lambda_{ik} f_{tk} + v_{it}, \tag{2}$$

where $r$ is the number of factors assumed for the estimation of the factor model. For convenience, model (2) is written in obvious vector notation as

$$X_t = \Lambda_r F_{rt} + V_{rt},\ (t = 1, \ldots, T),$$

where $X_t$ is an $N \times 1$ vector of observations,

$$\Lambda_r = \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1r} \\ \vdots & & \vdots \\ \lambda_{N1} & \cdots & \lambda_{Nr} \end{bmatrix} = \begin{bmatrix} \Lambda'_{r1} \\ \vdots \\ \Lambda'_{rN} \end{bmatrix}$$

and $F_{rt} = [f_{t1}, \ldots, f_{tr}]'$.

In matrix notation, model (2) can be written as

$$X = F_r \Lambda'_r + V_r,$$

where $X = \begin{bmatrix} X'_1 \\ \vdots \\ X'_T \end{bmatrix}$, $F_r = \begin{bmatrix} F'_{r1} \\ \vdots \\ F'_{rT} \end{bmatrix}$, $\Lambda_r = \begin{bmatrix} \Lambda'_{r1} \\ \vdots \\ \Lambda'_{rN} \end{bmatrix}$ and $V_r = \begin{bmatrix} V'_{r1} \\ \vdots \\ V'_{rT} \end{bmatrix}$. Similarly, model (1) is written as

$$X = F^o \Lambda^{o\prime} + \mathcal{E}$$

where $\mathcal{E} = \begin{bmatrix} e'_1 \\ \vdots \\ e'_T \end{bmatrix}$ and $e_t = [e_{1t}, \ldots, e_{Nt}]'$. We let $F^o \Lambda^{o\prime} = \Xi$ for later use.

Suppose that $V_{rt} \sim$ i. i. d. $N(0, \Omega_r)$ and assume for the moment that $\Omega_r$ is known. With the standardization $F'_r F_r = T \times I_r$, the conditional (on $F_r$) maximum likelihood estimator (CMLE) of $F_r$, denoted by $\hat{F}_r$, is $\sqrt{T}$ times the matrix consisting of the eigenvectors corresponding to the $r$ largest eigenvalues of the matrix

$X\Omega_r^{-1}X'$, and the CMLE of $\Lambda_r$ is given by $\hat{\Lambda}_r = \frac{1}{T}X'\hat{F}_r$, as explained in Choi (2012). The CMLE of $\Omega_r$ is $\hat{\Omega}_r = \text{diag}\left[\frac{1}{T}\sum_{t=1}^{T}\left(X_t - \hat{\Lambda}_r\hat{F}_{rt}\right)\left(X_t - \hat{\Lambda}_r\hat{F}_{rt}\right)'\right]$. We denote $\hat{\Omega}_r = \text{diag}\left[\hat{\sigma}_1^2(r), \ldots, \hat{\sigma}_N^2(r)\right]$ which will be used in Appendix II.

Alternatively, when the log-likelihood function is maximized with respect to $F$ first, the CMLE of $\Lambda_r$, $\breve{\Lambda}$, is $\sqrt{N}$ times the matrix consisting of the eigenvectors corresponding to the $r$ largest eigenvalues of the matrix $\frac{1}{N}\Omega_r^{-1/2}X'X\Omega_r^{-1/2}$. The CMLE of $F_r$ is given by $\breve{F} = \frac{1}{N}X\Omega_r^{-1}\breve{\Lambda}$. These estimators are suitable for use when $N$ is small relative to $T$, while $\hat{F}_r$ and $\hat{\Lambda}_r$ are for when $T$ is small relative to $N$. The two pairs of estimators yield numerically identical values for $\hat{\sigma}_i^2(r)$ $(i = 1, \ldots, N)$. In the discussions below, we will use $\hat{F}_r$ and $\hat{\Lambda}_r$, but their alternative versions can also be used without generating any significant differences.

The CMLEs are infeasible in practice since $\Omega_r$ is not known. In practice, the CMLEs can be calculated with the following steps.

Step 1: For a given $r$, calculate the principal component estimator of $F_r$, $\ddot{F}_r$, which is $\sqrt{T}$ times the matrix consisting of the eigenvectors corresponding to the $r$ largest eigenvalues of the matrix $XX'$. The principal component estimator of $\Lambda_r$ is given by $\ddot{\Lambda}_r = \frac{1}{T}X'\ddot{F}$ (see, for example, Bai, 2003). Using the resulting estimator of $V_r$, $\ddot{V}_r = X - \ddot{F}_r\ddot{\Lambda}_r'$, estimate $\sigma_i^2$ from $\ddot{\sigma}_i^2(r) = \frac{1}{T}\sum_{t=1}^{T}\ddot{v}_{rit}^2$, where $\ddot{v}_{rit}$ is the $t$-th element of the $i$-th column of $\ddot{V}_r$.

Step 2: Using $\ddot{\Omega}_r = \text{diag}\left[\ddot{\sigma}_1^2(r), \ldots, \ddot{\sigma}_N^2(r)\right]$, calculate feasible CMLEs, $\hat{F}_r^f$ and $\hat{\Lambda}_r^f$, by replacing $\Omega_r$ with $\ddot{\Omega}_r$.

Step 3: Using an estimator of $V_r$, $\hat{V}_r = X - \hat{F}_r^f\hat{\Lambda}_r^{f\prime}$, estimate $\sigma_i^2$ from $\hat{\sigma}_{fi}^2(r) = \frac{1}{T}\sum_{t=1}^{T}\hat{v}_{rit}^2$, where $\hat{v}_{rit}$ is the $t$-th element of the $i$-th column of $\hat{V}_r$.

Step 2 can be repeated using $\left\{\hat{\sigma}_{fi}^2(r)\right\}$ instead of $\left\{\ddot{\sigma}_i^2(r)\right\}$. The estimators $\left\{\hat{\sigma}_{fi}^2(r)\right\}$ will be used for some of the model selection criteria introduced later.

# 3 Model selection criteria

This section introduces model selection criteria for the previous section's factor model. Some of them are new, while the others were already introduced in the literature.

## 3.1 Akaike information criterion

Akaike's (1973) AIC is an estimator of the Kullback–Leibler information measure. It has often been used for regression and time series models. In deriving AIC and its variants, it is usually assumed that candidate models include the true model (cf. Akaike, 1973; Hurvich and Tsai, 1989), though it does not have to be so because the Kullback–Leibler information measure is a distance between the true model and any candidate model. In this section, we will derive the AIC for the factor model of Section 2 without assuming $r \geq r^o$.

We use the general formula for the AIC given in Burnham and Anderson (2002). Assume $V_{rt} \mid F_r \sim \text{i.i.d.}\, N(0, \Omega_r)$ where $\Omega_r = \text{diag}\left[\sigma_1^2, \ldots, \sigma_N^2\right]$ and denote the conditional (on $\Xi$) log-likelihood function of the candidate model as $l(\Lambda_r, F_r, \Omega_r) = l(\delta)$ where $\delta = \left[\text{vec}(\Lambda_r')', \text{vec}(F_r')', \left[\sigma_1^2, \ldots, \sigma_N^2\right]\right]'$. The general formula is given by

$$T \sum_{i=1}^{N} \ln(\hat{\sigma}_{fi}^2(r)) + 2 \,\text{tr}\left[J(\delta_o) I(\delta_o)^{-1}\right],$$

where $\hat{\sigma}_{fi}^2(r)$ is the estimator of $\sigma_i^2$ introduced in Section 2, $\delta_o$ minimizes the Kullback–Leibler information measure, $J(\delta_o) = E\left[\frac{\partial l(\delta)}{\partial \delta} \frac{\partial l(\delta)}{\partial \delta}_{|\delta=\delta_o}\right]$ and $I(\delta_o) = E\left[\frac{\partial^2 l(\delta)}{\partial \delta \partial \delta'}_{|\delta=\delta_o}\right]$. The AIC is derived by calculating $J(\delta_o)$ and $I(\delta_o)$ as shown in Appendix I:

$$AIC = T \sum_{i=1}^{N} \ln\left(\hat{\sigma}_{fi}^2(r)\right) + 2\left(r(N+T) + N\right).$$

The AIC derived here is similar to the usual AIC used for time series and regression analyses. It is the sum of a goodness-of-fit term and twice the number of parameters. Note that it is different from the three AICs introduced in Bai and Ng (2002).

### 3.2 Corrected Akaike information criterion

The CAIC developed by Hurvich and Tsai (1989) has often been used for model selection and shows good finite-sample properties (see, for example, Hurvich and Tsai, 1989) It is a bias-corrected estimator of the Kullback–Leibler information measure that overcomes the negative-bias problem of the AIC.

As shown in equation (I.2) of Appendix I, the Kullback–Leibler information measure (×2) is written as

$$-2E_F(l(\Lambda_r, F_r, \Omega_r) \mid \Xi) = T \ln |\Omega_r| + \text{tr}\left\{\Omega_r^{-1}\left(\Xi - F_r\Lambda_r'\right)'\left(\Xi - F_r\Lambda_r'\right)\right\} + T\,\text{tr}\left\{\Omega_r^{-1}\Omega_e\right\}.$$

Replacing the unknown parameters with their (infeasible) CMLEs provides the corrected AIC we seek. The formula for the CAIC is

$$T\sum_{i=1}^{N} \ln\left(\hat{\sigma}_{fi}^2(r)\right) + \frac{NT(r+T)}{T-r-2}.$$

This is remarkably similar to Hurvich and Tsai's (1989) CAIC except that there is an extra parameter $N$.[1] The formula is derived under the assumption $r \geq r^o$ unlike AIC. Without this assumption, the CAIC cannot be derived properly. See Appendix II for the formal derivation.

### 3.3 Bayesian information criterion

The BIC of Akaike (1978) and Schwarz (1978) approximates a transformation of the Bayesian posterior probability of a candidate model. In deriving the BIC, we need not assume that the true model is included in the set of candidate models (cf. Cavanaugh and Neath, 1999). The formula for the BIC for the factor model follows the general rule of the BIC and is given by

$$BIC = T\sum_{i=1}^{N} \ln\left(\hat{\sigma}_{fi}^2(r)\right) + \ln(NT)\left[r(N+T)+N\right].$$

[1] In fact, if $N = 1$ and if $r$ is considered as the number of regressors, this is exactly the same as Hurvich and Tsai's (1989) CAIC for linear regression.

The difference between $AIC$ and $BIC$ lies their penalty terms, which are 2 and $\ln(NT)$, respectively. When $N$ and $T$ are large, this brings a huge difference, unlike in time-series regression with a moderate $T$ and their properties are expected to differ much for large $N$ and $T$.

### 3.4 Hannan and Quinn's (1979) criterion

Observing that the AIC tends to overestimate the true order of an autoregression, Hannan and Quinn (1979) propose a model selection criterion with a higher penalty for the complexity of the model. Hannan and Quinn's criterion estimates the true model, if there is one, with probability one in the limit (i.e., the model selection criterion is strongly consistent). For the factor model in this paper, Hannan and Quinn's criterion is analogously defined as

$$HQ_c = T\sum_{i=1}^{N}\ln\left(\hat{\sigma}_{fi}^2(r)\right) + c\ln\ln(NT)\left[r(N+T)+N\right],$$

where $c$ is a constant of our choice.

### 3.5 Bai and Ng's (2002) information criteria

Bai and Ng (2002) suggest a few information criteria that select the true number of factors with probability one as $N, T \to \infty$. These criteria use the principal component estimators unlike $AIC$, $CAIC$ and $BIC$ introduced above. The estimators are $\ddot{F}_r$ and $\ddot{\Lambda}_r$ discussed in Section 2. In Bai and Ng's simulation study, each time series is standardized such that its variance becomes one before the principal component estimation. Letting

$$V_{NT}(r) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(x_{it} - \ddot{\lambda}_{ri}'\ddot{F}_{rt}\right)^2,$$

where $\ddot{\lambda}'_{ri}$ and $\ddot{F}'_{rt}$ are the $i$-th and $t$-th row of $\ddot{F}_r$ and $\ddot{\Lambda}_r$, respectively, Bai and Ng's information criteria are

$$\begin{aligned}
PC_{p1} &= V_{NT}(r) + r\hat{\sigma}^2 \left(\frac{N+T}{NT}\right) \ln\left(\frac{NT}{N+T}\right) \\
PC_{p2} &= V_{NT}(r) + r\hat{\sigma}^2 \left(\frac{N+T}{NT}\right) \ln(\min\{N,T\}) \\
PC_{p3} &= V_{NT}(r) + r\hat{\sigma}^2 \left(\frac{\ln(\min\{N,T\})}{\min\{N,T\}}\right) \\
IC_{p^1} &= \ln\left(V_{NT}(r)\right) + r\left(\frac{N+T}{NT}\right) \ln\left(\frac{NT}{N+T}\right) \\
IC_{p2} &= \ln\left(V_{NT}(r)\right) + r\left(\frac{N+T}{NT}\right) \ln(\min\{N,T\}) \\
IC_{p3} &= \ln\left(V_{NT}(r)\right) + r\left(\frac{\ln(\min\{N,T\})}{\min\{N,T\}}\right).
\end{aligned}$$

Here, $\hat{\sigma}^2$ is an estimator of $\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} E\left(e_{it}^2\right)$ that uses the maximum number of factors designated for the model selection – denoted here as $r_{\max}$. That is, $\hat{\sigma}^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(x_{it} - \ddot{\lambda}'_{r_{\max}i}\ddot{F}_{r_{\max}t})^2$. The first three information criteria are motivated by Mallows' (1973) $C_p$ criterion. The latter three are similar to the first three, but they do not depend on $r_{\max}$ for they do not use $\hat{\sigma}^2$. Because these two sets of information criteria show similar performance, we will consider only the $IC$ criteria in our simulation study. Bai and Ng show that the probability of these model selection criteria selecting $r^o$ converges to one as $N, T \to \infty$.

Bai and Ng (2002) also suggest a model selection criterion,

$$BIC_3 = V_{NT}(r) + r\hat{\sigma}^2 \left(\frac{(N+T-r)\ln(NT)}{NT}\right),$$

with good finite sample properties according to their simulation results. This is certainly different from the BIC introduced in Subsection 3.3. Note that $BIC_3$ does not satisfy the sufficient conditions for the consistency of model selection given in Theorem 2 of Bai and Ng (2002).

### 3.6 Onatski's (2010) criterion

Onatski (2010) develops a factor selection method that uses the eigenvalues of $XX'/T$. Denoting the eigenvalues of $XX'/T$ in descending order as $\mu_i$ and given an upper bound of $r^o$, $r_{\max}$, Onatski's estimator of $r^o$ is defined as

$$\hat{r}^o = \max\{i \leq r_{\max} : \mu_i - \mu_{i+1} \geq \eta\},$$

where $\eta > 0$. That is, it equals the number of the eigenvalues greater than the threshold value $\eta$. The estimator is based on the observation that $\mu_{r^o}$ diverges to infinity in probability while $\mu_i = O_p(1)$ for $i > r^o$. The unknown threshold value $\eta$ is estimated from the empirical distribution of the eigenvalues and $r_{\max}$, which is chosen such that $r^o < r_{\max}$. Onatski's estimator of $r^o$ is called an edge distribution (ED) estimator, which originates from the square root shape of the edge of the eigenvalue distribution. Onatski (2010) shows that the ED estimator is a consistent estimator of $r^o$ under the assumption that the idiosyncratic errors are both cross-sectionally and serially correlated. Onatski (2009) also develops a testing procedure for $r^o$, which is based on the similar observation on the properties of the eigenvalues. We will study only the model selection criterion of Onatski (2010) in our simulation study.

### 3.7 Ahn and Horenstein's (2009) criterion

In order to estimate the true number of factors, Ahn and Horenstein (2009) consider maximizing the ratio of two adjacent eigenvalues of $XX'/NT$,

$$ER(r) = \frac{\mu_r}{\mu_{r+1}},$$

or the growth ratio

$$GR(r) = \frac{\ln(1 + \mu_r^*)}{\ln(1 + \mu_{r+1}^*)},$$

where $\mu_r^* = \frac{\mu_r}{\sum_{j=r+1}^{\min(N,T)} \mu_j}$. The value of $r$ that maximizes each ratio is the estimator of $r^o$ from the ratio. When $r = r^0$, these ratios are $O_p(\min(N, T))$. But when $r \neq r^o$, the ratios are $O_p(1)$. Thus, the ratios can select $r^o$ consistently as $N, T \to \infty$.

## 4 Consistency

This section studies consistency of the model selection criteria introduced in this paper. Consistency in model selection implies that the probability of choosing the true model becomes one as sample size grows. The concept of model selection consistency is based on the assumptions that data were generated by a true model and that candidate models include the true model. If these assumptions are not valid, proving consistency is of little use. In the literature on model selection, however, consistency has been regarded as one of the most important touchstones in the evaluation of model selection criteria. In fact, in many simulation exercises where a true model is included in a set of candidate models, consistent model selection criteria tend to perform better than inconsistent ones.[2] For deeper discussions on the issue of model selection consistency, the reader is referred to Section 6.3 of Burnham and Anderson (2002).

In deriving the model selection criteria in Section 3, model (1) is defined to be the true model. In reality, however, it is more reasonable to assume that the idiosyncratic errors are cross-sectionally and serially correlated. Thus, we allow the idiosyncratic errors of the true model to be cross-sectionally and serially correlated and assume that Assumptions A, B, C and D of Bai and Ng (2002) hold. When $r < r^o$ (the case of underfitting), there will be too strong cross-sectional correlation that is not satisfied by the assumptions of Bai and Ng. When $r > r^o$ (the case of overfitting), all the models have the same level of fit in the sense that $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r) - \hat{\sigma}^2_{fi}(r^o)\right) = o_p(1)$. Therefore, our focus in this case will be on selecting a model with the fewest factors.

Sufficient conditions for the model selection criterion, including $AIC$, $CAIC$, $BIC$ and $HQ_c$ as special cases, are given in Appendix III. If the penalty terms of these criteria are written as $g(r, N, T)$, the conditions are (i) $\frac{g(r^o,N,T)-g(r,N,T)}{NT} \to 0$ for $r < r^o$ and (ii) $\frac{C^2_{N,T}(g(r^o,N,T)-g(r,N,T))}{NT} \to -\infty$ for $r \geq r^o$, where $C_{N,T} = \min\left\{\sqrt{N}, \sqrt{T}\right\}$. If $g(r, N, T) = rh(N, T) + k(N, T)$ as for $AIC$, $BIC$ and $HQ_c$, the conditions are simplified to (i) $\frac{h(N,T)}{NT} \to 0$ and (ii) $\frac{C^2_{N,T}h(N,T)}{NT} \to \infty$.

Now we verify the sufficient conditions are satisfied by these new model selection criteria. For $AIC$, it is easy to check that condition (ii) is not satisfied, although condition (i) is. Thus, $AIC$ does not satisfy the sufficient conditions for consistency.

[2] An exception is $CAIC$, which performs as well as $BIC$ for linear regression in finite samples (see Hurvich and Tsai, 1989).

For $CAIC$, condition (i) is satisfied because $\frac{g(r^o,N,T)-g(r,N,T)}{NT} = \frac{r^o+T}{T-r^o-2} - \frac{r+T}{T-r-2} \to 0$. However, since $\frac{C_{N,T}^2(g(r^o,N,T)-g(r,N,T))}{NT} = 2(r^o - r)\frac{\min\{N,T\}(T-1)}{(T-r^o-2)(T-r-2)}$, condition (ii) cannot be satisfied. Thus, $CAIC$ does not satisfy the sufficient conditions for consistency.

For $BIC$, $\frac{h(N,T)}{NT} = \frac{\ln(NT)(N+T)}{NT} \to 0$ if $\frac{\ln(N)}{T}, \frac{\ln(T)}{N} \to 0$. Moreover, $\frac{C_{N,T}^2 h(N,T)}{NT} = \frac{(\ln(NT)N+\ln(NT)T)\min\{N,T\}}{NT} \to \infty$. These show that $BIC$ satisfies the sufficient conditions for consistency if $\frac{\ln(N)}{T}, \frac{\ln(T)}{N} \to 0$.

For $HQ_c$, $\frac{h(N,T)}{NT} = \frac{c\ln\ln(NT)(N+T)}{NT} = \frac{c\ln\ln(NT)}{T} + \frac{c\ln\ln(NT)}{N}$. But we have an inequality,[3] for $N,T > e^2$, $\ln(N)+\ln(T) \leq \ln(N)\ln(T)$. Thus, for $N,T > e^2$, $\frac{\ln\ln(NT)}{T} = \frac{\ln(\ln(N)+\ln(T))}{T} \leq \frac{\ln(\ln(N)\ln(T))}{T} = \frac{\ln\ln(N)}{T} + \frac{\ln\ln(T)}{T}$. Likewise, $\frac{\ln\ln(NT)}{N} \leq \frac{\ln\ln(N)}{N} + \frac{\ln\ln(T)}{N}$. Thus, if $\frac{\ln\ln(N)}{T}, \frac{\ln\ln(T)}{N} \to 0$, condition (i) is satisfied. Additionally, $\frac{C_{N,T}^2 h(N,T)}{NT} = \frac{c\min\{N,T\}\ln\ln(NT)(N+T)}{NT} \to \infty$. We find from these that $HQ_c$ satisfies the sufficient conditions if $\frac{\ln\ln(N)}{T}, \frac{\ln\ln(T)}{N} \to 0$. Note that these conditions are weaker than those for $BIC$.

In summary, $AIC$ and $CAIC$ do not satisfy the sufficient conditions for consistency, while $BIC$ and $HQ_c$ do under the additional conditions $\frac{\ln(N)}{T}, \frac{\ln(T)}{N} \to 0$.

## 5 Simulation

This section reports the finite sample properties of the model selection criteria considered in Section 3. Bai and Ng (2002) suggest standardizing the data before applying their model selection criteria. That is, they divide the data for each individual by the estimate of its standard deviation such that each individual has unit variance. We report the results for both standardized and unstandardized data in this section to show effects of the standardization on model selection criteria. Among Bai and Ng's criteria, we will consider only $IC_{p2}$, $IC_{p3}$ and $BIC_3$ because these work better than the others. For the standardized version of these, we will use notation $IC_{p2s}$, $IC_{p3s}$ and $BIC_{3s}$. For the $HQ_c$ criterion, we will consider $c = 2, 3, 4, 5$.

The tables below report sample means of the estimated number of factors out of

[3] Note that $x + y \leq xy$ for $x, y \geq 2$.

1,000 iterations for each sample size and set of parameter values. At the end of each table, we report sample means and root mean squared errors (RMSE) of each column. These will summarize the simulation results in a compact manner. Between the two, the RMSEs will be a major criterion in evaluating the model selection criteria.

## 5.1 Homoskedastic idiosyncratic errors

The first DGP for our simulation is

$$\begin{aligned} x_{it} &= \sum_{k=1}^{r^o} \lambda_{ik} f_{tk} + \sqrt{\theta} e_{it}, (t = 1, \ldots, T;\ i = 1, \ldots, N) \\ \lambda_{ik} &\sim \text{i. i. d.}\, N\left(0, \sigma_\lambda^2\right) \\ f_{tk} &= \alpha f_{t-1,k} + w_{tk},\ w_{tk} \sim \text{i. i. d.}\, N(0,1) \\ e_{it} &= \rho e_{i,t-1} + \varepsilon_{it} + \beta \sum_{1 \le |j| \le 8} \varepsilon_{i-j,t},\ \varepsilon_{ik} \sim \text{i. i. d.}\, N(0,1). \end{aligned} \tag{3}$$

This DGP allows serially correlated factors, and serially and cross-sectionally correlated idiosyncratic errors. Since

$$\begin{aligned} \text{Var}\left(\sum_{k=1}^{r^o} \lambda_{ik} f_{tk}\right) &= \sum_{k=1}^{r^o} E(\lambda_{ik} f_{tk})^2 \\ &= \sum_{k=1}^{r^o} E\left(\lambda_{ik}^2\right) E\left(f_{tk}^2\right) \\ &= \frac{r^o \sigma_\lambda^2}{1-\alpha^2} \end{aligned}$$

and

$$\text{Var}(e_{it}) = \frac{1+16\beta^2}{1-\rho^2},$$

the signal-to-noise ratio (SNR) defined by $\text{Var}\left(\sum_{k=1}^{r^o} \lambda_{ik} f_{tk}\right) / \text{Var}(e_{it})$ becomes one when we set $\theta = \left(\frac{r^o \sigma_\lambda^2}{1-\alpha^2}\right) / \left(\frac{1+16\beta^2}{1-\rho^2}\right)$. The parameter $\sigma_\lambda^2$ in DGP (3) controls variability in the factor loading, and $\alpha$ the degree of serial correlation in the factors. As the value of $\sigma_\lambda^2$ increases, there will be greater variability in the factor loadings. The factors become more serially correlated as $\alpha$ increases. In our simulation, we try $\left(\sigma_\lambda^2, \alpha\right) = (1, 0.5), (1, 0.85), (3, 0.5), (3, 0.85)$. The parameters $\rho$ and $\beta$ set the degrees of serial and cross-sectional correlations of the idiosyncratic errors, respectively.

We consider $(\rho, \beta) = (0, 0), (0.5, 0.2)$. For the sample size, we consider the combinations of $N = 20, 30, 100, 200$ and $T = 30, 60, 100, 200$. The true numbers of factors considered are 1, 3, and 5, and the maximum number of factors is set to 8.

Simulation results for DGP (3) are reported in Table 1. These results can be summarized as follows.

(i) When there are no serial and cross-sectional correlations in the idiosyncratic errors – $(\rho, \beta) = (0, 0)$ – Parts I, III, V and VII show that $BIC$, all of $HQ_c$, $IC_{p2}$, $IC_{p2s}$, $ER$, $GR$ and $ED$ select the true number quite well for $r^o = 1$ at all sample sizes. But when $r^o = 3$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $ER$, $GR$ and $ED$ tend to perform better than the others. When $r^o = 5$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $IC_{p3}$, $IC_{p3s}$, $ER$, $GR$ and $ED$ tend to perform well relative to the others.

(ii) When the idiosyncratic errors are serially and cross-sectionally correlated – $(\rho, \beta) = (0.5, 0.2)$ – Parts II, IV, VI and VIII show that the performance of all the model selection criteria tends to deteriorate. When $r^o = 1$ and 3, $BIC$, $HQ_4$, $HQ_5$, $ER$, $GR$ and $ED$ perform best in terms of RMSE. When $r^o = 5$, $HQ_2$, $BIC_3$, $IC_{p2s}$, $ER$, and $GR$ perform well.

(iii) Comparing Parts I, II, V, VI with Parts III, IV, VII, VIII, respectively, we find that the degree of serial correlation in the factors does not seem to affect the performance of the model selection criteria in any significant way although most of the model selection criteria tend to perform slightly worse when the factors are more serially correlated.

(iv) Comparing Parts I, II, III, IV with Parts V, VI, VII, VIII, respectively, we find that the degree of variability in the factor loadings embodied in the values of $\sigma^2_\lambda$ does not seem to affect the performance of the model selection criteria in any significant manner.

(v) As $r^o$ increases, most criteria become less accurate in determining the true order. Exceptions are $IC_{p3}$ and $IC_{p3s}$.

(vi) In most cases, $IC_{p2}$, $IC_{p3}$ and $BIC_3$ perform better than $IC_{p2s}$, $IC_{p3s}$ and $BIC_{3s}$, respectively. It seems that the usual standardization is not warranted at least for DGP (3).

(vii) $BIC_3$ tends to perform better than $IC_{p2}$ and $IC_{p3}$ when the idiosyncratic errors are serially and cross-sectionally correlated. Under the serially and cross-sectionally correlated errors, performance of $IC_{p2}$ and $IC_{p3}$ tends to deteriorate.

(viii) $AIC$ and $CAIC$ do not perform well compared to the others. They tend to select the maximum number of factors allowed.

### 5.2 Heteroskedastic idiosyncratic errors

The second DGP we consider is the same as DGP (3) except that the idiosyncratic errors are generated by

$$e_{it} = \rho_i e_{i,t-1} + \varepsilon_{it} + \beta \sum_{1\leq|j|\leq 8} \varepsilon_{i-j,t}, \ \varepsilon_{ik} \sim \text{i.i.d.}\, N(0,1). \quad (4)$$

Here, $e_{it}$ are heteroskedastic since the parameters $\rho_i$ change over individuals. In our experiments, we let $\rho_i \sim \text{i.i.d.}\, U(0, 0.85)$ and $\theta_i = \left(\frac{r^o\sigma_\lambda^2}{1-\alpha^2}\right) / \left(\frac{1+16\beta^2}{1-\rho_i^2}\right)$. The SNR is as before. For the other parameters, we follow the same design as for DGP (3).

Simulation results for DGP (4) are reported in Table 2. These results can be summarized as follows.

(i) When there are no cross-sectional correlations in the idiosyncratic errors ($\beta = 0$), Parts I, III, V and VII show that $BIC$, all of $HQ_c$, $IC_{p2}$, $IC_{p2s}$, $ER$, $GR$ and $ED$ perform well for $r^o = 1$ relative to the others at all sample sizes. But when $r^o = 3$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $ER$, $GR$ and $ED$ tend to perform better than the others. When $r^o = 5$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $IC_{p3}$, $IC_{p3s}$, $ER$ and $GR$ tend to perform well. Parts III and VII show that the eigenvalue-based methods perform worse than Bai and Ng's (2002) information criteria at $r^o = 5$.

(ii) According to Parts II, IV, VI and VIII, performance of all the model selection criteria tends to become worse with cross-sectionally correlated ($\beta = 0.2$) idiosyncratic errors. When $r^o = 1$, $BIC$, $HQ_4$, $HQ_5$, $ER$, $GR$ and $ED$ perform best in terms of RMSE. When $r^o = 3$, $BIC$, $HQ_4$, $BIC_{3s}$, $ER$, $GR$ and $ED$ perform well. When $r^o = 5$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $BIC_{3s}$, $ER$ and $GR$ perform well, but $ED$ performs worse than Bai and Ng's (2002) information criteria at $r^o = 5$.

(iii) As in Table 1, the degree of serial correlation in the factors does not seem to affect the performance of the model selection criteria in any significant way.

(iv) The degree of variability in the factor loadings does not seem to significantly affect the performance of the model selection criteria.

(v) As $r^o$ increases, most criteria become less accurate in determining the true order. Exceptions are $IC_{p3}$ and $IC_{p3s}$.

(vi) In most cases, $IC_{p2}$, $IC_{p3}$ and $BIC_3$ perform better than $IC_{p2s}$, $IC_{p3s}$ and $BIC_{3s}$, respectively. The usual standardization is not warranted for DGP (4).

(vii) $BIC_3$ tends to perform better than $IC_{p2}$ and $IC_{p3}$ when the idiosyncratic errors are cross-sectionally correlated. Performance of $IC_{p2}$ and $IC_{p3}$ tends to deteriorate with cross-sectionally correlated idiosyncratic errors.

(viii) $AIC$ and $CAIC$ do not perform well compared to the others. They tend to select the maximum number of factors allowed.

### 5.3 Fat-tailed, heteroskedastic idiosyncratic errors

The third DGP is the same as DGP (4) except that we assume $\sqrt{3}\varepsilon_{ik} \sim \text{i.i.d.}\, t(3)$, where $t(3)$ denotes the Student's t-distribution with three degrees of freedom. In this DGP, the idiosyncratic errors have unit variance as in the previous DGPs, but have fatter tails that those of DGP (4). The other parameters are set in the same way as for DGP (4).

Simulation results for the third DGP are reported in Table 3. These results can be summarized as follows.

(i) Compared to Table 2, the performance of all the criteria tends to deteriorate.

(ii) When the idiosyncratic errors are not cross-sectionally correlated (i.e., $\beta = 0$) and $r^o = 1$, Parts I, III, V and VII show that $BIC$, most of $HQ_c$, $IC_{p2s}$, $ER$ and $GR$ select the true number quite well for all sample sizes. When $r^o = 3$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $ER$ and $GR$ show good performance. When $r^o = 5$, $HQ_2$, $IC_{p2}$, $IC_{p3}$, $IC_{p3s}$, $ER$ and $GR$ select the true number of factors well.

(iii) According to Parts II, IV, VI and VIII, the performance of all the model selection criteria tends to become worse with cross-sectionally correlated idiosyncratic errors (i.e., $\beta = 0.2$). When $r^o = 1$, $BIC$, $HQ_4$, $HQ_5$, $ER$, $GR$ and $ED$ perform best in terms of RMSE. When $r^o = 3$, $HQ_4$, $BIC_3$, $BIC_{3s}$, $ER$, $GR$ and $ED$ perform well. When $r^o = 5$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $IC_{p3}$, $IC_{p3s}$ and $GR$ perform well.

(iv) The degree of serial correlation in the factors does not seem to affect the performance of the model selection criteria in any significant way.

(v) The degree of variability in the factor loadings does not seem to significantly affect the performance of the model selection criteria.

(vi) As $r^o$ increases, most criteria become less accurate in determining the true order. Exceptions are $IC_{p3}$ and $IC_{p3s}$.

(vii) Unlike in the previous tables, the standardized versions of $IC_{p2}$, $IC_{p3}$ and $BIC_3$ tend to perform slightly better than the unstandardized versions.

(viii) When the idiosyncratic errors are cross-sectionally correlated, $BIC_3$ performs better than $IC_{p2}$ and $IC_{p3}$.

(ix) $AIC$ and $CAIC$ do not perform well compared to the others. They tend to select the maximum number of factors allowed as in Tables 1 and 2.

(x) Unlike in the previous tables, the performance of $ED$ is not particularly good when $r^o = 5$.

### 5.4 Dynamic factors and heteroskedastic idiosyncratic errors

The fourth DGP we consider assumes a dynamic factor structure

$$\begin{aligned}
x_{it} &= \sum_{k=1}^{r^o} \lambda_{ik} f_{tk} + \sum_{k=1}^{r^o} \delta_{ik} f_{t-1,k} + \sqrt{\theta_i} e_{it}, (t = 1, \ldots, T;\ i = 1, \ldots, N) \\
\lambda_{ik} &\sim \text{i.i.d.}\, N\left(0, \sigma_\lambda^2\right), \delta_{ik} \sim \text{i.i.d.}\, N\left(0, \sigma_\delta^2\right) \\
f_{tk} &= w_{tk}, w_{tk} \sim \text{i.i.d.}\, N(0,1) \\
e_{it} &= \rho_i e_{i,t-1} + \varepsilon_{it} + \beta \sum_{1 \le |j| \le 8} \varepsilon_{i-j,t}, \varepsilon_{ik} \sim \text{i.i.d.}\, N(0,1).
\end{aligned}$$

The true number of factors of this DGP is $2 \times r^o$. In this DGP, $\theta_i = r^o \left(\sigma_\lambda^2 + \sigma_\delta^2\right) / \left(\frac{1+16\beta^2}{1-\rho_i^2}\right)$ makes the SNR equal to one. We consider $r^o = 1, 3$. For $\sigma_\lambda^2$ and $\sigma_\delta^2$, we consider the combinations $\left(\sigma_\lambda^2, \sigma_\delta^2\right) = (1,1), (3,3)$. The other parameters are set equal to those for DGP (4).

Simulation results for the fourth DGP are reported in Table 4. These results can be summarized as follows.

(i) When there are no cross-sectional correlations of the idiosyncratic errors and $r^o = 1$, $HQ_2$, $IC_{p2}$, $IC_{p2s}$, $ER$, $GR$ and $ED$ select the true number quite well for all sample sizes. But when $r^o = 3$, $IC_{p2}$, $IC_{p3}$, $BIC_3$, $ER$ and $GR$ show good performance.

(ii) When the idiosyncratic errors are cross-sectionally correlated, $BIC$, $HQ_4$, $HQ_5$, $ER$, $GR$ and $ED$ perform well for the case $r^o = 1$. When $r^o = 3$, $HQ_2$, $IC_{p2}$, $BIC_3$, $BIC_{3s}$, $ER$ and $GR$ perform well. As in Tables 1, 2 and 3, $HQ_c$ with a small constant performs well in the small sample while $HQ_c$ with a large constant performs better in the large sample.

(iii) The degree of variability in the factor loadings – the values of $\sigma_\lambda^2$ and $\sigma_\delta^2$ – does not seem to engender any noticeable changes in the performance of the model selection criteria.

(iv) As $r^o$ increases, most criteria become less accurate in determining the true order. Exceptions are $IC_{p3}$ and $IC_{p3s}$.

(v) In most cases, $IC_{p2}$, $IC_{p3}$ and $BIC_3$ perform better than $IC_{p2s}$, $IC_{p3s}$ and $BIC_{3s}$, respectively.

(vi) $IC_{p2}$ tends to perform better than $IC_{p3}$. But when the idiosyncratic errors are cross-sectionally correlated, $BIC_3$ performs better than $IC_{p2}$.

(vii) $AIC$ does not perform well compared to the others. $CAIC$ performs well when $r^o = 3$, but this may have been caused by the maximum number of factors being set to 8.

### 5.5 Sensitivity to signal-to-noise ratio

In the DGPs considered so far, the SNR is commonly one. But if $\sqrt{\theta}$ is replaced by $\sqrt{2\theta}$, it becomes $\frac{1}{2}$. Likewise, it becomes 2 when $\sqrt{\theta}$ is replaced by $\sqrt{\frac{\theta}{2}}$. We examine in this section how sensitive the model selection criteria are to these changes. To this end, we consider the cases $SNR = \frac{1}{2}, 2$ for DGP (3) with $r^o = 1, 3, 5$; $\sigma_\lambda^2 = 1, 3$; and $(\alpha, \rho, \beta) = (0.5, 0, 0)$. The simulation results are reported in Table 5. The performance of most of the model selection criteria improves as $SNR$ increases, and vice versa. With the exception of this variation, evaluations of the model selection criteria do not change essentially from Table 1.

For other values of $SNR$, Figure 1 plots RMSE for DGP (3) with $\left(r^o, \sigma_\lambda^2, \alpha, \rho, \beta\right) = (3, 1, 0.5, 0, 0)$. As expected, Figure 1 shows that most of the selection criteria improve as SNR increases. Exceptions are $AIC$, $CAIC$ and $IC_{p3}$ and $IC_{p3s}$. In particular, the RMSE of $IC_{p3s}$ increases as $SNR$ increases, which makes it unsuitable for practical use.

### 5.6 Large variances for some cross-sectional units

Empirical economists using factor models have adopted the practice of standardization that makes each individual have a unit variance. However, it is not certain why such practice has to be adopted, although simulation results in Subsection 5.3 show that the standardized versions of $IC_{p2}$, $IC_{p3}$ and $BIC_3$ perform slightly better than the unstandardized versions.

This subsection studies how the standardization affects the performance of the information criteria studied in this paper. For this purpose, we employ DGP (3) with

$$\sigma_{ei}^2 = \begin{cases} 10 & \text{for the } p \text{ percent of all cross-sectional units} \\ 1 & \text{for the others.} \end{cases}$$

In this DGP, some cross-sectional unit have a much larger variance than the others. This is certainly different from the DGP considered in Subsection 5.3 because the idiosyncratic errors of all the cross-sectional units have the same variance in that subsection. In addition, the DGP of this subsection is relevant to empirical applications because, for example, some individual units of Stock and Watson's (2002, 2005) data sets have much larger variances than others. For the simulation of this subsection, we let $(\alpha, \rho, \beta) = (0.5, 0, 0)$. Parameter values of $\theta$ are adjusted so that the SNR is one. For $r^o$ and $p$, we consider $r^o = 3, 5, 8$ and $p = 2, 4$. We set $r_{\max} = 8$ for $r^o = 3, 5$ and $r_{\max} = 12$ for $r^o = 8$.

Simulation results of this subsection are contained in Table 6. The results are summarized as follows.

(i) Most of the information criteria perform better with the standardization.

(ii) With the standardization, $HQ_2$, $IC_{p2}$, $ER$, $GR$ and $ED$ tend to perform better than the others.

(iii) As $r^o$ increases, most criteria become less accurate in determining the true order.

### 5.7 Sensitivity to $r_{\max}$

This subsection reports how $r_{\max}$ affects the model selection criteria. For this, we consider two cases for DGP (3), $\left(r^o, \sigma_\lambda^2, \alpha, \rho, \beta\right) = (3, 1, 0.5, 0, 0)$ and $(3, 1, 0.5, 0.5, 0.2)$, and set $r_{\max} = 6$, 8, 10, 12, 14. The simulation results are given in Figure 2. These results can be summarized as follows.

(i) When there are no serial and cross-sectional correlations in the idiosyncratic errors, $BIC$, $HQ_c$, $IC_{p2}$, $IC_{p2s}$, $ER$, $GR$ and $ED$ perform steadily. However,

$AIC$, $CAIC$, $IC_{p3}$, $IC_{p3s}$, $BIC_3$ and $BIC_{3s}$ are sensitive to the values of $r_{\max}$. Especially, $BIC_3$ often leads to overestimation when $r_{\max}$ is large.

(ii) When the idiosyncratic errors are serially and cross-sectionally correlated, $BIC$, $HQ_3$, $HQ_4$, $HQ_5$, $ER$, $GR$ and $ED$ still choose the number of factors stably, but the others are sensitive to $r_{\max}$. It is shown that higher degrees of serial and cross-sectional correlations induce more criteria sensitivity to $r_{\max}$.

(iii) $BIC$, $HQ_c$, $ER$ $GR$ and $ED$ are less affected by $r_{\max}$ than the others.

### 5.8 Discussions

Properties of the selection criteria depend on the size of the penalty terms. Figure 3 plots the penalty terms for $(N, T) = (200, 200)$. Among the new criteria considered in this paper, $HQ_5$ tends to enforce the highest penalty and $AIC$ the lowest. As the number of factors increases, the CAIC penalty term does not increase as much as the others. This explains why $AIC$ and $CAIC$ favor larger numbers of factors compared to the others. Among Bai and Ng's (2002) criteria, $BIC_3$ enforces the highest penalty and $IC_{p3}$ the lowest. The observation that $IC_{p3}$ and $IC_{p3s}$ tend to choose larger models than Bai and Ng's other criteria is explained by this.

We find that the degree of serial and cross-sectional correlation affects the performance of the selection criteria significantly. According to our simulation results, selection criteria with higher penalty terms tend to work better than those with lower ones when the degree of serial and cross-sectional correlations is high.

Our simulation results show that the standardization that makes each individual have unit variance is not required in most cases. However, when the idiosyncratic errors have outliers, as in the DGP in Subsection 5.3, the standardization tends to yield better results for $IC_{p2}$, $IC_{p3}$ and $BIC$. Additionally, when some individual units have much larger variances than the others, the standardization improves the performance of the information criteria.

The simulation results reported so far show that it is hard to determine which criterion performs best for a particular model. The order of performance depends on the chosen DGP. However, $AIC$, $CAIC$, $IC_{p3}$ and $IC_{p3s}$ seem inferior to the others. In addition, $IC_{p3}$ tends to overestimate the number of factors when error terms are serially and cross-sectionally correlated. These criteria are also sensitive to the maximum number of factors. $BIC$ tends to underestimate the number of factors, but it performs well when the idiosyncratic errors are both serially and cross-sectionally correlated. Overall, it is found that $BIC$, $HQ_c$, $IC_{p2}$, $IC_{p2s}$, $BIC_3$, $BIC_{3s}$, $ER$, $GR$ and $ED$ are found to perform well and are recommended for applications.

# 6 Application to the Stock and Watson's (2002, 2005) data sets

This section applies the model selection criteria discussed in Section 3 to the data sets of Stock and Watson (2002, 2005). Stock and Watson (2002) employ a factor-augmented predictive regression model in order to forecast the Federal Reserve Board's index of industrial production. Their panel data include 146 monthly U.S. time series with the sampling period 1959:1–1998:12 ($T = 480$). The 14 categories of their panel data are real output, income, employment and hours, retail, manufacturing, trade sales, real inventories, inventory–sales ratios, orders, unfilled orders, stock prices, exchange rates, interest rates, money and credit quantity aggregates, price indexes, average hourly earnings, and miscellaneous. We estimated the number of factors with and without standardization, using Stock and Watson's (2002) data and following their transformation rules. The estimation results are reported in Part I of Table 7. For $r_{\max}$, we considered 6, 8, 10, 12, 14 and 16.

For the panel data without standardization, Part I of Table 7 shows that $AIC$, $CAIC$, $BIC$, $HQ_c$ and all of Bai and Ng's criteria yield the maximum number of factors specified by $r_{\max}$. $ED$ selects three factors when $r_{\max} = 6$, four factors when $r_{\max} = 8, 10$ and six factors when $r_{\max} = 12, 14, 16$. In contrast, $ER$ and $GR$ select four factors regardless of $r_{\max}$. Next, for the standardized panel data set, $AIC$, $CAIC$, $HQ_2$, $HQ_3$, $IC_{p2}$ and $IC_{p3}$ choose the maximum number of factors given by $r_{\max}$. $HQ_4$ selects four factors when $r_{\max} = 6$, seven factors when $r_{\max} = 8, 10$

and 12 factors when $r_{\max} = 12, 14, 16$. $BIC_3$ reports two, three, four, five, six and seven factors when $r_{\max} = 6, 8, 10, 12, 14, 16$, respectively. In accordance with the simulation results of Subsection 5.7, $BIC_3$ is sensitive to the values of $r_{\max}$. On the other hand, $ER$, $GR$, $ED$, $BIC$ and $HQ_5$ estimate one, two, two, four and four factors, respectively, regardless of the values of $r_{\max}$.

Stock and Watson's (2005) data set is an updated and refined version of the Stock and Watson's (2002). This data set contains 132 monthly U.S. macroeconomic variables with the sampling period 1960:1–2003:12 ($T = 540$). Stock and Watson (2005) report that there are seven factors in this data set using the $IC_{p2}$ criterion. Using Stock and Watson's (2005) data and adopting their transformation rules, we estimated the number of factors with and without standardization. The results are reported in Part II of Table 7. For the data without standardization, $AIC$, $CAIC$, $BIC$, $HQ_c$ and all of Bai and Ng's criteria perform similar to Part I: They estimate the number of factors as equal to $r_{\max}$, although there are a few exceptions. $GR$ estimates one factor when $r_{\max} = 6, 8$ and 10 factors when $r_{\max} = 10, 12, 14, 16$. $ED$ chooses three factors when $r_{\max} = 6$ and four when $r_{\max} = 8, 10, 12, 14, 16$. $ER$ selects one factor for all values of $r_{\max}$. For the standardized panel, $AIC$, $CAIC$, $HQ_2$ and $IC_{p3}$ choose the maximum number of factors set by $r_{\max}$. $HQ_3$ and $IC_{p2}$ estimate seven factors for all values of $r_{\max}$ except 6; and six factors when $r_{\max} = 6$. $BIC_3$ selects two, two, three, four, four and five factors when $r_{\max}$ changes from 6 to 16. By contrast, $ER$, $GR$, $ED$, $HQ_5$, $BIC$, $HQ_4$ select one, one, one, two, six and six factors, respectively, regardless of the values of $r_{\max}$.

Results in Table 7 are so varied that it is hard to arrive at any definitive decision on the number of factors. The results differ depending on the practice of standardization and the maximum number of factors allowed. Regarding the maximum number of factors, there seems to be no criterion for choosing it. But for the issue of standardization, it will be beneficial to examine the degree of heteroskedasticity in the data sets because simulation results in Subsection 5.6 indicate that standardization can be beneficial when a few individuals have much larger variance than the others. In both the data sets we used, it is found that a few of the variables have much higher variance than the others. The maximum and minimum standard deviation of the variables in Stock and Watson's (2002) data set are 934 and 0.0017, respectively. Those for Stock and Watson (2005) data set are 53 and 0.0016. These results indicate that the results based on standardization may be more reliable.

Even if we exclude the results that do not adopt standardization, the results are still too mixed to provide a definitive a conclusion. To clarify this situation, we present scree plots for standardized Stock and Watson's (2002, 2005) data sets in Figure 4. Scree plots show ordered eigenvalues of $\frac{X'X}{NT}$. The number of factors that triggers a sharp drop and level-off (called an elbow) in the plots is interpreted as the true number of factors. Unfortunately, there seem to be no elbows in the plots, but the level-off seems to start at four to six factors for Stock and Watson's (2002) data and at six to eight factors for Stock and Watson's (2005) data. These findings imply that the true number of factors may be larger than suggested by $ER$, $GR$ and $ED$.

Perhaps a reasonable rule of thumb for the choice of the number of factors is to select the results that are robust to the maximum number of factors allowed. According to this rule of-thumb, Stock and Watson's (2002) data set seems to have one, two or four factors. Excluding one and two as too extreme, four factors seem to be appropriate for this data set. Based on the same rule, Stock and Watson's (2005) data set seems to have six or seven factors.

In order to check whether one or two factors is too small for Stock and Watson's data sets, we may consider the degree of cross-sectional correlation in the idiosyncratic errors for each number of factors. If a chosen number of factors is too small, there will remain quite strong cross-sectional correlation in the idiosyncratic errors. To this end, we plot the absolute values of Moscone and Tosetti's (2009) test statistic for cross-sectional correlation in Figure 5. Moscone and Tosetti's simulation results show that the test is good at detecting cross-sectional correlation generated by a factor structure. The test statistic is defined as

$$CD_Z = \sqrt{\frac{2(T-3)}{N(N-1)}} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Z_{ij},$$

where $Z_{ij} = 0.5 \ln \left( \frac{1+\hat{\psi}_{ij}}{1-\hat{\psi}_{ij}} \right)$, $\hat{\psi}_{ij} = \sum_{t=1}^{T} \hat{e}_{it}\hat{e}_{jt} / \sqrt{\sum_{t=1}^{T} \hat{e}_{it}^2} \sqrt{\sum_{t=1}^{T} \hat{e}_{it}^2}$ and $\hat{e}_{it}$ is the principal component estimator of $e_{it}$. Under the null hypothesis of no cross-sectional correlation in the idiosyncratic errors, $CD_Z$ tends to a standard normal distribution in the limit.

Since cross-sectional correlation in the idiosyncratic errors is allowed for the model selection criteria we have considered, the plots themselves do not provide information on the true number of factors. However, we find in the plot using Stock and Watson's (2002) data set that the value of the test statistic decreases sharply at five factors. When it is one or two, the test statistic takes a large value. This can be interpreted as evidence that there remains quite strong cross-sectional correlation at one and two factors for Stock and Watson's (2002) data set. In the plot using Stock and Watson's (2005) data set, the value of the test statistic decreases sharply again at five factors. The test statistic takes large values again at one and two factors. These suggest that quite strong cross-sectional correlation is present at one and two factors, which can further be decreased by increasing the number of factors. In a nutshell, Figure 5 presents evidence against a very small number of factors for Stock and Watson's (2002, 2005) data sets, although admittedly this evidence is not based on formal hypothesis testing.

# 7 Summary and further remarks

This paper has proposed the AIC, CAIC, BIC and Hannan and Quinn's (1979) information criterion for selecting the number of factors in a model. They have conventionally been used in the literature on model selection, but have not been considered in the literature on factor models. It is shown that BIC and Hannan and Quinn's information criterion are consistent, while AIC and CAIC are not.

This paper also reports extensive simulation results that compare performance of the extant and new procedures for the selection of the number of factors. The DGP for the simulation consists of serially correlated factors, and serially and cross-sectionally correlated idiosyncratic errors. The idiosyncratic errors are either homoskedastic or heteroskedastic, and fat-tailed idiosyncratic errors are also considered as are those with fat tails and outliers having a much larger variance than the others. The simulation results demonstrate the difficulty of determining which criterion performs best. The order of performance depends on the chosen DGP. In practice, therefore, it is advised to consider several criteria at the same time, especially $BIC$, $HQ_c$, $IC_{p2}$, $BIC_3$, $ER$, $GR$ and $ED$ because they show relatively good performance.

This paper also reports applications of these model selection criteria to Stock and Watson's (2002, 2005) data sets. The results differ widely depending on the model selection criterion in use. However, evidence exists to suggest four factors for Stock and Watson's (2002) data and six or seven factors for Stock and Watson's (2005).

In using $HQ_c$ in practice, the choice of $c$ is important, but its optimal value is unknown. Since $HQ_c$ performs well according to our simulation results, studies on how to choose $c$ in an adaptive manner seem to be practically important and should be pursued.

# 8 Appendix I: Derivation of AIC

First, we will derive the minimizer of the Kullback–Leibler information measure. The conditional (on $\Xi$) log-likelihood function for model (2) under the assumption $V_{rt} \mid F_r \sim \text{i. i. d.}\, N(0, \Omega_r)$ where $\Omega_r = \text{diag}\left[\sigma_1^2, \dots, \sigma_N^2\right]$ is

$$\begin{aligned} l(\Lambda_r, F_r, \Omega_r) &= -\frac{1}{2}TN\ln(2\pi) - \frac{1}{2}T\ln|\Omega_r| - \frac{1}{2}\operatorname{tr}\left\{\Omega_r^{-1}(X - F_r\Lambda_r')'(X - F_r\Lambda_r')\right\} \\ &= -\frac{1}{2}TN\ln(2\pi) - \frac{1}{2}T\sum_{i=1}^{N}\ln\left(\sigma_i^2\right) - \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N}\frac{\left(x_{it} - \sum_{k=1}^{r}\lambda_{ik}f_{tk}\right)^2}{\sigma_i^2}. \end{aligned} \tag{I.1}$$

Using this formula and ignoring a constant, the Kullback–Leibler information measure is written as

$$\begin{aligned} -E_F(l(\Lambda_r, F_r, \Omega_r) \mid \Xi) &= E_F\left(\frac{1}{2}T\ln|\Omega_r| + \frac{1}{2}\operatorname{tr}\left\{\Omega_r^{-1}(X - F_r\Lambda_r')'(X - F_r\Lambda_r')\right\} \mid \Xi\right) \\ &= \frac{1}{2}T\ln|\Omega_r| + \frac{1}{2}\operatorname{tr}\left\{\Omega_r^{-1}(\Xi - F_r\Lambda_r')'(\Xi - F_r\Lambda_r')\right\} \qquad \text{(I.2)} \\ &\quad + \frac{1}{2}T\operatorname{tr}\left\{\Omega_r^{-1}\Omega_e\right\}, \end{aligned}$$

where $E_F(\cdot)$ denotes the expectation operator using the true model (1). Estimators of $\Lambda_r, F_r$ and $\Omega_r$ that minimize the Kullback–Leibler information measure (I.2) will be called the Kullback–Leibler best estimators (KLBEs).

Assume for the moment that the matrix $\Omega_r$ is known. Then, we need to minimize $\mathrm{tr}\left\{\Omega_r^{-1}(\Xi - F_r\Lambda_r')'(\Xi - F_r\Lambda_r')\right\}$ with respect to $F_r$ and $\Lambda_r$. A standard theory of multivariate regression yields a preliminary KLBE of $\Lambda_r$, $\hat{\Lambda}_r = \Xi' F_r(F_r'F_r)^{-1}$. Plugging this in (I.2) and omitting the first term, we obtain $\mathrm{tr}\left\{\Omega_r^{-1}\Xi'(I - P_{F_r})\Xi\right\}$ with $P_{F_r} = F_r(F_r'F_r)^{-1}F_r'$. The KLBE of $F_r$ is obtained by maximizing $\mathrm{tr}\left\{\frac{1}{T}\Omega_r^{-1}\Xi' F_r F_r'\Xi\right\} = \mathrm{tr}\left\{F_r'\left(\frac{1}{T}\Xi\Omega_r^{-1}\Xi'\right)F_r\right\}$. Therefore, the KLBE of $F_r$, denoted by $\tilde{F}_r$, is the matrix consisting of the eigenvectors corresponding to the $r$ largest eigenvalues of the matrix $\frac{1}{T}\Xi\Omega_r^{-1}\Xi'$ with the standardization $\tilde{F}_r'\tilde{F}_r = T \times I_r$. Equivalently, it is $\sqrt{T}$ times the matrix consisting of eigenvectors corresponding to the $r$ largest eigenvalues of the matrix $\Xi\Omega_r^{-1}\Xi'$. The KLBE of $\Lambda_r$ is given by $\tilde{\Lambda}_r = \frac{1}{T}\Xi'\tilde{F}_r$. The KLBEs introduced so far assume that $\Omega_r$ is known. The first-order condition for the KLBE of $\Omega_r$ is

$$\begin{aligned}
\frac{1}{2}T\,\mathrm{tr}\left(\Omega_r^{-1}\right)d\Omega_r - \frac{1}{2}\,\mathrm{tr}&\left\{\Omega_r^{-1}d\Omega_r\Omega_r^{-1}(\Xi - F_r\Lambda_r')'(\Xi - F_r\Lambda_r')\right\} \\
&- \frac{1}{2}T\,\mathrm{tr}\left\{\Omega_r^{-1}d\Omega_r\Omega_r^{-1}\Omega_e\right\} \\
&= \frac{1}{2}\,\mathrm{tr}\left[\Omega_r^{-1}(T\Omega_r - (\Xi - F_r\Lambda_r')'(\Xi - F_r\Lambda_r') - T\Omega_e)\right]\Omega_r^{-1}d\Omega_r \\
&= 0,
\end{aligned}$$

which gives the KLBE of $\Omega_r$ as $\tilde{\Omega}_r = \frac{1}{T}(\Xi - \tilde{F}_r\tilde{\Lambda}_r')'(\Xi - \tilde{F}_r\tilde{\Lambda}_r') + \Omega_e$.

Next, we will derive $J(\delta_o)$. Partial differentiations of the likelihood function (I.1) give

$$\begin{aligned}
\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \Lambda_{rk}} &= \sum_{t=1}^{T}(x_{kt} - \Lambda_{rk}'F_{rt})F_{rt}/\sigma_k^2, \quad (k = 1, \ldots, N); \qquad \text{(I.3)} \\
\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial F_{rs}} &= \sum_{i=1}^{N}(x_{is} - \Lambda_{ri}'F_{rs})\Lambda_{ri}/\sigma_i^2, \quad (s = 1, \ldots, T); \\
\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \sigma_k^2} &= \frac{1}{2}\left(-\frac{T}{\sigma_k^2} + \sum_{t=1}^{T}(x_{kt} - \Lambda_{rk}'F_{rt})^2/\sigma_k^4\right), \quad (k = 1, \ldots, N).
\end{aligned}$$

Letting $\theta = [\mathrm{vec}(\Lambda_r')', \mathrm{vec}(F_r')', \left[\sigma_1^2, \ldots, \sigma_N^2\right]]'$ and using relations (I.3), we have

$$\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \theta} \frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \theta'} = \begin{bmatrix} A^{\Lambda\Lambda} & A^{\Lambda F} & A^{\Lambda\sigma} \\ A^{F\Lambda} & A^{FF} & A^{F\sigma} \\ A^{\sigma\Lambda} & A^{\sigma F} & A^{\sigma\sigma} \end{bmatrix}, \tag{I.4}$$

with

$$A^{\Lambda\Lambda} = \left[\left(\sum_{t=1}^{T}(x_{kt} - \Lambda_{rk}'F_{rt})F_{rt}/\sigma_k^2\right)\left(\sum_{s=1}^{T}(x_{js} - \Lambda_{rj}'F_{rs})F_{rs}'/\sigma_j^2\right)\right]_{k,j=1,\ldots,N}$$

$$A^{\Lambda F} = \left[\left(\sum_{t=1}^{T}(x_{kt} - \Lambda_{rk}'F_{rt})F_{rt}/\sigma_k^2\right)\left(\sum_{i=1}^{N}(x_{is} - \Lambda_{ri}'F_{rs})\Lambda_{ri}'/\sigma_i^2\right)\right]_{\substack{k=1,\ldots,N \\ s=1,\ldots,T}}$$

$$A^{\Lambda\sigma} = \left[\left(\sum_{t=1}^{T}\frac{(x_{kt} - \Lambda_{rk}'F_{rt})F_{rt}}{\sigma_k^2}\right)\left(\frac{1}{2}\left(-\frac{T}{\sigma_j^2} + \sum_{s=1}^{T}\frac{(x_{js} - \Lambda_{rj}'F_{rs})^2}{\sigma_j^4}\right)\right)\right]_{k,j=1,\ldots,N}$$

$$A^{FF} = \left[\left(\sum_{i=1}^{N}(x_{it} - \Lambda_{ri}'F_{rt})\Lambda_{ri}/\sigma_i^2\right)\left(\sum_{j=1}^{N}(x_{js} - \Lambda_{rj}'F_{rs})\Lambda_{rj}'/\sigma_j^2\right)\right]_{t,s=1,\ldots,T}$$

$$A^{F\sigma} = \left[\left(\sum_{i=1}^{N}(x_{it} - \Lambda_{ri}'F_{rt})\Lambda_{ri}/\sigma_i^2\right)\left(\frac{1}{2}\left(-\frac{T}{\sigma_j^2} + \sum_{t=1}^{T}(x_{jt} - \Lambda_{rj}'F_{rt})^2/\sigma_j^4\right)\right)\right]_{\substack{t=1,\ldots,T \\ j=1,\ldots,N}}$$

$$A^{\sigma\sigma} = \left[\frac{1}{4}\left(-\frac{T}{\sigma_k^2} + \sum_{t=1}^{T}\frac{(x_{kt} - \Lambda_{rk}'F_{rt})^2}{\sigma_k^4}\right)\left(-\frac{T}{\sigma_j^2} + \sum_{s=1}^{T}\frac{(x_{js} - \Lambda_{rj}'F_{rs})^2}{\sigma_j^4}\right)\right]_{k,j=1,\ldots,N}.$$

The following lemma reports expected values of the elements in relation (I.4) evaluated at the KLBEs.

**Lemma I.1** *(i)* $E\left(\tilde{A}_{kk}^{\Lambda\Lambda} \mid \Xi\right) = \frac{T\sigma_{ek}^2}{\tilde{\sigma}_k^4} I_r; E\left(\tilde{A}_{kj}^{\Lambda\Lambda} \mid \Xi\right) = 0\ (k \neq j).$

*(ii)* $E\left(\tilde{A}_{ks}^{\Lambda F} \mid \Xi\right) = \frac{\sigma_{ek}^2}{\tilde{\sigma}_k^4}\tilde{F}_{rs}\tilde{\Lambda}_{rk}'.$

*(iii)* $E\left(\tilde{A}_{kj}^{\Lambda\sigma} \mid \Xi\right) = 0.$

*(iv)* $E\left(\tilde{A}_{tt}^{FF} \mid \Xi\right) = \sum_{i=1}^{N}\frac{\sigma_{ei}^2}{\tilde{\sigma}_i^4}\tilde{\Lambda}_{ri}\tilde{\Lambda}_{ri}'; E\left(A_{ts}^{FF} \mid \Xi\right) = 0(t \neq s).$

*(v)* $E\left(\tilde{A}_{tj}^{F\sigma} \mid \Xi\right) = \frac{\sigma_{ej}^2}{\tilde{\sigma}_j^6}\left(\xi_{jt} - \tilde{\Lambda}_{rj}'\tilde{F}_{rt}\right)\tilde{\Lambda}_{rj}.$

*(vi)* $E\left(\tilde{A}_{kk}^{\sigma\sigma} \mid \Xi\right) = \frac{T\sigma_{ek}^2\left(2\tilde{\sigma}_k^2 - \sigma_{ek}^2\right)}{2\tilde{\sigma}_k^8}; E\left(\tilde{A}_{kj}^{\sigma\sigma} \mid \Xi\right) = 0(k \neq j).$

**Proof:**

(i) Using $x_{kt} = \xi_{kt} + e_{kt}$, write

$$
\begin{aligned}
E\left(\tilde{A}^{\Lambda\Lambda}_{kj} \mid \Xi\right) &= E\left(\sum_{t=1}^{T}\sum_{s=1}^{T} \frac{(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt} + e_{kt})(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs} + e_{js})\tilde{F}_{rt}\tilde{F}'_{rs}}{\left(\tilde{\sigma}^2_k\tilde{\sigma}^2_j\right)} \mid \Xi\right) \\
&= E\left(\left(\sum_{t=1}^{T}(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt})\tilde{F}_{rt}\right)\left(\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})\tilde{F}'_{rs}\right)/\left(\tilde{\sigma}^2_k\tilde{\sigma}^2_j\right) \mid \Xi\right) \\
&\quad + E\left(\sum_{t=1}^{T}\sum_{s=1}^{T} e_{kt}e_{js}\tilde{F}_{rt}\tilde{F}'_{rs}/\left(\tilde{\sigma}^2_k\tilde{\sigma}^2_j\right) \mid \Xi\right) \\
&= B_1 + B_2, \text{ say.}
\end{aligned} \tag{I.5}
$$

But $B_1 = 0$ due to part (i) Lemma I.3 given below; and $B_2 = \frac{1}{\tilde{\sigma}^4_k}\sum_{t=1}^{T} E(e^2_{kt} \mid \Xi)\tilde{F}_{rt}\tilde{F}'_{rt} = \frac{T\sigma^2_{ek}}{\tilde{\sigma}^4_k} I_r$ when $k = j$ and $B_2 = 0$ when $k \neq j$. Note that the cross-product terms in (I.5) are equal to zero since $E(e_{kt}(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})\tilde{F}_{rt}\tilde{F}'_{rs}/(\tilde{\sigma}^2_k\tilde{\sigma}^2_j) \mid \Xi) = E\left(e_{kt} \mid \Xi\right)\left(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs}\right)\tilde{F}_{rt}\tilde{F}'_{rs}/\left(\tilde{\sigma}^2_k\tilde{\sigma}^2_j\right) = 0.$ ∎

(ii) Using Lemma I.3 below, we obtain

$$
\begin{aligned}
E\left(\tilde{A}^{\Lambda F}_{ks}\right) &= E\left(\left(\sum_{t=1}^{T} \frac{(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt} + e_{kt})\tilde{F}_{rt}}{\tilde{\sigma}^2_k}\right)\left(\sum_{i=1}^{N} \frac{(\xi_{is} - \tilde{\Lambda}'_{ri}\tilde{F}_{rs} + e_{is})\tilde{\Lambda}'_{ri}}{\tilde{\sigma}^2_i}\right) \mid \Xi\right) \\
&= E\left(\left(\sum_{t=1}^{T} e_{kt}\tilde{F}_{rt}/\tilde{\sigma}^2_k\right)\left(\sum_{i=1}^{N} e_{is}\tilde{\Lambda}'_{ri}/\tilde{\sigma}^2_i\right) \mid \Xi\right).
\end{aligned}
$$

But this reduces to

$$
\sum_{t=1}^{T}\sum_{i=1}^{N} \frac{\tilde{F}_{rt}\tilde{\Lambda}'_{ri}}{\tilde{\sigma}^2_k\tilde{\sigma}^2_i} E\left(e_{kt}e_{is} \mid \Xi\right) = \sum_{i=1}^{N} \frac{\tilde{F}_{rs}\tilde{\Lambda}'_{ri}}{\tilde{\sigma}^2_k\tilde{\sigma}^2_i} E\left(e_{ks}e_{is} \mid \Xi\right) = \frac{\sigma^2_{ek}}{\tilde{\sigma}^4_k}\tilde{F}_{rs}\tilde{\Lambda}'_{rk},
$$

because $\{e_{kt}\}$ are serially and cross-sectionally uncorrelated. ∎

(iii) Since $\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})^2 = T(\tilde{\sigma}_j^2 - \sigma_{ej}^2)$, we have

$$
\begin{aligned}
&-\frac{T}{\tilde{\sigma}_j^2} + \sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs} + e_{js})^2/\tilde{\sigma}_j^4 \qquad \text{(I.6)}\\
&= -\frac{T}{\tilde{\sigma}_j^2} + \sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})^2/\tilde{\sigma}_j^4 + \sum_{s=1}^{T} e_{js}^2/\tilde{\sigma}_j^4 + 2\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}/\tilde{\sigma}_j^4\\
&= -\frac{T}{\tilde{\sigma}_j^2} + \frac{T(\tilde{\sigma}_j^2 - \sigma_{ej}^2)}{\tilde{\sigma}_j^4} + \sum_{s=1}^{T} e_{js}^2/\tilde{\sigma}_j^4 + 2\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}/\tilde{\sigma}_j^4\\
&= \frac{\sigma_{ej}^2}{\tilde{\sigma}_j^4}\left(\sum_{s=1}^{T} e_{js}^2/\sigma_{ej}^2 - T\right) + 2\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}/\tilde{\sigma}_j^4.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
E\left(\tilde{A}_{kj}^{\Lambda\sigma} \mid \Xi\right) &= \frac{1}{2}E\left[\left(\sum_{t=1}^{T}\frac{e_{kt}\tilde{F}_{rt}}{\tilde{\sigma}_k^2}\right)\left(\frac{\sigma_{ej}^2}{\tilde{\sigma}_j^4}\left(\sum_{s=1}^{T}\frac{e_{js}^2}{\sigma_{ej}^2} - T\right) + 2\sum_{s=1}^{T}\frac{(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}}{\tilde{\sigma}_j^4}\right) \mid \Xi\right]\\
&= \frac{\sigma_{ej}^2}{2}E\left[\left(\sum_{t=1}^{T}\sum_{s=1}^{T} e_{kt}\left(\sum_{s=1}^{T} e_{js}^2/\sigma_{ej}^2 - T\right)\tilde{F}_{rt}/\left(\tilde{\sigma}_k^2\tilde{\sigma}_j^4\right)\right) \mid \Xi\right] \qquad \text{(I.7)}\\
&\quad + E\left[\left(\sum_{t=1}^{T}\sum_{s=1}^{T} e_{kt}e_{js}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})\tilde{F}_{rt}/\left(\tilde{\sigma}_k^2\tilde{\sigma}_j^4\right)\right) \mid \Xi\right]\\
&= \frac{\sigma_{ej}^2}{2}\sum_{t=1}^{T}\sum_{s=1}^{T} E\left(e_{kt}\left(\sum_{s=1}^{T} e_{js}^2/\sigma_{ej}^2 - T\right) \mid \Xi\right)\tilde{F}_{rt}/\left(\tilde{\sigma}_k^2\tilde{\sigma}_j^4\right)\\
&\quad + E(e_{k1}e_{j1} \mid \Xi)\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}_{rj}'\tilde{F}_{rs})\tilde{F}_{rs}/\left(\tilde{\sigma}_k^2\tilde{\sigma}_j^4\right)\\
&= 0.
\end{aligned}
$$

Note that part (i) of Lemma I.3 is used for the first and fourth equalities in relation (I.7). ∎

(iv) We have

$$
\begin{aligned}
E\left(\tilde{A}_{ts}^{FF} \mid \Xi\right) &= E\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{(\xi_{it}-\tilde{\Lambda}_{ri}'\tilde{F}_{rt}+e_{it})(\xi_{js}-\tilde{\Lambda}_{rj}'\tilde{F}_{rs}+e_{js})\tilde{\Lambda}_{ri}\tilde{\Lambda}_{rj}'}{\left(\tilde{\sigma}_i^2\tilde{\sigma}_j^2\right)} \mid \Xi\right) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} E\left(e_{it}e_{js} \mid \Xi\right)\tilde{\Lambda}_{ri}\tilde{\Lambda}_{rj}'/\left(\tilde{\sigma}_i^2\tilde{\sigma}_j^2\right) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}(\xi_{it}-\tilde{\Lambda}_{ri}'\tilde{F}_{rt})(\xi_{js}-\tilde{\Lambda}_{rj}'\tilde{F}_{rs})\tilde{\Lambda}_{ri}\tilde{\Lambda}_{rj}'/\left(\tilde{\sigma}_i^2\tilde{\sigma}_j^2\right) \\
&= \sum_{i=1}^{N} E\left(e_{it}e_{is} \mid \Xi\right)\tilde{\Lambda}_{ri}\tilde{\Lambda}_{ri}'/\left(\tilde{\sigma}_i^4\right),
\end{aligned}
$$

where part (ii) of Lemma I.3 is used for the third equality. The stated results follows from this. ▮

(v) Using relation (I.6) and the assumptions on $\{e_t\}$, we obtain

$$
\begin{aligned}
&E\left(\tilde{A}_{tj}^{F\sigma} \mid \Xi\right) \\
&= \frac{1}{2}E\left(\sum_{i=1}^{N}\frac{(\xi_{it}-\tilde{\Lambda}_{ri}'\tilde{F}_{rt}+e_{it})\tilde{\Lambda}_{ri}}{\tilde{\sigma}_i^2}\left(\frac{\sigma_{ej}^2}{\tilde{\sigma}_j^4}\left(\sum_{s=1}^{T}\frac{e_{js}^2}{\sigma_{ej}^2}-T\right)+2\sum_{s=1}^{T}\frac{(\xi_{js}-\tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}}{\tilde{\sigma}_j^4}\right) \mid \Xi\right) \\
&= \frac{1}{2}E\left(\left(\sum_{i=1}^{N} e_{it}\tilde{\Lambda}_{ri}/\tilde{\sigma}_i^2\right)\left(\frac{\sigma_{ej}^2}{\tilde{\sigma}_j^4}\left(\sum_{s=1}^{T} e_{js}^2/\sigma_{ej}^2-T\right)+2\sum_{s=1}^{T}(\xi_{js}-\tilde{\Lambda}_{rj}'\tilde{F}_{rs})e_{js}/\tilde{\sigma}_j^4\right) \mid \Xi\right) \\
&= \frac{\sigma_{ej}^2}{2\tilde{\sigma}_j^4}E\left(\left(\sum_{i=1}^{N} e_{it}\tilde{\Lambda}_{ri}/\tilde{\sigma}_i^2\right)\left(\sum_{s=1}^{T} e_{js}^2/\sigma_{ej}^2-T\right) \mid \Xi\right) \\
&\quad + \sum_{i=1}^{N}\sum_{s=1}^{T} E\left(e_{it}e_{js} \mid \Xi\right)(\xi_{js}-\tilde{\Lambda}_{rj}'\tilde{F}_{rs})\tilde{\Lambda}_{ri}/\left(\tilde{\sigma}_i^2\tilde{\sigma}_j^4\right) \\
&= \frac{1}{2\tilde{\sigma}_j^4}\sum_{i=1}^{N} E\left(e_{it}e_{jt}^2 \mid \Xi\right)\tilde{\Lambda}_{ri}/\tilde{\sigma}_i^2 + \sum_{i=1}^{N} E\left(e_{it}e_{jt} \mid \Xi\right)(\xi_{jt}-\tilde{\Lambda}_{rj}'\tilde{F}_{rt})\tilde{\Lambda}_{ri}/\left(\tilde{\sigma}_i^2\tilde{\sigma}_j^4\right) \\
&= E\left(e_{jt}^2 \mid \Xi\right)(\xi_{jt}-\tilde{\Lambda}_{rj}'\tilde{F}_{rt})\tilde{\Lambda}_{rj}/\tilde{\sigma}_j^6 \\
&= \frac{\sigma_{ej}^2}{\tilde{\sigma}_j^6}(\xi_{jt}-\tilde{\Lambda}_{rj}'\tilde{F}_{rt})\tilde{\Lambda}_{rj}.
\end{aligned}
$$

(vi) Using relation (I.6), we obtain

$$
\begin{aligned}
E(\tilde{A}^{\sigma\sigma}_{kj} \mid \Xi) =& \frac{1}{4}E( \left(\frac{\sigma^2_{ek}}{\tilde{\sigma}^4_k}\left(\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T\right) + 2\sum_{t=1}^{T}(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt})e_{kt}/\tilde{\sigma}^4_k\right) \\
&\times \left(\frac{\sigma^2_{ej}}{\tilde{\sigma}^4_j}\left(\sum_{s=1}^{T} e^2_{js}/\sigma^2_{ej} - T\right) + 2\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})e_{js}/\tilde{\sigma}^4_j\right) \mid \Xi) \\
=& \frac{\sigma^2_{ek}\sigma^2_{ej}}{4\tilde{\sigma}^4_k\tilde{\sigma}^4_j}E\left(\left(\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T\right)\left(\left(\sum_{s=1}^{T} e^2_{js}/\sigma^2_{ej} - T\right)\right) \mid \Xi\right) \\
&+ \frac{1}{\tilde{\sigma}^4_k\tilde{\sigma}^4_j}E\left(\left(\sum_{t=1}^{T}\sum_{s=1}^{T}(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt})(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})e_{kt}e_{js}\right) \mid \Xi\right) \\
&+ \frac{\sigma^2_{ek}}{2\tilde{\sigma}^4_k\tilde{\sigma}^4_j}E\left(\left(\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T\right)\left(\sum_{s=1}^{T}(\xi_{js} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})e_{js}\right) \mid \Xi\right) \\
&+ \frac{\sigma^2_{ej}}{2\tilde{\sigma}^4_k\tilde{\sigma}^4_j}E\left(\left(\sum_{t=1}^{T}(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt})e_{kt}\right)\left(\sum_{s=1}^{T} e^2_{js}/\sigma^2_{ej} - T\right) \mid \Xi\right) \\
=& C_1 + C_2 + C_3 + C_4, \text{ say.}
\end{aligned}
$$

Since $\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T$ is distributed as $\chi^2(T) - E\left(\chi^2(T)\right)$ conditional on $\Xi$, $C_1 = 0$ if $k \neq j$ and $C_1 = \frac{T\sigma^4_{ek}}{2\tilde{\sigma}^8_k}$ otherwise. Next, using the assumption that $\{e_t\}$ are uncorrelated, write

$$
C_2 = \frac{1}{\tilde{\sigma}^4_k\tilde{\sigma}^4_j}\left(\sum_{t=1}^{T}(\xi_{kt} - \tilde{\Lambda}'_{rk}\tilde{F}_{rt})(\xi_{jt} - \tilde{\Lambda}'_{rj}\tilde{F}_{rs})E\left(e_{kt}e_{jt} \mid \Xi\right)\right).
$$

Thus, $C_2 = 0$ when $k \neq j$ and $C_2 = \frac{T\sigma^2_{ek}(\tilde{\sigma}^2_k - \sigma^2_{ek})}{\tilde{\sigma}^8_k}$ when $k = j$. Using the fact that $E\left(\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T\right) = 0$, we obtain $C_3 = 0$ when $k \neq j$. When $k = j$,

$$
\begin{aligned}
C_3 =& \frac{\sigma^2_{ek}}{2\tilde{\sigma}^8_k}E\left(\left(\sum_{t=1}^{T} e^2_{kt}/\sigma^2_{ek} - T\right)\left(\sum_{s=1}^{T}(\xi_{ks} - \tilde{\Lambda}'_{rk}\tilde{F}_{rs})e_{ks}\right) \mid \Xi\right) \\
=& \frac{1}{2\tilde{\sigma}^8_k}\left(\sum_{t=1}^{T}\sum_{s=1}^{T} E\left(e^2_{kt}e_{ks} \mid \Xi\right)(\xi_{ks} - \tilde{\Lambda}'_{rk}\tilde{F}_{rs})\right) \\
=& 0.
\end{aligned}
$$

Likewise, $C_4 = 0$ for any $k$ and $j$. ∎

Last, we will derive $I(\delta_o)$. Using relations (I.3), we obtain

$$-\frac{\partial^2 l(\Lambda_r, F_r, \Omega_r)}{\partial\theta\partial\theta'} = \begin{bmatrix} H^{\Lambda\Lambda} & H^{\Lambda F} & H^{\Lambda\sigma} \\ H^{F\Lambda} & H^{FF} & H^{F\sigma} \\ H^{\sigma\Lambda} & H^{\sigma F} & H^{\sigma\sigma} \end{bmatrix},$$

where

$$\begin{aligned} H^{\Lambda\Lambda} &= \sum_{t=1}^{T} F_{rt}F_{rt}'/\sigma_1^2 \oplus \cdots \oplus \sum_{t=1}^{T} F_{rt}F_{rt}'/\sigma_N^2 \qquad \text{(I.8)} \\ H^{\Lambda F} &= \left[\left(\Lambda_{rk}'F_{rs}I_r + F_{rs}\Lambda_{rk}' - x_{ks}I_r\right)/\sigma_k^2\right]_{\substack{k=1,\dots,N \\ s=1,\dots,T}} \\ H^{\Lambda\sigma} &= \sum_{t=1}^{T}(x_{1t} - \Lambda_{r1}'F_{rt})F_{rt}/\sigma_1^4 \oplus \cdots \oplus \sum_{t=1}^{T}(x_{Nt} - \Lambda_{rN}'F_{rt})F_{rt}/\sigma_N^4 \\ H^{FF} &= \sum_{i=1}^{N} \Lambda_{ri}\Lambda_{ri}'/\sigma_i^2 \oplus \cdots \oplus \sum_{i=1}^{N} \Lambda_{ri}\Lambda_{ri}'/\sigma_i^2 \\ H^{F\sigma} &= \left[(x_{jt} - \Lambda_{rj}'F_{rt})\Lambda_{rj}/\sigma_j^4\right]_{\substack{t=1,\dots,T \\ j=1,\dots,N}} \\ H^{\sigma\sigma} &= \text{diag}\left[\frac{1}{\sigma_1^6}\sum_{t=1}^{T}(x_{1t} - \Lambda_{r1}'F_{rt})^2 - \frac{T}{2\sigma_1^4}, \dots, \frac{1}{\sigma_N^6}\sum_{t=1}^{T}(x_{Nt} - \Lambda_{rN}'F_{rt})^2 - \frac{T}{2\sigma_N^4}\right]. \end{aligned}$$

Expected values of the above elements evaluated at the KLBEs are reported in the following lemma.

**Lemma I.2** *(i)* $E(\tilde{H}^{\Lambda\Lambda} \mid \Xi) = T\,\text{diag}\left[I_r/\tilde{\sigma}_1^2, \dots, I_r/\tilde{\sigma}_N^2\right]$.

*(ii)* $E(H^{\Lambda F} \mid \Xi) = \left[\left(\left(\tilde{\Lambda}_{rk}'\tilde{F}_{rs} - \xi_{ks}\right)I_r + \tilde{F}_{rs}\tilde{\Lambda}_{rk}'\right)/\tilde{\sigma}_k^2\right]_{\substack{k=1,\dots,N \\ s=1,\dots,T}}$

*(iii)* $E(H^{\Lambda\sigma} \mid \Xi) = 0$.

*(iv)* $E(H^{FF} \mid \Xi) = \text{diag}\left[\sum_{i=1}^{N}\tilde{\Lambda}_{ri}\tilde{\Lambda}_{ri}'/\tilde{\sigma}_i^2, \dots, \sum_{i=1}^{N}\tilde{\Lambda}_{ri}\tilde{\Lambda}_{ri}'/\tilde{\sigma}_i^2\right]$.

*(v)* $E(H^{F\sigma} \mid \Xi) = \left[(\xi_{jt} - \tilde{\Lambda}_{rj}'\tilde{F}_{rt})\tilde{\Lambda}_{rj}/\tilde{\sigma}_j^4\right]_{\substack{t=1,\dots,T \\ j=1,\dots,N}}$.

*(vi)* $E(H^{\sigma\sigma} \mid \Xi) = \text{diag}\left[\frac{T}{2\tilde{\sigma}_1^4}, \dots, \frac{T}{2\tilde{\sigma}_N^4}\right]$.

**Proof:** These follow straightforwardly from relations (I.8) along with the definitions of the KLBEs and Lemma I.3 (i). ∎

The AIC can derived using the results obtained thus far. Since $\tilde{\sigma}_i^2 \xrightarrow{p} \sigma_i^2$ for $i = 1, \ldots, N$ and $\tilde{\Lambda}'_{rk}\tilde{F}_{rs} - \xi_{ks} \simeq 0$, we obtain using Lemmas I.1 and I.2

$$\operatorname{tr}\left(E\left(\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \theta}\frac{\partial l(\Lambda_r, F_r, \Omega_r)}{\partial \theta'}_{|\theta=\tilde{\theta}}\right)\left[E\left(-\frac{\partial^2 l(\Lambda_r, F_r, \Omega_r)}{\partial \theta \partial \theta'}_{|\theta=\tilde{\theta}}\right)\right]^{-1}\right)$$
$$\simeq r(N+T) + N,$$

which gives

$$AIC = T\sum_{i=1}^{N} \ln\left(\hat{\sigma}_i^2\right) + 2\left(r(N+T) + N\right).$$

Now, replacing $\hat{\sigma}_i^2$ with its feasible version, $\hat{\sigma}_{fi}^2(r)$, provides the AIC we desire.

The following lemma is used to prove Lemma I.1.

**Lemma I.3** *(i)* $\tilde{F}'_r\left(\Xi - \tilde{F}_r\tilde{\Lambda}'_r\right) = 0$*;*

*(ii)* $\left(\Xi - \tilde{F}_r\tilde{\Lambda}'_r\right)\tilde{\Omega}_r^{-1}\tilde{\Lambda}_r = 0.$

**Proof:**.

(i) This follows from the definitional relation $\tilde{\Lambda}_r = \frac{1}{T}\Xi'\tilde{F}_r$.

(ii) Let $\eta_i$ be the $i$-th largest eigenvalue of matrix $\Xi\tilde{\Omega}_r^{-1}\Xi'/T$. Then, the definitions of $\tilde{\Lambda}_r$ and $\tilde{F}_r$ yield

$$\begin{aligned}(\Xi - \tilde{F}_r\tilde{\Lambda}'_r)\tilde{\Omega}_r^{-1}\tilde{\Lambda}_r &= \left(\Xi\tilde{\Omega}_r^{-1}\Xi'\right)\tilde{F}_r/T - \tilde{F}_r\left[\tilde{F}'_r\left(\Xi\tilde{\Omega}_r^{-1}\Xi'/T\right)\tilde{F}_r\right]/T \\ &= \left(\Xi\tilde{\Omega}_r^{-1}\Xi'\right)\tilde{F}_r/T - \tilde{F}_r\operatorname{diag}(\eta_1, \ldots, \eta_r)/T \\ &= 0. \quad \blacksquare\end{aligned}$$

# 9 Appendix II: Derivation of the corrected AIC

The formula of the corrected AIC given in Subsection 3.2 follows from the lemma below upon replacing $\hat{\sigma}_i^2$ with its feasible version, $\hat{\sigma}_{fi}^2(r)$.

**Lemma II.1** *(i)* $E\left[\operatorname{tr}\left\{\hat{\Omega}_r^{-1}(\Xi - \hat{F}_r\hat{\Lambda}'_r)'(\Xi - \hat{F}_r\hat{\Lambda}'_r)\right\}\right] = \frac{NTr}{T-r-2}$

*(ii)* $E\left[\mathrm{tr}\left(\hat{\Omega}_r^{-1}\Omega_e\right)\right] = \frac{NT}{T-r-2}$.

**Proof:**

(i) Let $P_A = A(A'A)^{-1}A'$ and $M_A = I - P_A$. Since $\Xi - \hat{F}_r\hat{\Lambda}_r' = \Xi - P_{\hat{F}_r}X = \Xi - P_{\hat{F}_r}(\Xi + \mathcal{E}) = M_{\hat{F}_r}\Xi - P_{\hat{F}_r}\mathcal{E}$, we have

$$\mathrm{tr}\left\{\hat{\Omega}_r^{-1}(\Xi - \hat{F}_r\hat{\Lambda}_r')'(\Xi - \hat{F}_r\hat{\Lambda}_r')\right\} = \mathrm{tr}\{\hat{\Omega}_r^{-1}\mathcal{E}'P_{\hat{F}_r}\mathcal{E}\} + \mathrm{tr}\{\hat{\Omega}_r^{-1}\Xi'M_{\hat{F}_r}\Xi\}. \quad \text{(II.1)}$$

The first term on the right-hand side of this equation can be written as

$$\mathrm{tr}\{\hat{\Omega}_r^{-1}\mathcal{E}'P_{\hat{F}_r}\mathcal{E}\} = \sum_{i=1}^{N} w_i'P_{\hat{F}_r}w_i/\hat{\sigma}_i^2,$$

where $w_i = [e_{i1}, \ldots e_{iT}]'$. We may write

$$w_i'P_{\hat{F}_r}w_i/\hat{\sigma}_{ei}^2 = \frac{Tr}{T-r}\frac{\left(\frac{w_i'P_{\hat{F}_r}w_i}{\sigma_{ei}^2}\right)/r}{\left(\frac{T\hat{\sigma}_i^2}{\sigma_{ei}^2}\right)/(T-r)}.$$

Due to a standard theory of econometrics, $\frac{w_i'P_{\hat{F}_r}w_i}{\sigma_{ei}^2} \sim \chi^2(r)$. Furthermore, letting $q_i = [x_{i1}, \ldots, x_{iT}]'$ and $s_i = [\xi_{i1}, \ldots, \xi_{iT}]'$ (note that $q_i = s_i + w_i$),

$$\frac{T\hat{\sigma}_i^2}{\sigma_{ei}^2} = \frac{q_i'M_{\hat{F}_r}q_i}{\sigma_{ei}^2} = \frac{w_i'M_{\hat{F}_r}w_i}{\sigma_{ei}^2} + \frac{s_i'M_{\hat{F}_r}s_i}{\sigma_{ei}^2} + \frac{2w_i'M_{\hat{F}_r}s_i}{\sigma_{ei}^2}. \quad \text{(II.2)}$$

The first term on the right-hand side of this equation, $\frac{w_i'M_{\hat{F}_r}w_i}{\sigma_{ei}^2}$, is distributed as $\chi^2(T-r)$ and the other terms are $o_p(1)$ as shown in Lemma II.2 below. In addition, $\frac{w_i'P_{\hat{F}_r}w_i}{\sigma_{ei}^2}$ and $\frac{w_i'M_{\hat{F}_r}w_i}{\sigma_{ei}^2}$ are independent due to a standard theory. Thus,

$$\mathrm{tr}\{\hat{\Omega}_r^{-1}\mathcal{E}'P_{\hat{F}_r}\mathcal{E}\} \sim \sum_{i=1}^{N}\frac{Tr}{T-r}z_i \text{ with } z_i \sim \text{i. i. d. } F(r, T-r).$$

For the second term in equation (II.1), write $\Xi'M_{\hat{F}_r}\Xi = \Lambda^oF^{o\prime}M_{\hat{F}_r}F^o\Lambda^{o\prime}$. But $M_{\hat{F}_r}F^o = o_p(1)$ as shown in Lemma II.2. Thus, the mean value of $\mathrm{tr}\left\{\hat{\Omega}_r^{-1}(\Xi - \hat{F}_r\hat{\Lambda}_r')'(\Xi - \hat{F}_r\hat{\Lambda}_r')\right\}$ is $\frac{NTr}{T-r-2}$.

(ii) Using relation (II.2) and the results following it, we obtain

$$\mathrm{tr}\left\{\hat{\Omega}_r^{-1}\Omega_e\right\} = \sum_{i=1}^{N}\frac{1}{\left(\frac{T\hat{\sigma}_i^2}{\sigma_{ei}^2}\right)/T} \sim \sum_{i=1}^{N}\frac{T}{c_i} \text{ with } c_i \sim \text{i. i. d. } \chi^2(T-r),$$

which implies $E\left[\operatorname{tr}\left(\hat{\Omega}_r^{-1}\Omega_e\right)\right] = \frac{NT}{T-r-2}$ (cf. Bernardo and Smith, 1994, 431) as required. ∎

**Lemma II.2** *Take Assumptions 1, 2 and 3 of Choi (2012) for $r^o$, $F^o$ and $\Lambda^o$, and let $\Sigma_{\Lambda^o} = \lim_{N\to\infty} \frac{\Lambda^{o\prime}\Omega_e^{-1}\Lambda^o}{N}$ and $\Sigma_{F^o} = p\lim_{T\to\infty} \frac{F^{o\prime}F^o}{T}$. Then, when $r \geq r^o$,*

*(i)* $M_{\hat{F}_r} F^o = o_p(1)$

*(ii)* $M_{\hat{F}_r} s_i = o_p(1)$.

**Proof:**

(i) Suppose that $r \geq r^o$. Let $\hat{F}_r = \begin{bmatrix} \overset{r^o}{\hat{F}_{r^o}} & \overset{r-r^o}{\hat{F}_{r-r^o}} \end{bmatrix}$. Then, we may write

$$M_{\hat{F}_r} F^o = F^o - \hat{F}_r \frac{\hat{F}_r' F^o}{T} = F^o - \left( \hat{F}_{r^o} \frac{\hat{F}_{r^o}' F^o}{T} + \hat{F}_{r-r^o} \frac{\hat{F}_{r-r^o}' F^o}{T} \right). \tag{II.3}$$

Due to Lemma B.3 of Choi (2012), $\frac{\hat{F}_{r^o}' F^o}{T} \xrightarrow{p} R_{r^o} = W_{r^o}^{\frac{1}{2}} \Theta' \Sigma_{\Lambda^o}^{-\frac{1}{2}}$, where $W_{r^o} = \operatorname{diag}[w_1, \cdots, w_{r^o}]$, $w_1 > \cdots > w_{r^o} > 0$ are eigenvalues of $\Sigma_{\Lambda^o}\Sigma_{F^o}$ and $\Theta_{r^o}$ is the corresponding eigenvector matrix: $\Theta_{r^o}'\Theta_{r^o} = I_{r^o}$. Moreover, $\hat{F}_{r^o} \xrightarrow{p} F^o J_{r^o}$ as Theorem 1 of Choi (2012) shows, where $J_{r^o} = \left(\frac{\Lambda_0'\Omega_e^{-1}\Lambda_0}{N}\right)\left(\frac{F^{o\prime}\hat{F}_{r^o}}{T}\right) W_{r^o NT}^{-1}$ and $W_{r^o NT}$ is an $r^o \times r^o$ diagonal matrix consisting of the first $r^o$ largest eigenvalues of $\frac{1}{NT} X \Omega_e^{-1} X'$ in descending order. Since $J_{r^o} \xrightarrow{p} R_{r^o}^{-1}$, as shown in Choi (2012), we have

$$\hat{F}_{r^o} \frac{\hat{F}_{r^o}' F^o}{T} \xrightarrow{p} F^o. \tag{II.4}$$

From the identity relation $\left(\frac{X\Omega_e^{-1}X'}{TN}\right)\hat{F}_r = \hat{F}_r W_{rNT}$ where $W_{rNT}$ is an $r \times r$ diagonal matrix consisting of the first $r$ largest eigenvalues of $\frac{1}{NT} X\Omega_e^{-1}X'$ in descending order, we obtain $\frac{1}{T} F^{o\prime} \left(\frac{X\Omega_e^{-1}X'}{TN}\right) \hat{F}_{r-r^o} = \frac{F^{o\prime}\hat{F}_{r-r^o}}{T} W_{(r-r^o)NT}$ with $W_{(r-r^o)NT}$ denoting the southeast block of the matrix $W_{rNT}$ of size $(r-r^o) \times (r-r^o)$. Using the same arguments as in the proof of Bai's (2003) proposition 1, we have

$$\frac{1}{NT} F^{o\prime} F^o \Lambda^{o\prime} \Omega_e^{-1} \Lambda^o \frac{F^{o\prime}\hat{F}_{r-r^o}}{T} + o_p(1) = \frac{F^{o\prime}\hat{F}_{r-r^o}}{T} W_{(r-r^o)NT}.$$

Because $W_{NT}^{r-r^o} \xrightarrow{p} 0$ as proven in Lemma II.3 and because $\frac{1}{NT} F^{o\prime} F^o \Lambda^{o\prime} \Omega_e^{-1} \Lambda^o$ is of full rank in the limit, it follows that

$$\frac{F^{o\prime} \hat{F}_{r-r^o}}{T} \xrightarrow{p} 0. \tag{II.5}$$

Using relations (II.3), (II.4) and (II.5), we obtain the stated result. ∎

(ii) This follows immediately from part (i). ∎

**Lemma II.3** *Under the same assumptions and notation as in Lemma II.2,*

$$W_{rNT} = T^{-1} \hat{F}_r' \left( \frac{X \Omega_e^{-1} X'}{TN} \right) \hat{F}_r \xrightarrow{p} \begin{bmatrix} W_{r^o} & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} r^o \\ r - r^o \end{matrix} \quad \textit{as } T, N \to \infty.$$

**Proof:** The off-diagonal elements of the matrix $\hat{F}_r' \left( \frac{X \Omega_e^{-1} X'}{TN} \right) \hat{F}_r$ are zeros by definition. Lemma B.3 of Choi (2012) proves that $T^{-1} \hat{F}_{r^o}' \left( \frac{X \Omega_e^{-1} X'}{TN} \right) \hat{F}_{r^o} \xrightarrow{p} W_{r^o}$. Since the rank of $\frac{X \Omega_e^{-1} X'}{TN}$ is $r^o$ in the limit, $T^{-1} \hat{F}_{r-r^o}' \left( \frac{X \Omega_e^{-1} X'}{TN} \right) \hat{F}_{r-r^o} \xrightarrow{p} 0$ as desired. ∎

# 10 Appendix III: Consistency

The information criteria $AIC$, $CAIC$, $BIC$ and $HQ_c$ can be written as

$$MSC(r) = T \sum_{i=1}^{N} \ln(\hat{\sigma}_{fi}^2(r)) + g(r, N, T),$$

where $g(r, N, T)$ is a function of $r, N$ and $T$. For $MSC(r)$ to be consistent, we should have $\lim_{N,T\to\infty} P\left[MSC(r) - MSC(r^o) < 0\right] = 0$ for all $r \neq r^o$ and $r \leq r_{\max}$. This relation is tantamount to

$$\lim_{N,T\to\infty} P\left[ \frac{1}{N} \sum_{i=1}^{N} \left( \ln\!\left(\hat{\sigma}_{fi}^2(r)\right) - \ln\!\left(\hat{\sigma}_{fi}^2(r^o)\right) \right) < \frac{g(r^o, N, T) - g(r, N, T)}{NT} \right] = 0 \tag{III.1}$$

for all $r \neq r^o$.

As discussed in Section 2, the variance estimator $\hat{\sigma}_{fi}^2(r)$ is obtained by running the principal component analysis on the model $x_{it}/\ddot{\sigma}_i(r) = \sum_{k=1}^{r} \lambda_{ik} f_{tk}/\ddot{\sigma}_i(r) + v_{it}/\ddot{\sigma}_i(r)$ where $\ddot{\sigma}_i^2(r)$ is the estimator of $\sigma_i^2$ defined in Section 2. Since $\ddot{\sigma}_i^2(r) \xrightarrow{p} \gamma_i$ as $N, T \to \infty$ $(0 < \gamma_i < \infty)$, as can be deduced from the proofs of Lemmas 2, 3 and 4 of Bai and Ng (2002), dividing the original model by $\ddot{\sigma}_i(r)$ does not change the asymptotic properties of the principal component estimators.

The mean-value theorem gives the following inequality

$$\begin{aligned}\sum_{i=1}^{N}\left(\ln\left(\hat{\sigma}^2_{fi}(r)\right)-\ln\left(\hat{\sigma}^2_{fi}(r^o)\right)\right) &= \sum_{i=1}^{N}\ln\left(1+\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)/\hat{\sigma}^2_{fi}(r^o)\right)\\ &= \sum_{i=1}^{N}\frac{1}{1+\delta_i}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)/\hat{\sigma}^2_{fi}(r^o)\\ &\geq \frac{1}{1+\max_i|\delta_i|}\frac{1}{\max_i\hat{\sigma}^2_{fi}(r^o)}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right), \quad \text{(III.2)}\end{aligned}$$

where $\delta_i$ lies on the line joining 0 and $\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)/\hat{\sigma}^2_{fi}(r^o)$. In addition, since $\ln(1+x)\leq x$ for all $x$ on the real line,

$$\begin{aligned}\sum_{i=1}^{N}\left(\ln\left(\hat{\sigma}^2_{fi}(r)\right)-\ln\left(\hat{\sigma}^2_{fi}(r^o)\right)\right) &= \sum_{i=1}^{N}\ln\left(1+\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)/\hat{\sigma}^2_{fi}(r^o)\right)\\ &\leq \sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)/\hat{\sigma}^2_{fi}(r^o)\\ &\leq \frac{1}{\min_i\hat{\sigma}^2_{fi}(r^o)}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right). \quad \text{(III.3)}\end{aligned}$$

Since $\hat{\sigma}^2_{fi}(r^o)\xrightarrow{p}\sigma^2_i$ for all $i$ and $\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)=O_p(1)$ for all $i$ as can be deduced from the proofs of Lemmas 2, 3 and 4 of Bai and Ng (2002), inequalities (III.2) and (III.3) imply that $\sum_{i=1}^{N}\left(\ln\left(\hat{\sigma}^2_{fi}(r)\right)-\ln\left(\hat{\sigma}^2_{fi}(r^o)\right)\right)$ and $\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)$ have the same stochastic order of magnitude.

When $r<r^o$, Lemmas 2 and 3 of Bai and Ng (2002) imply $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)\xrightarrow{p}\tau\ (>0)$. Thus, a sufficient condition for relation (III.1) to hold is

$$\frac{g(r^o,N,T)-g(r,N,T)}{NT}\to 0.$$

By contrast, when $r\geq r^o$, Lemma 4 of Bai and Ng (2002) implies $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)=O_p(C^{-2}_{N,T})$ with $C_{N,T}=\min\left\{\sqrt{N},\sqrt{T}\right\}$. In addition, $\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\sigma}^2_{fi}(r)-\hat{\sigma}^2_{fi}(r^o)\right)<0$

almost surely. Thus, a sufficient condition for relation (III.1) to hold is

$$\frac{C_{N,T}^2 \left[g(r^o, N, T) - g(r, N, T)\right]}{NT} \to -\infty.$$

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Table 1: Homoskedastic idiosyncratic errors

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 1, 0.5, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.15 | 1.01 | 8.00 | 4.35 | 1.00 | 1.00 | 1.07 |
| 20 | 60 | 7.88 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 6.47 | 1.19 | 1.00 | 1.00 | 1.04 |
| 20 | 100 | 7.83 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 |
| 20 | 200 | 7.82 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 30 | 30 | 7.90 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.02 | 1.00 | 8.00 | 1.06 | 1.00 | 1.00 | 1.06 |
| 30 | 60 | 7.54 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 4.39 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 100 | 6.95 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 200 | 5.62 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 30 | 5.57 | 1.87 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 60 | 3.22 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 100 | 2.92 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.18 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 200 | 1.33 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.03 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 60 | 1.07 | 3.75 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.31 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 200 | 1.76 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 4.85 | 6.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 2.44 | 1.23 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.78 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.29 | 0.00 | 2.95 | 0.84 | 0.00 | 0.00 | 0.03 |
| Part I.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 1, 0.5, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.98 | 1.00 | 1.18 | 1.00 | 1.00 | 1.00 | 1.80 | 8.00 | 3.30 | 1.27 | 8.00 | 3.40 | 1.71 | 1.86 | 1.86 |
| 20 | 60 | 7.90 | 8.00 | 1.00 | 2.40 | 1.03 | 1.00 | 1.00 | 2.98 | 3.62 | 2.98 | 2.91 | 6.18 | 2.95 | 2.78 | 2.89 | 3.01 |
| 20 | 100 | 7.92 | 8.00 | 1.00 | 2.15 | 1.00 | 1.00 | 1.00 | 3.00 | 3.00 | 2.90 | 2.97 | 3.09 | 2.83 | 2.89 | 2.96 | 3.00 |
| 20 | 200 | 7.88 | 8.00 | 1.00 | 2.99 | 1.04 | 1.00 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 | 3.00 | 3.00 | 3.00 | 3.00 |
| 30 | 30 | 7.92 | 7.90 | 1.00 | 1.57 | 1.00 | 1.00 | 1.00 | 2.08 | 8.00 | 2.73 | 1.68 | 8.00 | 2.63 | 2.05 | 2.22 | 2.57 |
| 30 | 60 | 7.80 | 8.00 | 1.00 | 2.49 | 1.03 | 1.00 | 1.00 | 2.94 | 3.09 | 2.82 | 2.71 | 3.75 | 2.46 | 2.70 | 2.85 | 3.00 |
| 30 | 100 | 7.70 | 8.00 | 1.00 | 2.36 | 1.24 | 1.00 | 1.00 | 2.91 | 3.00 | 2.24 | 2.76 | 3.00 | 2.08 | 2.57 | 2.80 | 3.01 |
| 30 | 200 | 7.09 | 8.00 | 1.00 | 3.00 | 1.33 | 1.00 | 1.00 | 3.00 | 3.00 | 2.94 | 3.00 | 3.00 | 2.56 | 3.00 | 3.00 | 3.01 |
| 100 | 30 | 7.71 | 3.15 | 1.01 | 2.58 | 1.23 | 1.00 | 1.00 | 2.94 | 3.00 | 2.56 | 2.83 | 2.99 | 2.21 | 2.79 | 2.88 | 3.01 |
| 100 | 60 | 6.46 | 8.00 | 1.66 | 3.00 | 2.77 | 1.68 | 1.05 | 3.00 | 3.00 | 2.95 | 3.00 | 3.00 | 2.84 | 3.00 | 3.00 | 3.01 |
| 100 | 100 | 5.88 | 8.00 | 2.34 | 3.00 | 2.99 | 2.57 | 1.19 | 3.00 | 3.00 | 3.00 | 3.00 | 3.04 | 2.97 | 3.00 | 3.00 | 3.02 |
| 100 | 200 | 3.77 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.97 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 30 | 4.03 | 3.00 | 1.01 | 2.74 | 1.32 | 1.01 | 1.00 | 2.98 | 3.00 | 2.49 | 2.92 | 2.98 | 2.00 | 2.91 | 2.94 | 3.01 |
| 200 | 60 | 3.30 | 4.58 | 2.26 | 3.00 | 2.97 | 2.45 | 1.41 | 3.00 | 3.00 | 2.98 | 3.00 | 3.00 | 2.91 | 3.00 | 3.00 | 3.01 |
| 200 | 100 | 3.74 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.94 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 200 | 4.26 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.33 | 7.16 | 1.58 | 2.59 | 1.81 | 1.61 | 1.41 | 2.85 | 3.67 | 2.87 | 2.75 | 3.88 | 2.74 | 2.78 | 2.84 | 2.91 |
| | $RMSE$ | 3.78 | 4.52 | 1.63 | 0.68 | 1.49 | 1.62 | 1.76 | 0.38 | 1.77 | 0.28 | 0.55 | 1.95 | 0.46 | 0.43 | 0.35 | 0.30 |
| Part I.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 1, 0.5, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.98 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.63 | 8.00 | 4.04 | 1.18 | 8.00 | 3.72 | 2.31 | 2.58 | 1.14 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.15 | 1.00 | 1.00 | 1.00 | 2.32 | 6.56 | 2.84 | 1.65 | 7.47 | 2.55 | 1.50 | 1.76 | 1.93 |
| 20 | 100 | 7.91 | 8.00 | 1.00 | 1.17 | 1.00 | 1.00 | 1.00 | 3.45 | 4.75 | 2.60 | 1.70 | 4.82 | 1.84 | 2.88 | 3.33 | 3.21 |
| 20 | 200 | 7.93 | 8.00 | 1.00 | 1.24 | 1.00 | 1.00 | 1.00 | 4.04 | 4.74 | 2.65 | 3.56 | 4.52 | 2.20 | 4.30 | 4.63 | 4.91 |
| 30 | 30 | 7.95 | 7.93 | 1.00 | 1.37 | 1.00 | 1.00 | 1.00 | 2.04 | 8.00 | 3.40 | 1.37 | 8.00 | 3.10 | 2.35 | 2.74 | 1.74 |
| 30 | 60 | 7.91 | 8.00 | 1.00 | 1.31 | 1.00 | 1.00 | 1.00 | 2.48 | 5.39 | 2.36 | 1.70 | 5.67 | 1.86 | 2.92 | 3.35 | 2.61 |
| 30 | 100 | 7.88 | 8.00 | 1.00 | 3.31 | 1.06 | 1.00 | 1.00 | 4.62 | 4.99 | 3.28 | 4.30 | 5.01 | 3.07 | 4.49 | 4.78 | 4.96 |
| 30 | 200 | 7.61 | 8.00 | 1.00 | 2.61 | 1.00 | 1.00 | 1.00 | 4.26 | 4.84 | 2.23 | 3.47 | 4.64 | 1.92 | 4.79 | 4.90 | 5.01 |
| 100 | 30 | 7.97 | 4.93 | 1.00 | 2.33 | 1.01 | 1.00 | 1.00 | 3.91 | 4.82 | 2.69 | 3.40 | 4.73 | 2.15 | 3.48 | 3.95 | 4.43 |
| 100 | 60 | 7.72 | 8.00 | 1.01 | 4.33 | 1.47 | 1.01 | 1.00 | 4.72 | 5.00 | 3.06 | 4.52 | 5.00 | 2.53 | 4.88 | 4.96 | 5.00 |
| 100 | 100 | 7.57 | 8.00 | 1.11 | 4.99 | 3.35 | 1.18 | 1.00 | 4.99 | 5.01 | 3.84 | 4.96 | 5.09 | 3.26 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 6.39 | 8.00 | 1.48 | 5.00 | 4.65 | 2.13 | 1.02 | 5.00 | 5.00 | 4.16 | 5.00 | 5.00 | 3.50 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.49 | 4.77 | 1.00 | 2.24 | 1.01 | 1.00 | 1.00 | 3.98 | 4.55 | 2.13 | 3.54 | 4.34 | 1.58 | 3.81 | 4.23 | 4.90 |
| 200 | 60 | 5.97 | 5.72 | 1.02 | 4.91 | 2.82 | 1.05 | 1.00 | 4.98 | 5.00 | 3.54 | 4.95 | 5.00 | 2.90 | 5.00 | 5.00 | 5.00 |
| 200 | 100 | 6.37 | 8.00 | 1.96 | 5.00 | 4.90 | 2.88 | 1.07 | 5.00 | 5.00 | 4.63 | 5.00 | 5.00 | 4.17 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.76 | 8.00 | 4.93 | 5.00 | 5.00 | 4.99 | 4.60 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 4.99 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 7.46 | 7.46 | 1.34 | 2.94 | 2.02 | 1.45 | 1.23 | 3.90 | 5.42 | 3.28 | 3.46 | 5.46 | 2.83 | 3.92 | 4.14 | 4.05 |
| | $RMSE$ | 2.54 | 2.70 | 3.78 | 2.61 | 3.35 | 3.70 | 3.87 | 1.58 | 1.15 | 1.92 | 2.10 | 1.26 | 2.35 | 1.58 | 1.34 | 1.66 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part II.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 1, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.00 | 1.79 | 1.04 | 1.00 | 1.00 | 2.58 | 8.00 | 5.06 | 4.30 | 8.00 | 5.98 | 1.04 | 1.11 | 1.83 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.78 | 1.01 | 1.00 | 1.00 | 2.04 | 7.89 | 2.43 | 4.92 | 8.00 | 4.16 | 1.01 | 1.04 | 1.99 |
| 20 | 100 | 7.95 | 8.00 | 1.00 | 1.81 | 1.00 | 1.00 | 1.00 | 2.00 | 2.71 | 2.00 | 2.42 | 7.80 | 2.29 | 1.01 | 1.08 | 2.04 |
| 20 | 200 | 7.94 | 8.00 | 1.00 | 1.95 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.99 | 1.99 | 2.31 | 1.96 | 1.34 | 1.65 | 2.03 |
| 30 | 30 | 8.00 | 8.00 | 1.25 | 2.55 | 1.56 | 1.10 | 1.01 | 2.74 | 8.00 | 4.28 | 3.11 | 8.00 | 4.94 | 1.20 | 1.38 | 2.30 |
| 30 | 60 | 8.00 | 8.00 | 1.21 | 2.54 | 1.67 | 1.10 | 1.00 | 2.74 | 8.00 | 2.79 | 3.05 | 8.00 | 3.12 | 1.12 | 1.31 | 2.49 |
| 30 | 100 | 8.00 | 8.00 | 1.50 | 2.66 | 1.95 | 1.40 | 1.02 | 2.86 | 4.92 | 2.48 | 2.69 | 7.99 | 2.29 | 1.27 | 1.59 | 2.80 |
| 30 | 200 | 7.99 | 8.00 | 1.78 | 2.56 | 2.00 | 1.80 | 1.06 | 2.91 | 3.00 | 2.04 | 2.36 | 4.80 | 2.00 | 1.11 | 1.54 | 2.99 |
| 100 | 30 | 8.00 | 7.89 | 1.01 | 4.58 | 1.25 | 1.01 | 1.00 | 7.27 | 8.00 | 4.41 | 7.16 | 8.00 | 4.15 | 1.00 | 1.02 | 1.82 |
| 100 | 60 | 8.00 | 8.00 | 1.04 | 6.67 | 2.22 | 1.05 | 1.00 | 7.02 | 8.00 | 4.44 | 6.52 | 8.00 | 3.99 | 1.00 | 1.00 | 1.93 |
| 100 | 100 | 8.00 | 8.00 | 1.14 | 7.59 | 4.19 | 1.25 | 1.00 | 7.34 | 8.00 | 4.96 | 7.29 | 8.00 | 4.79 | 1.00 | 1.00 | 2.06 |
| 100 | 200 | 8.00 | 8.00 | 1.68 | 7.98 | 6.09 | 2.73 | 1.02 | 7.95 | 8.00 | 5.45 | 7.88 | 8.00 | 5.16 | 1.00 | 1.00 | 2.37 |
| 200 | 30 | 8.00 | 7.54 | 1.00 | 2.84 | 1.01 | 1.00 | 1.00 | 7.55 | 8.00 | 3.20 | 7.40 | 8.00 | 2.81 | 1.00 | 1.00 | 1.29 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.63 | 1.11 | 1.00 | 1.00 | 7.77 | 8.00 | 2.69 | 7.62 | 8.00 | 2.32 | 1.00 | 1.00 | 1.15 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.99 | 1.78 | 1.00 | 1.00 | 7.99 | 8.00 | 2.94 | 7.98 | 8.00 | 2.59 | 1.00 | 1.00 | 1.09 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 6.62 | 1.07 | 1.00 | 8.00 | 8.00 | 3.83 | 8.00 | 8.00 | 3.42 | 1.00 | 1.00 | 1.03 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.16 | 4.37 | 2.22 | 1.22 | 1.01 | 5.05 | 6.78 | 3.44 | 5.29 | 7.43 | 3.50 | 1.07 | 1.17 | 1.95 |
| | $RMSE$ | 6.99 | 6.97 | 0.30 | 4.20 | 2.13 | 0.49 | 0.02 | 4.80 | 6.17 | 2.69 | 4.88 | 6.61 | 2.78 | 0.13 | 0.29 | 1.11 |
| Part II.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 1, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.12 | 3.93 | 1.49 | 1.01 | 1.00 | 5.03 | 8.00 | 5.64 | 5.94 | 8.00 | 6.12 | 2.56 | 2.96 | 3.03 |
| 20 | 60 | 7.98 | 8.00 | 1.07 | 3.59 | 1.83 | 1.01 | 1.00 | 4.18 | 7.99 | 3.98 | 5.62 | 8.00 | 4.45 | 2.93 | 3.18 | 3.52 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 3.96 | 1.28 | 1.00 | 1.00 | 4.05 | 6.24 | 3.96 | 4.86 | 7.93 | 3.89 | 3.60 | 3.80 | 3.94 |
| 20 | 200 | 7.96 | 8.00 | 1.01 | 3.87 | 2.74 | 1.01 | 1.00 | 4.00 | 4.01 | 3.83 | 4.98 | 6.68 | 3.95 | 3.62 | 3.81 | 4.02 |
| 30 | 30 | 8.00 | 8.00 | 1.25 | 4.16 | 1.69 | 1.09 | 1.00 | 4.63 | 8.00 | 5.15 | 4.66 | 8.00 | 5.39 | 2.36 | 2.86 | 2.96 |
| 30 | 60 | 8.00 | 8.00 | 1.42 | 4.61 | 2.98 | 1.15 | 1.00 | 4.86 | 8.00 | 4.40 | 5.39 | 8.00 | 4.31 | 3.60 | 3.96 | 4.12 |
| 30 | 100 | 8.00 | 8.00 | 2.48 | 4.23 | 3.75 | 2.20 | 1.14 | 4.50 | 7.12 | 4.01 | 5.22 | 7.99 | 4.01 | 3.74 | 3.95 | 4.29 |
| 30 | 200 | 8.00 | 8.00 | 1.04 | 4.69 | 3.86 | 1.05 | 1.00 | 4.96 | 5.02 | 4.00 | 5.11 | 7.46 | 4.00 | 3.53 | 4.11 | 4.95 |
| 100 | 30 | 8.00 | 7.98 | 1.11 | 6.36 | 2.04 | 1.06 | 1.00 | 7.86 | 8.00 | 5.21 | 7.81 | 8.00 | 4.93 | 2.63 | 3.20 | 1.78 |
| 100 | 60 | 8.00 | 8.00 | 2.25 | 7.85 | 3.95 | 2.27 | 1.16 | 7.94 | 8.00 | 5.45 | 7.88 | 8.00 | 5.17 | 2.52 | 2.98 | 2.23 |
| 100 | 100 | 8.00 | 8.00 | 3.10 | 8.00 | 6.03 | 3.21 | 2.66 | 7.99 | 8.00 | 5.99 | 7.98 | 8.00 | 5.70 | 2.95 | 3.22 | 2.66 |
| 100 | 200 | 8.00 | 8.00 | 3.46 | 8.00 | 7.83 | 4.42 | 2.86 | 8.00 | 8.00 | 6.73 | 8.00 | 8.00 | 6.63 | 2.59 | 2.88 | 2.42 |
| 200 | 30 | 8.00 | 7.82 | 1.14 | 4.90 | 2.23 | 1.15 | 1.00 | 7.86 | 8.00 | 4.08 | 7.80 | 8.00 | 3.72 | 2.32 | 2.60 | 2.35 |
| 200 | 60 | 8.00 | 8.00 | 2.68 | 7.64 | 3.10 | 2.82 | 1.81 | 7.97 | 8.00 | 3.76 | 7.96 | 8.00 | 3.48 | 2.87 | 2.94 | 2.98 |
| 200 | 100 | 8.00 | 8.00 | 2.99 | 8.00 | 3.74 | 3.00 | 2.90 | 8.00 | 8.00 | 3.91 | 8.00 | 8.00 | 3.59 | 2.95 | 2.97 | 3.04 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 7.65 | 3.07 | 3.00 | 8.00 | 8.00 | 4.51 | 8.00 | 8.00 | 4.31 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 7.99 | 7.99 | 1.88 | 5.74 | 3.51 | 1.91 | 1.53 | 6.24 | 7.40 | 4.66 | 6.58 | 7.88 | 4.60 | 2.99 | 3.28 | 3.21 |
| | $RMSE$ | 4.99 | 4.99 | 1.43 | 3.27 | 2.04 | 1.53 | 1.67 | 3.67 | 4.56 | 1.88 | 3.83 | 4.89 | 1.85 | 0.47 | 0.54 | 0.86 |
| Part II.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 1, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.99 | 8.00 | 1.00 | 3.94 | 1.06 | 1.00 | 1.00 | 6.05 | 8.00 | 6.08 | 6.04 | 8.00 | 6.34 | 2.44 | 2.98 | 2.09 |
| 20 | 60 | 8.00 | 8.00 | 1.02 | 4.24 | 1.30 | 1.00 | 1.00 | 5.96 | 8.00 | 4.89 | 6.90 | 8.00 | 5.25 | 2.95 | 3.54 | 2.79 |
| 20 | 100 | 7.99 | 8.00 | 1.00 | 3.58 | 1.00 | 1.00 | 1.00 | 5.62 | 7.73 | 4.31 | 6.48 | 8.00 | 4.07 | 3.04 | 3.75 | 2.37 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 3.68 | 1.00 | 1.00 | 1.00 | 5.91 | 6.15 | 3.94 | 6.11 | 7.35 | 3.82 | 2.12 | 3.24 | 4.78 |
| 30 | 30 | 8.00 | 8.00 | 1.01 | 5.35 | 1.13 | 1.00 | 1.00 | 6.16 | 8.00 | 5.95 | 6.27 | 8.00 | 6.04 | 3.49 | 4.07 | 2.06 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 5.19 | 1.11 | 1.00 | 1.00 | 6.33 | 8.00 | 5.13 | 6.40 | 8.00 | 4.95 | 3.91 | 4.50 | 2.17 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 5.92 | 1.69 | 1.00 | 1.00 | 6.78 | 7.92 | 5.24 | 7.12 | 8.00 | 5.21 | 5.18 | 5.83 | 3.62 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 5.83 | 2.23 | 1.00 | 1.00 | 6.70 | 7.04 | 5.01 | 6.52 | 7.71 | 4.97 | 5.28 | 5.65 | 5.66 |
| 100 | 30 | 8.00 | 8.00 | 1.00 | 7.24 | 1.31 | 1.00 | 1.00 | 7.97 | 8.00 | 5.60 | 7.97 | 8.00 | 5.30 | 3.25 | 3.77 | 1.03 |
| 100 | 60 | 8.00 | 8.00 | 1.10 | 7.99 | 4.56 | 1.12 | 1.00 | 8.00 | 8.00 | 6.07 | 8.00 | 8.00 | 5.82 | 3.84 | 4.47 | 0.47 |
| 100 | 100 | 8.00 | 8.00 | 1.94 | 8.00 | 7.15 | 2.45 | 1.03 | 8.00 | 8.00 | 6.51 | 8.00 | 8.00 | 6.18 | 3.43 | 3.97 | 0.44 |
| 100 | 200 | 8.00 | 8.00 | 4.08 | 8.00 | 8.00 | 5.42 | 1.95 | 8.00 | 8.00 | 7.24 | 8.00 | 8.00 | 6.85 | 3.23 | 3.77 | 0.29 |
| 200 | 30 | 8.00 | 7.94 | 1.00 | 5.95 | 1.33 | 1.00 | 1.00 | 7.97 | 8.00 | 4.56 | 7.94 | 8.00 | 4.11 | 2.74 | 3.25 | 1.17 |
| 200 | 60 | 8.00 | 8.00 | 1.27 | 7.93 | 4.58 | 1.58 | 1.00 | 8.00 | 8.00 | 5.01 | 8.00 | 8.00 | 4.84 | 3.81 | 4.23 | 1.71 |
| 200 | 100 | 8.00 | 8.00 | 4.09 | 8.00 | 5.58 | 4.65 | 1.90 | 8.00 | 8.00 | 5.12 | 8.00 | 8.00 | 5.06 | 4.55 | 4.79 | 3.46 |
| 200 | 200 | 8.00 | 8.00 | 5.00 | 8.00 | 7.92 | 5.05 | 4.97 | 8.00 | 8.00 | 5.42 | 8.00 | 8.00 | 5.29 | 4.99 | 4.99 | 4.83 |
| | $r_{mean}$ | 8.00 | 8.00 | 1.72 | 6.18 | 3.18 | 1.89 | 1.37 | 7.09 | 7.80 | 5.38 | 7.23 | 7.94 | 5.26 | 3.64 | 4.18 | 2.43 |
| | $RMSE$ | 3.00 | 3.00 | 3.53 | 2.04 | 3.16 | 3.48 | 3.76 | 2.29 | 2.84 | 0.90 | 2.37 | 2.95 | 0.86 | 1.64 | 1.15 | 3.03 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Part III.1 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (1, 1, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.11 | 1.05 | 8.00 | 3.97 | 1.00 | 1.00 | 1.06 |
| 20 | 60 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 3.66 | 1.05 | 1.00 | 1.00 | 1.04 |
| 20 | 100 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.53 | 1.00 | 1.00 | 1.00 | 1.02 |
| 20 | 200 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 |
| 30 | 30 | 7.90 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.08 | 1.00 | 1.00 | 1.04 |
| 30 | 60 | 7.53 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 100 | 6.91 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 200 | 5.43 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.59 | 1.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 60 | 3.28 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 100 | 2.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 200 | 1.29 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 60 | 1.06 | 3.73 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.33 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| | $r_{mean}$ | 4.85 | 6.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 2.09 | 1.19 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.77 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.28 | 0.01 | 2.57 | 0.74 | 0.00 | 0.00 | 0.02 |
| | Part III.2 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 1, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.91 | 7.98 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.36 | 8.00 | 2.90 | 1.14 | 8.00 | 2.87 | 1.40 | 1.47 | 1.49 |
| 20 | 60 | 7.92 | 8.00 | 1.00 | 1.35 | 1.01 | 1.00 | 1.00 | 2.04 | 3.30 | 2.17 | 1.84 | 5.44 | 2.13 | 1.39 | 1.55 | 2.38 |
| 20 | 100 | 7.87 | 8.00 | 1.00 | 1.58 | 1.02 | 1.00 | 1.00 | 2.18 | 2.56 | 2.01 | 1.95 | 2.49 | 1.86 | 1.89 | 2.03 | 2.67 |
| 20 | 200 | 7.91 | 8.00 | 1.00 | 2.16 | 1.05 | 1.00 | 1.00 | 2.96 | 2.99 | 2.31 | 2.97 | 3.00 | 2.48 | 2.79 | 2.88 | 3.00 |
| 30 | 30 | 7.93 | 7.91 | 1.00 | 1.19 | 1.01 | 1.00 | 1.00 | 1.43 | 8.00 | 2.15 | 1.23 | 8.00 | 1.98 | 1.49 | 1.58 | 1.91 |
| 30 | 60 | 7.78 | 8.00 | 1.01 | 1.64 | 1.06 | 1.00 | 1.00 | 2.08 | 2.99 | 1.95 | 1.81 | 3.26 | 1.76 | 1.80 | 1.94 | 2.65 |
| 30 | 100 | 7.68 | 8.00 | 1.01 | 2.25 | 1.14 | 1.00 | 1.00 | 2.80 | 2.99 | 2.23 | 2.58 | 2.97 | 1.96 | 2.49 | 2.67 | 3.01 |
| 30 | 200 | 7.11 | 8.00 | 1.02 | 2.96 | 1.77 | 1.03 | 1.00 | 3.00 | 3.00 | 2.82 | 3.00 | 3.00 | 2.58 | 3.00 | 3.00 | 3.00 |
| 100 | 30 | 7.65 | 3.13 | 1.00 | 1.60 | 1.05 | 1.00 | 1.00 | 2.20 | 2.74 | 1.69 | 1.89 | 2.64 | 1.43 | 1.75 | 1.92 | 2.83 |
| 100 | 60 | 6.43 | 8.00 | 1.28 | 2.86 | 2.01 | 1.29 | 1.05 | 2.95 | 3.00 | 2.45 | 2.90 | 3.00 | 2.24 | 2.74 | 2.85 | 3.00 |
| 100 | 100 | 5.96 | 8.00 | 1.90 | 3.00 | 2.80 | 2.02 | 1.33 | 3.00 | 3.00 | 2.86 | 2.99 | 3.08 | 2.65 | 2.98 | 2.99 | 3.01 |
| 100 | 200 | 3.77 | 8.00 | 2.70 | 3.00 | 3.00 | 2.82 | 2.27 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 2.95 | 3.00 | 3.00 | 3.01 |
| 200 | 30 | 4.04 | 2.95 | 1.00 | 1.67 | 1.08 | 1.00 | 1.00 | 2.36 | 2.66 | 1.55 | 2.09 | 2.48 | 1.27 | 1.95 | 2.14 | 2.98 |
| 200 | 60 | 3.31 | 4.65 | 1.32 | 2.87 | 2.07 | 1.42 | 1.09 | 2.95 | 3.00 | 2.23 | 2.90 | 3.00 | 1.95 | 2.73 | 2.86 | 3.01 |
| 200 | 100 | 3.77 | 8.00 | 2.18 | 3.00 | 2.93 | 2.44 | 1.72 | 3.00 | 3.00 | 2.87 | 3.00 | 3.00 | 2.61 | 2.99 | 3.00 | 3.01 |
| 200 | 200 | 4.32 | 8.00 | 2.98 | 3.00 | 3.00 | 3.00 | 2.95 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.34 | 7.16 | 1.40 | 2.20 | 1.69 | 1.44 | 1.28 | 2.52 | 3.58 | 2.39 | 2.39 | 3.71 | 2.23 | 2.34 | 2.43 | 2.75 |
| | $RMSE$ | 3.78 | 4.52 | 1.72 | 1.08 | 1.54 | 1.71 | 1.81 | 0.74 | 1.78 | 0.76 | 0.89 | 1.88 | 0.92 | 0.91 | 0.81 | 0.51 |
| | Part III.3 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (5, 1, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.94 | 7.99 | 1.00 | 1.12 | 1.00 | 1.00 | 1.00 | 1.49 | 8.00 | 3.51 | 1.21 | 8.00 | 3.51 | 1.63 | 1.74 | 1.38 |
| 20 | 60 | 7.92 | 8.00 | 1.00 | 1.14 | 1.00 | 1.00 | 1.00 | 1.85 | 4.77 | 2.19 | 1.52 | 5.90 | 2.06 | 1.64 | 1.80 | 1.88 |
| 20 | 100 | 7.92 | 8.00 | 1.00 | 1.18 | 1.00 | 1.00 | 1.00 | 2.46 | 3.61 | 2.14 | 2.01 | 3.43 | 1.88 | 1.99 | 2.28 | 2.28 |
| 20 | 200 | 7.91 | 8.00 | 1.00 | 2.03 | 1.00 | 1.00 | 1.00 | 3.84 | 4.36 | 2.62 | 3.12 | 3.82 | 2.07 | 3.04 | 3.59 | 4.70 |
| 30 | 30 | 7.94 | 7.91 | 1.00 | 1.13 | 1.00 | 1.00 | 1.00 | 1.31 | 8.00 | 2.24 | 1.10 | 8.00 | 1.95 | 1.53 | 1.64 | 1.54 |
| 30 | 60 | 7.87 | 8.00 | 1.00 | 1.61 | 1.01 | 1.00 | 1.00 | 2.35 | 4.75 | 2.21 | 1.95 | 5.39 | 1.96 | 1.91 | 2.14 | 2.54 |
| 30 | 100 | 7.88 | 8.00 | 1.00 | 1.66 | 1.00 | 1.00 | 1.00 | 2.63 | 3.82 | 1.92 | 2.26 | 3.78 | 1.57 | 2.18 | 2.44 | 3.02 |
| 30 | 200 | 7.69 | 8.00 | 1.00 | 1.38 | 1.00 | 1.00 | 1.00 | 3.23 | 4.23 | 1.42 | 2.26 | 3.63 | 1.07 | 3.22 | 3.75 | 4.91 |
| 100 | 30 | 7.94 | 3.96 | 1.00 | 1.26 | 1.00 | 1.00 | 1.00 | 1.84 | 2.99 | 1.41 | 1.49 | 2.66 | 1.17 | 1.62 | 1.75 | 2.35 |
| 100 | 60 | 7.66 | 8.00 | 1.03 | 2.87 | 1.38 | 1.03 | 1.00 | 3.24 | 4.85 | 2.18 | 2.88 | 4.81 | 1.80 | 2.53 | 3.03 | 4.74 |
| 100 | 100 | 7.50 | 8.00 | 1.15 | 4.14 | 2.20 | 1.20 | 1.01 | 4.15 | 5.00 | 2.73 | 3.92 | 5.05 | 2.39 | 4.39 | 4.68 | 5.01 |
| 100 | 200 | 6.46 | 8.00 | 1.83 | 5.00 | 4.29 | 2.40 | 1.18 | 5.00 | 5.00 | 3.99 | 5.00 | 5.00 | 3.51 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.16 | 3.47 | 1.00 | 1.39 | 1.01 | 1.00 | 1.00 | 2.19 | 2.70 | 1.36 | 1.82 | 2.37 | 1.12 | 1.69 | 1.87 | 3.30 |
| 200 | 60 | 5.93 | 5.69 | 1.04 | 3.09 | 1.55 | 1.07 | 1.00 | 3.60 | 4.52 | 2.04 | 3.28 | 4.35 | 1.65 | 2.99 | 3.44 | 4.98 |
| 200 | 100 | 6.40 | 8.00 | 2.13 | 4.94 | 3.90 | 2.51 | 1.47 | 4.96 | 5.00 | 3.69 | 4.92 | 5.00 | 3.33 | 4.90 | 4.96 | 5.01 |
| 200 | 200 | 6.76 | 8.00 | 3.89 | 5.00 | 4.99 | 4.45 | 3.16 | 5.00 | 5.00 | 4.69 | 5.00 | 5.00 | 4.50 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 7.43 | 7.31 | 1.32 | 2.43 | 1.77 | 1.42 | 1.18 | 3.07 | 4.79 | 2.52 | 2.73 | 4.76 | 2.22 | 2.83 | 3.07 | 3.60 |
| | $RMSE$ | 2.52 | 2.74 | 3.76 | 2.96 | 3.49 | 3.70 | 3.86 | 2.28 | 1.41 | 2.65 | 2.62 | 1.56 | 2.94 | 2.51 | 2.30 | 1.97 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Part IV.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 1, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.00 | 1.49 | 1.01 | 1.00 | 1.00 | 2.24 | 8.00 | 4.95 | 4.81 | 8.00 | 6.41 | 1.00 | 1.00 | 1.63 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.92 | 1.11 | 1.00 | 1.00 | 2.03 | 7.88 | 2.39 | 2.50 | 8.00 | 3.43 | 1.04 | 1.11 | 1.97 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.74 | 1.00 | 1.00 | 1.00 | 1.98 | 2.53 | 1.99 | 5.22 | 8.00 | 2.97 | 1.01 | 1.05 | 2.02 |
| 20 | 200 | 7.94 | 8.00 | 1.00 | 1.88 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.99 | 4.63 | 7.61 | 2.10 | 1.00 | 1.03 | 2.02 |
| 30 | 30 | 8.00 | 7.99 | 1.25 | 2.41 | 1.55 | 1.09 | 1.01 | 2.71 | 8.00 | 4.26 | 2.98 | 8.00 | 4.83 | 1.27 | 1.44 | 2.27 |
| 30 | 60 | 7.99 | 8.00 | 1.38 | 2.49 | 1.84 | 1.23 | 1.01 | 2.75 | 7.99 | 2.75 | 2.58 | 8.00 | 2.78 | 1.40 | 1.62 | 2.52 |
| 30 | 100 | 7.99 | 8.00 | 1.59 | 2.65 | 1.98 | 1.48 | 1.03 | 2.83 | 4.67 | 2.41 | 3.14 | 7.94 | 2.60 | 1.34 | 1.64 | 2.78 |
| 30 | 200 | 7.97 | 8.00 | 1.44 | 2.73 | 2.00 | 1.46 | 1.01 | 2.96 | 3.00 | 2.06 | 2.60 | 6.39 | 2.01 | 1.06 | 1.34 | 3.00 |
| 100 | 30 | 8.00 | 7.87 | 1.01 | 4.51 | 1.23 | 1.00 | 1.00 | 7.14 | 8.00 | 4.36 | 7.03 | 8.00 | 4.12 | 1.09 | 1.15 | 1.71 |
| 100 | 60 | 8.00 | 8.00 | 1.04 | 6.66 | 2.26 | 1.04 | 1.00 | 7.03 | 8.00 | 4.46 | 6.76 | 8.00 | 4.12 | 1.01 | 1.01 | 1.76 |
| 100 | 100 | 8.00 | 8.00 | 1.13 | 7.56 | 4.09 | 1.22 | 1.00 | 7.29 | 8.00 | 4.93 | 6.90 | 8.00 | 4.40 | 1.00 | 1.00 | 2.15 |
| 100 | 200 | 8.00 | 8.00 | 1.63 | 7.97 | 6.19 | 2.66 | 1.03 | 7.95 | 8.00 | 5.58 | 7.90 | 8.00 | 5.05 | 1.00 | 1.00 | 2.11 |
| 200 | 30 | 8.00 | 7.50 | 1.00 | 2.78 | 1.01 | 1.00 | 1.00 | 7.49 | 8.00 | 3.14 | 7.07 | 7.99 | 2.62 | 1.03 | 1.05 | 1.31 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.67 | 1.10 | 1.00 | 1.00 | 7.79 | 8.00 | 2.72 | 7.56 | 8.00 | 2.23 | 1.00 | 1.00 | 1.19 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.99 | 1.76 | 1.00 | 1.00 | 7.99 | 8.00 | 2.90 | 7.97 | 8.00 | 2.48 | 1.00 | 1.00 | 1.10 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 6.65 | 1.05 | 1.00 | 8.00 | 8.00 | 3.78 | 8.00 | 8.00 | 3.61 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.15 | 4.34 | 2.24 | 1.20 | 1.01 | 5.01 | 6.75 | 3.42 | 5.48 | 7.87 | 3.49 | 1.08 | 1.15 | 1.91 |
| | $RMSE$ | 6.99 | 6.96 | 0.27 | 4.19 | 2.14 | 0.45 | 0.01 | 4.78 | 6.16 | 2.68 | 4.94 | 6.88 | 2.76 | 0.15 | 0.27 | 1.07 |
| | Part IV.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 1, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.02 | 2.85 | 1.08 | 1.00 | 1.00 | 4.08 | 8.00 | 5.43 | 4.38 | 8.00 | 5.79 | 1.75 | 2.04 | 2.21 |
| 20 | 60 | 7.97 | 8.00 | 1.06 | 3.13 | 1.31 | 1.03 | 1.00 | 3.94 | 7.99 | 3.87 | 5.13 | 8.00 | 4.23 | 1.61 | 1.91 | 2.86 |
| 20 | 100 | 7.98 | 8.00 | 1.04 | 3.01 | 1.28 | 1.01 | 1.00 | 3.88 | 5.92 | 3.55 | 4.58 | 7.90 | 3.67 | 2.21 | 2.66 | 3.53 |
| 20 | 200 | 7.96 | 8.00 | 1.05 | 3.51 | 1.86 | 1.04 | 1.00 | 4.00 | 4.01 | 3.44 | 4.58 | 6.72 | 3.77 | 3.28 | 3.67 | 4.00 |
| 30 | 30 | 8.00 | 8.00 | 1.13 | 3.79 | 1.46 | 1.04 | 1.00 | 4.34 | 8.00 | 5.01 | 4.35 | 8.00 | 5.24 | 2.12 | 2.54 | 2.68 |
| 30 | 60 | 8.00 | 8.00 | 1.25 | 4.28 | 2.20 | 1.11 | 1.00 | 4.76 | 8.00 | 4.26 | 4.85 | 8.00 | 4.14 | 2.54 | 3.12 | 3.39 |
| 30 | 100 | 8.00 | 8.00 | 1.42 | 4.53 | 2.60 | 1.30 | 1.02 | 4.85 | 7.15 | 4.05 | 5.20 | 7.99 | 3.97 | 2.91 | 3.59 | 4.37 |
| 30 | 200 | 8.00 | 8.00 | 2.11 | 4.33 | 3.80 | 2.15 | 1.06 | 4.87 | 5.01 | 4.00 | 4.81 | 6.70 | 4.00 | 3.81 | 4.07 | 4.87 |
| 100 | 30 | 8.00 | 7.96 | 1.10 | 5.88 | 1.67 | 1.07 | 1.01 | 7.78 | 8.00 | 4.95 | 7.74 | 8.00 | 4.71 | 1.85 | 2.25 | 1.89 |
| 100 | 60 | 8.00 | 8.00 | 1.93 | 7.84 | 3.81 | 1.96 | 1.28 | 7.92 | 8.00 | 5.44 | 7.90 | 8.00 | 5.12 | 1.94 | 2.36 | 2.00 |
| 100 | 100 | 8.00 | 8.00 | 2.70 | 8.00 | 5.97 | 2.87 | 1.93 | 8.00 | 8.00 | 5.94 | 7.99 | 8.00 | 5.55 | 2.18 | 2.65 | 2.04 |
| 100 | 200 | 8.00 | 8.00 | 3.39 | 8.00 | 7.78 | 4.22 | 2.82 | 8.00 | 8.00 | 6.57 | 8.00 | 8.00 | 6.35 | 2.40 | 2.72 | 2.27 |
| 200 | 30 | 8.00 | 7.69 | 1.08 | 4.16 | 1.56 | 1.08 | 1.01 | 7.76 | 8.00 | 3.76 | 7.58 | 8.00 | 3.34 | 1.66 | 1.88 | 1.89 |
| 200 | 60 | 8.00 | 8.00 | 1.79 | 7.59 | 2.80 | 1.94 | 1.35 | 7.96 | 8.00 | 3.64 | 7.91 | 8.00 | 3.32 | 1.90 | 2.10 | 2.36 |
| 200 | 100 | 8.00 | 8.00 | 2.69 | 8.00 | 3.68 | 2.86 | 2.26 | 8.00 | 8.00 | 3.85 | 8.00 | 8.00 | 3.61 | 2.45 | 2.59 | 2.86 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 7.59 | 3.06 | 2.99 | 8.00 | 8.00 | 4.51 | 8.00 | 8.00 | 4.56 | 2.90 | 2.96 | 3.00 |
| | $r_{mean}$ | 7.99 | 7.98 | 1.74 | 5.43 | 3.15 | 1.80 | 1.42 | 6.13 | 7.38 | 4.52 | 6.31 | 7.83 | 4.46 | 2.34 | 2.69 | 2.89 |
| | $RMSE$ | 4.99 | 4.98 | 1.49 | 3.17 | 2.13 | 1.54 | 1.71 | 3.62 | 4.55 | 1.77 | 3.68 | 4.85 | 1.71 | 0.89 | 0.70 | 0.90 |
| | Part IV.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 1, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.99 | 8.00 | 1.01 | 3.36 | 1.07 | 1.00 | 1.00 | 5.38 | 8.00 | 5.85 | 5.70 | 8.00 | 6.18 | 2.17 | 2.54 | 2.09 |
| 20 | 60 | 7.99 | 8.00 | 1.01 | 3.12 | 1.07 | 1.00 | 1.00 | 5.23 | 7.99 | 4.47 | 6.18 | 8.00 | 4.70 | 2.12 | 2.54 | 2.19 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 3.77 | 1.05 | 1.00 | 1.00 | 5.76 | 7.74 | 4.33 | 6.35 | 7.98 | 4.21 | 2.71 | 3.35 | 2.88 |
| 20 | 200 | 7.98 | 8.00 | 1.00 | 5.36 | 1.11 | 1.00 | 1.00 | 5.98 | 6.14 | 4.52 | 6.57 | 7.76 | 4.02 | 4.58 | 5.22 | 5.58 |
| 30 | 30 | 8.00 | 8.00 | 1.05 | 4.08 | 1.27 | 1.01 | 1.00 | 4.88 | 8.00 | 5.34 | 4.63 | 8.00 | 5.43 | 2.30 | 2.87 | 2.24 |
| 30 | 60 | 8.00 | 8.00 | 1.09 | 5.12 | 1.77 | 1.03 | 1.00 | 6.22 | 8.00 | 5.01 | 6.47 | 8.00 | 5.02 | 2.68 | 3.33 | 2.66 |
| 30 | 100 | 8.00 | 8.00 | 1.03 | 5.20 | 1.72 | 1.01 | 1.00 | 6.49 | 7.91 | 4.56 | 6.66 | 7.99 | 4.47 | 3.30 | 4.19 | 2.91 |
| 30 | 200 | 8.00 | 8.00 | 1.01 | 5.82 | 1.67 | 1.01 | 1.00 | 6.59 | 7.00 | 4.35 | 6.56 | 7.67 | 3.84 | 4.29 | 5.19 | 4.72 |
| 100 | 30 | 8.00 | 7.98 | 1.02 | 5.88 | 1.36 | 1.01 | 1.00 | 7.84 | 8.00 | 4.95 | 7.78 | 8.00 | 4.65 | 2.24 | 2.64 | 1.36 |
| 100 | 60 | 8.00 | 8.00 | 1.32 | 7.96 | 3.34 | 1.34 | 1.02 | 7.99 | 8.00 | 5.51 | 7.97 | 8.00 | 5.11 | 2.06 | 2.53 | 1.18 |
| 100 | 100 | 8.00 | 8.00 | 1.85 | 8.00 | 6.08 | 2.10 | 1.14 | 8.00 | 8.00 | 6.09 | 8.00 | 8.00 | 5.74 | 2.32 | 2.81 | 0.84 |
| 100 | 200 | 8.00 | 8.00 | 3.57 | 8.00 | 7.99 | 4.71 | 1.79 | 8.00 | 8.00 | 7.07 | 8.00 | 8.00 | 6.59 | 2.63 | 3.08 | 0.57 |
| 200 | 30 | 8.00 | 7.77 | 1.02 | 4.48 | 1.33 | 1.02 | 1.00 | 7.85 | 8.00 | 3.92 | 7.66 | 8.00 | 3.42 | 1.83 | 2.09 | 1.58 |
| 200 | 60 | 8.00 | 8.00 | 1.27 | 7.82 | 2.91 | 1.38 | 1.05 | 7.99 | 8.00 | 4.16 | 7.97 | 8.00 | 3.74 | 1.91 | 2.26 | 1.44 |
| 200 | 100 | 8.00 | 8.00 | 3.17 | 8.00 | 5.34 | 3.66 | 2.19 | 8.00 | 8.00 | 5.02 | 8.00 | 8.00 | 4.85 | 2.63 | 3.08 | 2.34 |
| 200 | 200 | 8.00 | 8.00 | 4.65 | 8.00 | 7.93 | 4.98 | 4.11 | 8.00 | 8.00 | 5.43 | 8.00 | 8.00 | 5.22 | 3.76 | 4.14 | 3.37 |
| | $r_{mean}$ | 8.00 | 7.98 | 1.63 | 5.87 | 2.94 | 1.77 | 1.33 | 6.89 | 7.80 | 5.04 | 7.03 | 7.96 | 4.82 | 2.72 | 3.24 | 2.37 |
| | $RMSE$ | 3.00 | 2.98 | 3.54 | 1.99 | 3.17 | 3.50 | 3.75 | 2.21 | 2.84 | 0.80 | 2.27 | 2.96 | 0.87 | 2.42 | 1.99 | 2.93 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Part V.1 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (1, 3, 0.5, 0, 0)$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.94 | 7.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.12 | 1.04 | 8.00 | 4.81 | 1.00 | 1.00 | 1.06 |
| 20 | 60 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 5.15 | 1.05 | 1.00 | 1.00 | 1.03 |
| 20 | 100 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.40 | 1.00 | 1.00 | 1.00 | 1.02 |
| 20 | 200 | 7.85 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 30 | 30 | 7.89 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.18 | 1.00 | 1.00 | 1.04 |
| 30 | 60 | 7.51 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.14 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 100 | 6.95 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 200 | 5.46 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.63 | 1.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 60 | 3.24 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 100 | 2.88 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 200 | 1.29 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 60 | 1.06 | 3.72 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.34 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | $r_{mean}$ | 4.85 | 6.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 2.17 | 1.25 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.78 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.28 | 0.01 | 2.69 | 0.95 | 0.00 | 0.00 | 0.02 |
| | | Part V.2 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 3, 0.5, 0, 0)$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.91 | 7.98 | 1.00 | 1.48 | 1.01 | 1.00 | 1.00 | 1.96 | 8.00 | 3.26 | 1.55 | 8.00 | 3.42 | 1.66 | 1.82 | 2.08 |
| 20 | 60 | 7.90 | 8.00 | 1.00 | 1.72 | 1.00 | 1.00 | 1.00 | 2.65 | 3.65 | 2.69 | 2.36 | 6.58 | 2.57 | 1.45 | 1.77 | 2.90 |
| 20 | 100 | 7.88 | 8.00 | 1.00 | 1.92 | 1.02 | 1.00 | 1.00 | 2.43 | 2.88 | 2.09 | 2.21 | 2.82 | 2.05 | 2.14 | 2.27 | 2.93 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 2.40 | 1.08 | 1.00 | 1.00 | 3.00 | 3.00 | 2.64 | 3.00 | 3.00 | 2.88 | 2.97 | 2.99 | 3.00 |
| 30 | 30 | 7.93 | 7.91 | 1.00 | 1.87 | 1.03 | 1.00 | 1.00 | 2.43 | 8.00 | 2.85 | 1.94 | 8.00 | 2.75 | 2.13 | 2.35 | 2.75 |
| 30 | 60 | 7.81 | 8.00 | 1.00 | 2.13 | 1.15 | 1.00 | 1.00 | 2.66 | 3.12 | 2.37 | 2.31 | 3.60 | 2.13 | 2.43 | 2.59 | 2.98 |
| 30 | 100 | 7.67 | 8.00 | 1.00 | 2.86 | 1.33 | 1.00 | 1.00 | 3.00 | 3.00 | 2.75 | 2.96 | 3.00 | 2.43 | 2.94 | 2.98 | 3.02 |
| 30 | 200 | 7.09 | 8.00 | 1.01 | 3.00 | 2.39 | 1.01 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.95 | 3.00 | 3.00 | 3.01 |
| 100 | 30 | 7.69 | 3.13 | 1.03 | 2.74 | 1.38 | 1.01 | 1.00 | 2.97 | 3.00 | 2.70 | 2.92 | 3.00 | 2.44 | 2.80 | 2.90 | 3.01 |
| 100 | 60 | 6.50 | 8.00 | 2.02 | 3.00 | 2.95 | 2.04 | 1.11 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.97 | 3.00 | 3.00 | 3.01 |
| 100 | 100 | 5.92 | 8.00 | 2.89 | 3.00 | 3.00 | 2.95 | 1.97 | 3.00 | 3.00 | 3.00 | 3.00 | 3.12 | 3.00 | 3.00 | 3.00 | 3.01 |
| 100 | 200 | 3.76 | 8.00 | 2.99 | 3.00 | 3.00 | 3.00 | 2.84 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 30 | 4.03 | 3.00 | 1.02 | 2.83 | 1.52 | 1.02 | 1.00 | 2.99 | 3.00 | 2.58 | 2.96 | 2.99 | 2.17 | 2.92 | 2.96 | 3.00 |
| 200 | 60 | 3.30 | 4.63 | 1.99 | 3.00 | 2.92 | 2.20 | 1.30 | 3.00 | 3.00 | 2.95 | 3.00 | 3.00 | 2.78 | 3.00 | 3.00 | 3.00 |
| 200 | 100 | 3.78 | 8.00 | 2.97 | 3.00 | 3.00 | 2.99 | 2.72 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 200 | 4.35 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| | $r_{mean}$ | 6.34 | 7.17 | 1.62 | 2.56 | 1.92 | 1.64 | 1.43 | 2.82 | 3.67 | 2.81 | 2.70 | 3.88 | 2.72 | 2.65 | 2.73 | 2.92 |
| | $RMSE$ | 3.78 | 4.52 | 1.62 | 0.69 | 1.39 | 1.61 | 1.73 | 0.35 | 1.78 | 0.34 | 0.54 | 1.99 | 0.47 | 0.62 | 0.50 | 0.24 |
| | | Part V.3 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (5, 3, 0.5, 0, 0)$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 8.00 | 1.00 | 1.33 | 1.00 | 1.00 | 1.00 | 2.49 | 8.00 | 4.38 | 1.69 | 8.00 | 4.49 | 2.33 | 2.63 | 1.55 |
| 20 | 60 | 7.94 | 8.00 | 1.00 | 1.19 | 1.00 | 1.00 | 1.00 | 2.72 | 6.09 | 2.95 | 2.11 | 7.23 | 2.83 | 2.12 | 2.47 | 2.09 |
| 20 | 100 | 7.93 | 8.00 | 1.00 | 1.15 | 1.00 | 1.00 | 1.00 | 3.27 | 4.61 | 2.64 | 2.62 | 4.49 | 2.32 | 2.73 | 3.24 | 3.00 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 2.40 | 1.00 | 1.00 | 1.00 | 4.48 | 4.86 | 2.97 | 3.66 | 4.35 | 2.33 | 4.19 | 4.63 | 4.95 |
| 30 | 30 | 7.96 | 7.94 | 1.00 | 1.45 | 1.00 | 1.00 | 1.00 | 2.10 | 8.00 | 3.34 | 1.36 | 8.00 | 3.06 | 2.33 | 2.66 | 2.00 |
| 30 | 60 | 7.90 | 8.00 | 1.00 | 2.39 | 1.00 | 1.00 | 1.00 | 3.56 | 5.44 | 3.08 | 3.17 | 6.21 | 2.82 | 3.23 | 3.67 | 3.59 |
| 30 | 100 | 7.87 | 8.00 | 1.00 | 2.27 | 1.00 | 1.00 | 1.00 | 3.34 | 4.62 | 2.57 | 3.06 | 4.62 | 2.05 | 3.31 | 3.73 | 4.11 |
| 30 | 200 | 7.69 | 8.00 | 1.00 | 1.24 | 1.00 | 1.00 | 1.00 | 4.29 | 4.90 | 1.38 | 2.84 | 4.60 | 1.02 | 4.55 | 4.82 | 5.01 |
| 100 | 30 | 7.97 | 4.90 | 1.00 | 2.01 | 1.00 | 1.00 | 1.00 | 3.62 | 4.73 | 2.48 | 2.96 | 4.59 | 1.85 | 3.29 | 3.76 | 4.19 |
| 100 | 60 | 7.71 | 8.00 | 1.01 | 4.53 | 1.75 | 1.01 | 1.00 | 4.80 | 5.00 | 3.38 | 4.60 | 5.00 | 2.77 | 4.89 | 4.95 | 5.00 |
| 100 | 100 | 7.53 | 8.00 | 1.03 | 5.00 | 3.45 | 1.09 | 1.00 | 5.00 | 5.00 | 3.92 | 4.99 | 5.06 | 3.43 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 6.48 | 8.00 | 1.92 | 5.00 | 5.00 | 3.41 | 1.03 | 5.00 | 5.00 | 4.94 | 5.00 | 5.00 | 4.56 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.41 | 4.82 | 1.00 | 2.49 | 1.02 | 1.00 | 1.00 | 4.14 | 4.65 | 2.33 | 3.67 | 4.43 | 1.68 | 3.97 | 4.39 | 4.93 |
| 200 | 60 | 5.95 | 5.70 | 1.01 | 4.78 | 2.27 | 1.03 | 1.00 | 4.95 | 5.00 | 3.17 | 4.88 | 5.00 | 2.52 | 4.98 | 5.00 | 5.01 |
| 200 | 100 | 6.40 | 8.00 | 3.34 | 5.00 | 4.99 | 4.17 | 1.55 | 5.00 | 5.00 | 4.90 | 5.00 | 5.00 | 4.67 | 5.00 | 5.00 | 5.01 |
| 200 | 200 | 6.76 | 8.00 | 4.96 | 5.00 | 5.00 | 5.00 | 4.74 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 7.46 | 7.46 | 1.45 | 2.95 | 2.03 | 1.61 | 1.27 | 3.99 | 5.37 | 3.34 | 3.54 | 5.41 | 2.96 | 3.87 | 4.12 | 4.09 |
| | $RMSE$ | 2.54 | 2.71 | 3.71 | 2.57 | 3.36 | 3.62 | 3.84 | 1.40 | 1.12 | 1.94 | 1.90 | 1.27 | 2.33 | 1.55 | 1.28 | 1.52 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part VI.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 3, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.00 | 1.50 | 1.01 | 1.00 | 1.00 | 2.25 | 8.00 | 4.98 | 6.45 | 8.00 | 6.79 | 1.00 | 1.00 | 1.60 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.93 | 1.12 | 1.00 | 1.00 | 2.05 | 7.89 | 2.39 | 3.06 | 8.00 | 3.71 | 1.00 | 1.05 | 1.96 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.75 | 1.00 | 1.00 | 1.00 | 1.99 | 2.54 | 1.99 | 6.49 | 8.00 | 3.24 | 1.00 | 1.01 | 2.01 |
| 20 | 200 | 7.94 | 8.00 | 1.00 | 1.88 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.99 | 5.17 | 7.96 | 2.07 | 1.00 | 1.01 | 2.02 |
| 30 | 30 | 8.00 | 7.99 | 1.28 | 2.44 | 1.59 | 1.11 | 1.01 | 2.72 | 8.00 | 4.31 | 3.34 | 8.00 | 5.07 | 1.12 | 1.29 | 2.28 |
| 30 | 60 | 8.00 | 8.00 | 1.42 | 2.50 | 1.85 | 1.24 | 1.01 | 2.74 | 7.99 | 2.76 | 2.61 | 8.00 | 2.83 | 1.24 | 1.48 | 2.50 |
| 30 | 100 | 7.99 | 8.00 | 1.60 | 2.65 | 1.98 | 1.49 | 1.03 | 2.83 | 4.66 | 2.42 | 3.19 | 7.97 | 2.65 | 1.22 | 1.55 | 2.78 |
| 30 | 200 | 7.98 | 8.00 | 1.46 | 2.73 | 2.00 | 1.48 | 1.01 | 2.96 | 3.00 | 2.06 | 2.58 | 7.01 | 2.01 | 1.02 | 1.24 | 3.01 |
| 100 | 30 | 8.00 | 7.89 | 1.01 | 4.62 | 1.23 | 1.00 | 1.00 | 7.25 | 8.00 | 4.42 | 7.20 | 8.00 | 4.21 | 1.00 | 1.01 | 1.76 |
| 100 | 60 | 8.00 | 8.00 | 1.04 | 6.68 | 2.27 | 1.04 | 1.00 | 7.03 | 8.00 | 4.48 | 6.83 | 8.00 | 4.14 | 1.00 | 1.00 | 1.73 |
| 100 | 100 | 8.00 | 8.00 | 1.13 | 7.58 | 4.08 | 1.22 | 1.00 | 7.30 | 8.00 | 4.91 | 6.92 | 8.00 | 4.42 | 1.00 | 1.00 | 2.15 |
| 100 | 200 | 8.00 | 8.00 | 1.61 | 7.97 | 6.20 | 2.66 | 1.03 | 7.95 | 8.00 | 5.58 | 7.91 | 8.00 | 5.05 | 1.00 | 1.00 | 2.10 |
| 200 | 30 | 8.00 | 7.53 | 1.00 | 2.82 | 1.01 | 1.00 | 1.00 | 7.55 | 8.00 | 3.20 | 7.19 | 7.99 | 2.70 | 1.00 | 1.00 | 1.32 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.68 | 1.11 | 1.00 | 1.00 | 7.80 | 8.00 | 2.72 | 7.59 | 8.00 | 2.27 | 1.00 | 1.00 | 1.19 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.99 | 1.75 | 1.00 | 1.00 | 7.99 | 8.00 | 2.92 | 7.98 | 8.00 | 2.49 | 1.00 | 1.00 | 1.08 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 6.68 | 1.05 | 1.00 | 8.00 | 8.00 | 3.80 | 8.00 | 8.00 | 3.62 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.16 | 4.36 | 2.24 | 1.21 | 1.01 | 5.03 | 6.76 | 3.43 | 5.78 | 7.93 | 3.58 | 1.04 | 1.10 | 1.91 |
| | $RMSE$ | 6.99 | 6.96 | 0.28 | 4.21 | 2.15 | 0.46 | 0.01 | 4.80 | 6.16 | 2.70 | 5.19 | 6.94 | 2.88 | 0.09 | 0.21 | 1.07 |
| Part VI.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 3, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.06 | 3.77 | 1.23 | 1.01 | 1.00 | 4.88 | 8.00 | 5.57 | 5.74 | 8.00 | 6.08 | 2.01 | 2.41 | 2.62 |
| 20 | 60 | 7.97 | 8.00 | 1.05 | 3.62 | 1.41 | 1.01 | 1.00 | 4.15 | 7.98 | 4.00 | 5.72 | 8.00 | 4.41 | 1.62 | 2.14 | 3.30 |
| 20 | 100 | 7.97 | 8.00 | 1.03 | 3.27 | 1.48 | 1.01 | 1.00 | 4.01 | 5.99 | 3.82 | 4.86 | 7.92 | 3.91 | 2.55 | 2.94 | 3.80 |
| 20 | 200 | 7.97 | 8.00 | 1.07 | 3.67 | 2.13 | 1.04 | 1.00 | 4.00 | 4.01 | 3.51 | 4.58 | 6.81 | 3.85 | 3.53 | 3.77 | 4.00 |
| 30 | 30 | 8.00 | 8.00 | 1.19 | 4.45 | 1.93 | 1.05 | 1.00 | 4.79 | 8.00 | 5.19 | 5.32 | 8.00 | 5.53 | 2.80 | 3.31 | 3.33 |
| 30 | 60 | 8.00 | 8.00 | 1.30 | 4.61 | 2.68 | 1.12 | 1.00 | 4.88 | 8.00 | 4.42 | 5.07 | 8.00 | 4.34 | 3.34 | 3.85 | 3.92 |
| 30 | 100 | 8.00 | 8.00 | 1.66 | 4.64 | 3.08 | 1.52 | 1.02 | 4.88 | 7.14 | 4.11 | 5.41 | 8.00 | 4.06 | 3.63 | 4.10 | 4.55 |
| 30 | 200 | 8.00 | 8.00 | 2.65 | 4.34 | 3.94 | 2.70 | 1.02 | 4.88 | 5.01 | 4.00 | 4.85 | 6.72 | 4.00 | 3.89 | 4.06 | 4.88 |
| 100 | 30 | 8.00 | 7.98 | 1.30 | 6.49 | 2.57 | 1.23 | 1.01 | 7.88 | 8.00 | 5.24 | 7.88 | 8.00 | 5.08 | 2.46 | 2.95 | 2.38 |
| 100 | 60 | 8.00 | 8.00 | 2.80 | 7.87 | 4.20 | 2.82 | 1.67 | 7.95 | 8.00 | 5.58 | 7.92 | 8.00 | 5.29 | 2.87 | 3.20 | 2.62 |
| 100 | 100 | 8.00 | 8.00 | 3.10 | 8.00 | 6.12 | 3.20 | 2.76 | 8.00 | 8.00 | 6.00 | 7.99 | 8.00 | 5.65 | 2.97 | 3.13 | 2.73 |
| 100 | 200 | 8.00 | 8.00 | 3.46 | 8.00 | 7.81 | 4.34 | 3.01 | 8.00 | 8.00 | 6.64 | 8.00 | 8.00 | 6.40 | 2.93 | 3.05 | 2.75 |
| 200 | 30 | 8.00 | 7.80 | 1.24 | 4.86 | 2.40 | 1.25 | 1.01 | 7.89 | 8.00 | 4.04 | 7.77 | 8.00 | 3.69 | 2.36 | 2.61 | 2.52 |
| 200 | 60 | 8.00 | 8.00 | 2.59 | 7.66 | 3.07 | 2.71 | 1.83 | 7.98 | 8.00 | 3.75 | 7.94 | 8.00 | 3.44 | 2.72 | 2.84 | 2.97 |
| 200 | 100 | 8.00 | 8.00 | 3.00 | 8.00 | 3.73 | 3.00 | 2.94 | 8.00 | 8.00 | 3.89 | 8.00 | 8.00 | 3.65 | 2.99 | 2.99 | 3.03 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 7.64 | 3.06 | 3.00 | 8.00 | 8.00 | 4.54 | 8.00 | 8.00 | 4.62 | 3.00 | 3.00 | 3.00 |
| | $r_{mean}$ | 7.99 | 7.99 | 1.97 | 5.70 | 3.46 | 2.00 | 1.58 | 6.26 | 7.38 | 4.64 | 6.57 | 7.84 | 4.63 | 2.85 | 3.15 | 3.28 |
| | $RMSE$ | 4.99 | 4.99 | 1.36 | 3.28 | 2.06 | 1.45 | 1.64 | 3.69 | 4.55 | 1.88 | 3.83 | 4.86 | 1.86 | 0.59 | 0.56 | 0.78 |
| Part IV.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 3, 0.5, 0.5, 0.2)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.99 | 8.00 | 1.00 | 5.03 | 1.05 | 1.00 | 1.00 | 6.83 | 8.00 | 6.32 | 7.18 | 8.00 | 6.61 | 3.18 | 3.60 | 2.25 |
| 20 | 60 | 7.99 | 8.00 | 1.00 | 3.93 | 1.05 | 1.00 | 1.00 | 6.25 | 8.00 | 4.96 | 6.93 | 8.00 | 5.20 | 2.85 | 3.39 | 2.06 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 4.67 | 1.02 | 1.00 | 1.00 | 6.17 | 7.78 | 4.85 | 6.69 | 7.98 | 4.65 | 3.88 | 4.54 | 3.81 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 5.86 | 1.07 | 1.00 | 1.00 | 6.01 | 6.15 | 4.82 | 6.64 | 7.76 | 4.27 | 5.65 | 5.89 | 5.92 |
| 30 | 30 | 8.00 | 8.00 | 1.01 | 5.38 | 1.20 | 1.00 | 1.00 | 6.08 | 8.00 | 5.86 | 6.25 | 8.00 | 5.95 | 3.46 | 4.02 | 2.35 |
| 30 | 60 | 8.00 | 8.00 | 1.01 | 6.19 | 1.69 | 1.01 | 1.00 | 6.81 | 8.00 | 5.67 | 7.04 | 8.00 | 5.65 | 4.23 | 5.04 | 2.96 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 5.91 | 1.64 | 1.00 | 1.00 | 6.89 | 7.93 | 5.00 | 7.06 | 7.99 | 4.94 | 4.88 | 5.71 | 3.61 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 6.32 | 1.60 | 1.00 | 1.00 | 6.71 | 7.05 | 4.74 | 6.79 | 7.68 | 4.20 | 5.35 | 6.00 | 5.48 |
| 100 | 30 | 8.00 | 8.00 | 1.00 | 7.30 | 1.29 | 1.00 | 1.00 | 7.98 | 8.00 | 5.61 | 7.98 | 8.00 | 5.34 | 3.28 | 3.76 | 1.00 |
| 100 | 60 | 8.00 | 8.00 | 1.18 | 7.99 | 4.56 | 1.20 | 1.00 | 8.00 | 8.00 | 6.05 | 8.00 | 8.00 | 5.66 | 3.55 | 4.13 | 0.60 |
| 100 | 100 | 8.00 | 8.00 | 1.76 | 8.00 | 7.04 | 2.22 | 1.03 | 8.00 | 8.00 | 6.54 | 8.00 | 8.00 | 6.25 | 4.06 | 4.61 | 0.30 |
| 100 | 200 | 8.00 | 8.00 | 4.74 | 8.00 | 8.00 | 5.69 | 1.48 | 8.00 | 8.00 | 7.24 | 8.00 | 8.00 | 6.79 | 4.14 | 4.48 | 0.47 |
| 200 | 30 | 8.00 | 7.92 | 1.00 | 6.08 | 1.41 | 1.00 | 1.00 | 7.97 | 8.00 | 4.64 | 7.93 | 8.00 | 4.20 | 2.86 | 3.35 | 1.29 |
| 200 | 60 | 8.00 | 8.00 | 1.12 | 7.93 | 4.05 | 1.25 | 1.00 | 8.00 | 8.00 | 4.90 | 7.99 | 8.00 | 4.54 | 3.31 | 3.83 | 1.26 |
| 200 | 100 | 8.00 | 8.00 | 4.54 | 8.00 | 5.63 | 4.84 | 2.84 | 8.00 | 8.00 | 5.16 | 8.00 | 8.00 | 5.08 | 4.79 | 4.90 | 4.09 |
| 200 | 200 | 8.00 | 8.00 | 5.00 | 8.00 | 7.95 | 5.07 | 4.96 | 8.00 | 8.00 | 5.47 | 8.00 | 8.00 | 5.23 | 4.97 | 4.98 | 4.84 |
| | $r_{mean}$ | 8.00 | 8.00 | 1.77 | 6.54 | 3.14 | 1.89 | 1.39 | 7.23 | 7.81 | 5.49 | 7.41 | 7.96 | 5.29 | 4.03 | 4.51 | 2.64 |
| | $RMSE$ | 3.00 | 3.00 | 3.54 | 2.03 | 3.17 | 3.51 | 3.75 | 2.37 | 2.85 | 0.88 | 2.48 | 2.96 | 0.85 | 1.30 | 0.96 | 2.94 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part VII.1 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (1, 3, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.14 | 1.00 | 8.00 | 2.32 | 1.02 | 1.02 | 1.03 |
| 20 | 60 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 5.06 | 1.11 | 1.00 | 1.00 | 1.03 |
| 20 | 100 | 7.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 | 1.00 | 1.00 | 1.00 | 1.02 |
| 20 | 200 | 7.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 30 | 30 | 7.88 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.02 | 1.00 | 1.00 | 1.04 |
| 30 | 60 | 7.53 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 100 | 7.04 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 200 | 5.10 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.58 | 1.86 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 60 | 3.17 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 100 | 2.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 200 | 1.30 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.03 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 60 | 1.07 | 3.74 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.35 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.78 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| | $r_{mean}$ | 4.82 | 6.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 2.14 | 1.09 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.75 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.29 | 0.00 | 2.68 | 0.33 | 0.01 | 0.01 | 0.02 |
| Part VII.2 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 3, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.93 | 7.99 | 1.00 | 1.16 | 1.00 | 1.00 | 1.00 | 1.52 | 8.00 | 3.05 | 1.31 | 8.00 | 3.21 | 1.44 | 1.54 | 1.66 |
| 20 | 60 | 7.90 | 8.00 | 1.00 | 1.38 | 1.01 | 1.00 | 1.00 | 2.25 | 3.48 | 2.38 | 1.92 | 4.80 | 2.27 | 1.91 | 2.08 | 2.57 |
| 20 | 100 | 7.91 | 8.00 | 1.00 | 1.74 | 1.01 | 1.00 | 1.00 | 2.76 | 2.97 | 2.42 | 2.53 | 2.92 | 2.25 | 2.46 | 2.63 | 2.98 |
| 20 | 200 | 7.87 | 8.00 | 1.00 | 2.12 | 1.05 | 1.00 | 1.00 | 2.95 | 2.99 | 2.27 | 2.93 | 2.99 | 2.27 | 2.74 | 2.86 | 3.00 |
| 30 | 30 | 7.92 | 7.91 | 1.00 | 1.43 | 1.02 | 1.00 | 1.00 | 1.79 | 8.00 | 2.50 | 1.43 | 8.00 | 2.33 | 1.68 | 1.84 | 2.28 |
| 30 | 60 | 7.78 | 8.00 | 1.00 | 1.89 | 1.07 | 1.00 | 1.00 | 2.43 | 3.08 | 2.21 | 2.03 | 4.16 | 1.92 | 1.98 | 2.16 | 2.88 |
| 30 | 100 | 7.67 | 8.00 | 1.00 | 2.16 | 1.08 | 1.00 | 1.00 | 2.81 | 2.99 | 2.19 | 2.67 | 2.99 | 1.96 | 2.56 | 2.71 | 3.01 |
| 30 | 200 | 7.12 | 8.00 | 1.13 | 2.72 | 1.80 | 1.13 | 1.00 | 2.99 | 3.00 | 2.41 | 2.97 | 3.00 | 2.14 | 2.85 | 2.94 | 3.00 |
| 100 | 30 | 7.75 | 3.14 | 1.00 | 1.54 | 1.05 | 1.00 | 1.00 | 2.11 | 2.74 | 1.63 | 1.87 | 2.62 | 1.40 | 1.74 | 1.90 | 2.80 |
| 100 | 60 | 6.44 | 8.00 | 1.16 | 2.74 | 1.80 | 1.17 | 1.02 | 2.85 | 3.00 | 2.25 | 2.77 | 3.00 | 2.03 | 2.59 | 2.74 | 3.01 |
| 100 | 100 | 5.98 | 8.00 | 2.09 | 3.00 | 2.89 | 2.23 | 1.47 | 3.00 | 3.00 | 2.92 | 3.00 | 3.09 | 2.82 | 3.00 | 3.00 | 3.00 |
| 100 | 200 | 3.77 | 8.00 | 2.88 | 3.00 | 3.00 | 2.95 | 2.59 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 30 | 4.11 | 2.94 | 1.00 | 1.60 | 1.05 | 1.00 | 1.00 | 2.24 | 2.58 | 1.43 | 2.00 | 2.41 | 1.21 | 1.90 | 2.09 | 2.96 |
| 200 | 60 | 3.31 | 4.61 | 1.37 | 2.89 | 2.17 | 1.48 | 1.11 | 2.96 | 3.00 | 2.32 | 2.92 | 3.00 | 2.04 | 2.81 | 2.89 | 3.00 |
| 200 | 100 | 3.82 | 8.00 | 2.56 | 3.00 | 2.98 | 2.73 | 2.16 | 3.00 | 3.00 | 2.96 | 3.00 | 3.00 | 2.88 | 3.00 | 3.00 | 3.01 |
| 200 | 200 | 4.33 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| | $r_{mean}$ | 6.35 | 7.16 | 1.45 | 2.21 | 1.69 | 1.48 | 1.33 | 2.60 | 3.61 | 2.43 | 2.46 | 3.75 | 2.30 | 2.42 | 2.52 | 2.82 |
| | $RMSE$ | 3.79 | 4.52 | 1.71 | 1.03 | 1.55 | 1.69 | 1.78 | 0.61 | 1.78 | 0.73 | 0.79 | 1.86 | 0.89 | 0.79 | 0.68 | 0.40 |
| Part VII.3 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (5, 3, 0.85, 0, 0)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.99 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.27 | 8.00 | 3.14 | 1.08 | 8.00 | 3.09 | 1.49 | 1.56 | 1.30 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.40 | 1.01 | 1.00 | 1.00 | 2.35 | 4.82 | 2.52 | 2.00 | 6.62 | 2.41 | 1.78 | 2.00 | 2.35 |
| 20 | 100 | 7.90 | 8.00 | 1.00 | 1.24 | 1.00 | 1.00 | 1.00 | 2.57 | 3.79 | 2.21 | 1.80 | 3.36 | 1.77 | 1.93 | 2.19 | 2.52 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 1.46 | 1.00 | 1.00 | 1.00 | 3.40 | 3.91 | 2.26 | 2.97 | 3.88 | 1.86 | 2.73 | 3.29 | 4.20 |
| 30 | 30 | 7.92 | 7.90 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.11 | 8.00 | 1.83 | 1.03 | 8.00 | 1.58 | 1.54 | 1.62 | 1.28 |
| 30 | 60 | 7.88 | 8.00 | 1.00 | 1.44 | 1.01 | 1.00 | 1.00 | 2.05 | 4.29 | 1.98 | 1.67 | 4.75 | 1.68 | 1.73 | 1.91 | 2.38 |
| 30 | 100 | 7.85 | 8.00 | 1.00 | 2.18 | 1.05 | 1.00 | 1.00 | 3.37 | 4.57 | 2.40 | 3.16 | 4.66 | 2.21 | 2.53 | 3.01 | 4.16 |
| 30 | 200 | 7.68 | 8.00 | 1.00 | 3.59 | 1.21 | 1.00 | 1.00 | 4.77 | 4.96 | 2.95 | 4.62 | 4.91 | 2.84 | 4.43 | 4.80 | 5.00 |
| 100 | 30 | 7.95 | 4.08 | 1.00 | 1.37 | 1.01 | 1.00 | 1.00 | 2.02 | 3.19 | 1.55 | 1.65 | 2.90 | 1.27 | 1.68 | 1.87 | 2.47 |
| 100 | 60 | 7.68 | 8.00 | 1.01 | 2.65 | 1.27 | 1.01 | 1.00 | 3.03 | 4.79 | 2.00 | 2.65 | 4.73 | 1.63 | 2.35 | 2.76 | 4.57 |
| 100 | 100 | 7.56 | 8.00 | 1.23 | 4.26 | 2.43 | 1.31 | 1.03 | 4.25 | 5.01 | 2.89 | 4.06 | 5.08 | 2.55 | 4.30 | 4.60 | 5.00 |
| 100 | 200 | 6.42 | 8.00 | 1.69 | 5.00 | 4.14 | 2.20 | 1.13 | 5.00 | 5.00 | 3.85 | 4.99 | 5.00 | 3.38 | 5.00 | 5.00 | 5.01 |
| 200 | 30 | 7.17 | 3.48 | 1.00 | 1.37 | 1.01 | 1.00 | 1.00 | 2.11 | 2.65 | 1.31 | 1.80 | 2.35 | 1.11 | 1.66 | 1.83 | 3.25 |
| 200 | 60 | 5.98 | 5.71 | 1.06 | 3.34 | 1.71 | 1.10 | 1.01 | 3.81 | 4.69 | 2.19 | 3.53 | 4.57 | 1.79 | 3.24 | 3.77 | 5.00 |
| 200 | 100 | 6.33 | 8.00 | 1.78 | 4.81 | 3.42 | 2.10 | 1.26 | 4.84 | 5.00 | 3.27 | 4.79 | 5.00 | 2.98 | 4.86 | 4.94 | 5.00 |
| 200 | 200 | 6.78 | 8.00 | 3.80 | 5.00 | 4.96 | 4.34 | 3.22 | 5.00 | 5.00 | 4.61 | 5.00 | 5.00 | 4.42 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 7.43 | 7.32 | 1.29 | 2.57 | 1.76 | 1.38 | 1.17 | 3.18 | 4.85 | 2.56 | 2.93 | 4.93 | 2.29 | 2.89 | 3.13 | 3.66 |
| | $RMSE$ | 2.52 | 2.74 | 3.78 | 2.84 | 3.47 | 3.72 | 3.87 | 2.22 | 1.38 | 2.58 | 2.49 | 1.51 | 2.85 | 2.49 | 2.28 | 1.91 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Part VIII.1 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (1, 3, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.00 | 1.57 | 1.01 | 1.00 | 1.00 | 2.23 | 8.00 | 4.94 | 3.36 | 8.00 | 5.77 | 1.04 | 1.09 | 1.69 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.76 | 1.02 | 1.00 | 1.00 | 2.05 | 7.85 | 2.41 | 4.11 | 8.00 | 3.85 | 1.06 | 1.13 | 1.96 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 1.69 | 1.00 | 1.00 | 1.00 | 1.99 | 2.59 | 1.99 | 5.54 | 8.00 | 3.11 | 1.00 | 1.01 | 2.01 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 1.97 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.99 | 1.96 | 2.16 | 1.77 | 1.24 | 1.50 | 2.02 |
| 30 | 30 | 8.00 | 7.99 | 1.16 | 2.44 | 1.45 | 1.07 | 1.00 | 2.67 | 8.00 | 4.28 | 2.72 | 8.00 | 4.81 | 1.18 | 1.32 | 2.24 |
| 30 | 60 | 8.00 | 8.00 | 1.43 | 2.46 | 1.86 | 1.27 | 1.02 | 2.71 | 7.99 | 2.76 | 3.82 | 8.00 | 3.45 | 1.19 | 1.39 | 2.46 |
| 30 | 100 | 8.00 | 8.00 | 1.40 | 2.60 | 1.93 | 1.30 | 1.01 | 2.83 | 4.87 | 2.46 | 2.95 | 7.89 | 2.48 | 1.32 | 1.60 | 2.80 |
| 30 | 200 | 7.99 | 8.00 | 1.19 | 2.69 | 1.97 | 1.21 | 1.00 | 2.96 | 3.00 | 2.08 | 3.00 | 4.94 | 2.19 | 1.07 | 1.34 | 3.00 |
| 100 | 30 | 8.00 | 7.87 | 1.00 | 4.40 | 1.19 | 1.00 | 1.00 | 7.08 | 8.00 | 4.30 | 6.88 | 8.00 | 4.02 | 1.09 | 1.15 | 1.82 |
| 100 | 60 | 8.00 | 8.00 | 1.03 | 6.64 | 2.30 | 1.03 | 1.00 | 7.02 | 8.00 | 4.55 | 7.14 | 8.00 | 4.35 | 1.00 | 1.00 | 1.74 |
| 100 | 100 | 8.00 | 8.00 | 1.14 | 7.60 | 4.18 | 1.24 | 1.00 | 7.32 | 8.00 | 4.95 | 7.19 | 8.00 | 4.71 | 1.00 | 1.00 | 2.18 |
| 100 | 200 | 8.00 | 8.00 | 1.64 | 7.96 | 6.21 | 2.71 | 1.02 | 7.95 | 8.00 | 5.59 | 7.91 | 8.00 | 5.19 | 1.00 | 1.00 | 2.18 |
| 200 | 30 | 8.00 | 7.49 | 1.00 | 2.77 | 1.01 | 1.00 | 1.00 | 7.51 | 8.00 | 3.15 | 7.19 | 7.99 | 2.66 | 1.02 | 1.02 | 1.37 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.69 | 1.11 | 1.00 | 1.00 | 7.83 | 8.00 | 2.71 | 7.61 | 8.00 | 2.19 | 1.00 | 1.00 | 1.16 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 8.00 | 1.75 | 1.00 | 1.00 | 7.99 | 8.00 | 2.96 | 7.99 | 8.00 | 2.55 | 1.00 | 1.00 | 1.05 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 6.71 | 1.07 | 1.00 | 8.00 | 8.00 | 3.82 | 8.00 | 8.00 | 3.50 | 1.00 | 1.00 | 1.04 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.12 | 4.33 | 2.23 | 1.18 | 1.00 | 5.01 | 6.77 | 3.43 | 5.46 | 7.44 | 3.54 | 1.08 | 1.16 | 1.92 |
| | $RMSE$ | 6.99 | 6.96 | 0.23 | 4.19 | 2.16 | 0.45 | 0.01 | 4.78 | 6.17 | 2.69 | 4.96 | 6.62 | 2.79 | 0.12 | 0.25 | 1.08 |
| | Part VIII.2 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 3, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.02 | 2.64 | 1.08 | 1.00 | 1.00 | 3.87 | 8.00 | 5.35 | 3.95 | 8.00 | 5.77 | 1.89 | 2.17 | 2.24 |
| 20 | 60 | 7.98 | 8.00 | 1.02 | 3.16 | 1.16 | 1.01 | 1.00 | 4.11 | 7.98 | 3.94 | 5.33 | 8.00 | 4.24 | 2.12 | 2.50 | 3.08 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 3.14 | 1.05 | 1.00 | 1.00 | 3.95 | 6.05 | 3.67 | 4.15 | 7.71 | 3.66 | 2.73 | 3.16 | 3.52 |
| 20 | 200 | 7.95 | 8.00 | 1.00 | 3.90 | 1.34 | 1.00 | 1.00 | 4.00 | 4.00 | 3.75 | 4.28 | 6.56 | 3.50 | 3.50 | 3.87 | 4.01 |
| 30 | 30 | 8.00 | 8.00 | 1.11 | 3.50 | 1.45 | 1.03 | 1.00 | 4.14 | 8.00 | 4.90 | 3.92 | 8.00 | 5.02 | 2.07 | 2.46 | 2.70 |
| 30 | 60 | 8.00 | 8.00 | 1.25 | 4.16 | 2.41 | 1.12 | 1.01 | 4.63 | 8.00 | 4.19 | 4.82 | 8.00 | 4.22 | 2.91 | 3.41 | 3.73 |
| 30 | 100 | 8.00 | 8.00 | 1.55 | 4.47 | 3.40 | 1.36 | 1.02 | 4.84 | 7.08 | 4.10 | 5.17 | 7.99 | 4.05 | 3.27 | 3.82 | 4.53 |
| 30 | 200 | 7.99 | 8.00 | 1.38 | 4.73 | 3.80 | 1.42 | 1.00 | 4.97 | 5.01 | 4.01 | 4.92 | 6.72 | 4.00 | 4.00 | 4.33 | 4.97 |
| 100 | 30 | 8.00 | 7.97 | 1.11 | 5.91 | 1.70 | 1.08 | 1.00 | 7.77 | 8.00 | 4.98 | 7.72 | 8.00 | 4.75 | 1.83 | 2.22 | 1.81 |
| 100 | 60 | 8.00 | 8.00 | 1.69 | 7.76 | 3.49 | 1.71 | 1.16 | 7.91 | 8.00 | 5.25 | 7.86 | 8.00 | 4.95 | 1.98 | 2.47 | 1.79 |
| 100 | 100 | 8.00 | 8.00 | 2.77 | 8.00 | 5.88 | 2.94 | 2.01 | 7.99 | 8.00 | 5.95 | 7.99 | 8.00 | 5.74 | 2.25 | 2.69 | 2.12 |
| 100 | 200 | 8.00 | 8.00 | 3.50 | 8.00 | 7.72 | 4.30 | 2.86 | 8.00 | 8.00 | 6.53 | 8.00 | 8.00 | 6.38 | 2.61 | 2.95 | 2.15 |
| 200 | 30 | 8.00 | 7.70 | 1.06 | 4.24 | 1.53 | 1.07 | 1.00 | 7.79 | 8.00 | 3.78 | 7.63 | 8.00 | 3.34 | 1.68 | 1.90 | 1.84 |
| 200 | 60 | 8.00 | 8.00 | 1.96 | 7.56 | 2.92 | 2.14 | 1.41 | 7.96 | 8.00 | 3.73 | 7.89 | 8.00 | 3.36 | 1.99 | 2.22 | 2.58 |
| 200 | 100 | 8.00 | 8.00 | 2.79 | 8.00 | 3.69 | 2.88 | 2.46 | 8.00 | 8.00 | 3.86 | 8.00 | 8.00 | 3.62 | 2.34 | 2.54 | 2.89 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 7.69 | 3.08 | 3.00 | 8.00 | 8.00 | 4.58 | 8.00 | 8.00 | 4.32 | 2.96 | 2.98 | 3.01 |
| | $r_{mean}$ | 7.99 | 7.98 | 1.70 | 5.45 | 3.14 | 1.76 | 1.43 | 6.12 | 7.38 | 4.54 | 6.23 | 7.81 | 4.43 | 2.51 | 2.86 | 2.94 |
| | $RMSE$ | 4.99 | 4.98 | 1.53 | 3.17 | 2.16 | 1.58 | 1.72 | 3.62 | 4.55 | 1.75 | 3.65 | 4.83 | 1.69 | 0.81 | 0.69 | 0.96 |
| | Part VIII.3 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (5, 3, 0.85, 0.5, 0.2)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.01 | 2.68 | 1.05 | 1.00 | 1.00 | 4.43 | 8.00 | 5.62 | 4.54 | 8.00 | 5.83 | 2.08 | 2.36 | 1.89 |
| 20 | 60 | 7.97 | 8.00 | 1.01 | 2.94 | 1.08 | 1.00 | 1.00 | 4.60 | 8.00 | 4.15 | 4.17 | 8.00 | 3.99 | 1.99 | 2.31 | 1.93 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 3.33 | 1.05 | 1.00 | 1.00 | 5.40 | 7.68 | 4.05 | 6.38 | 8.00 | 3.99 | 2.31 | 2.83 | 2.53 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 4.71 | 1.02 | 1.00 | 1.00 | 5.83 | 6.12 | 4.07 | 6.26 | 7.60 | 3.71 | 3.28 | 4.15 | 4.67 |
| 30 | 30 | 8.00 | 8.00 | 1.02 | 3.80 | 1.15 | 1.00 | 1.00 | 4.67 | 8.00 | 5.30 | 4.04 | 8.00 | 5.26 | 2.30 | 2.78 | 2.18 |
| 30 | 60 | 8.00 | 8.00 | 1.04 | 4.50 | 1.53 | 1.01 | 1.00 | 5.63 | 8.00 | 4.62 | 5.72 | 8.00 | 4.55 | 2.79 | 3.35 | 2.25 |
| 30 | 100 | 8.00 | 8.00 | 1.02 | 5.89 | 1.77 | 1.01 | 1.00 | 6.64 | 7.91 | 5.22 | 7.00 | 8.00 | 5.09 | 4.38 | 5.21 | 3.72 |
| 30 | 200 | 8.00 | 8.00 | 1.01 | 6.26 | 2.59 | 1.01 | 1.00 | 6.76 | 7.03 | 5.23 | 6.69 | 7.55 | 4.94 | 5.47 | 6.04 | 5.93 |
| 100 | 30 | 8.00 | 7.99 | 1.02 | 5.76 | 1.33 | 1.01 | 1.00 | 7.82 | 8.00 | 4.92 | 7.68 | 8.00 | 4.50 | 2.09 | 2.48 | 1.47 |
| 100 | 60 | 8.00 | 8.00 | 1.13 | 7.95 | 2.73 | 1.14 | 1.00 | 7.99 | 8.00 | 5.30 | 7.97 | 8.00 | 4.88 | 2.18 | 2.64 | 1.01 |
| 100 | 100 | 8.00 | 8.00 | 1.83 | 8.00 | 6.24 | 2.10 | 1.15 | 8.00 | 8.00 | 6.14 | 8.00 | 8.00 | 5.74 | 2.30 | 2.75 | 0.96 |
| 100 | 200 | 8.00 | 8.00 | 3.73 | 8.00 | 7.99 | 5.03 | 1.98 | 8.00 | 8.00 | 7.12 | 8.00 | 8.00 | 7.05 | 2.64 | 3.09 | 0.59 |
| 200 | 30 | 8.00 | 7.78 | 1.01 | 4.29 | 1.28 | 1.01 | 1.00 | 7.83 | 8.00 | 3.85 | 7.67 | 8.00 | 3.38 | 1.84 | 2.12 | 1.50 |
| 200 | 60 | 8.00 | 8.00 | 1.33 | 7.84 | 3.09 | 1.46 | 1.07 | 7.99 | 8.00 | 4.25 | 7.98 | 8.00 | 3.89 | 1.92 | 2.21 | 1.54 |
| 200 | 100 | 8.00 | 8.00 | 2.45 | 8.00 | 5.01 | 2.96 | 1.62 | 8.00 | 8.00 | 4.82 | 8.00 | 8.00 | 4.54 | 2.40 | 2.71 | 1.79 |
| 200 | 200 | 8.00 | 8.00 | 4.75 | 8.00 | 7.91 | 5.00 | 4.28 | 8.00 | 8.00 | 5.37 | 8.00 | 8.00 | 5.21 | 3.94 | 4.34 | 3.76 |
| | $r_{mean}$ | 7.99 | 7.99 | 1.59 | 5.75 | 2.93 | 1.73 | 1.32 | 6.72 | 7.80 | 5.00 | 6.76 | 7.95 | 4.78 | 2.74 | 3.21 | 2.36 |
| | $RMSE$ | 2.99 | 2.99 | 3.58 | 2.10 | 3.17 | 3.53 | 3.77 | 2.20 | 2.84 | 0.83 | 2.25 | 2.95 | 0.93 | 2.46 | 2.10 | 3.00 |

Table 2: Heteroskedastic idiosyncratic errors

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.21 | 8.00 | 4.20 | 1.20 | 8.00 | 4.71 | 1.00 | 1.00 | 1.25 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.05 | 5.25 | 1.33 | 1.09 | 7.90 | 1.96 | 1.00 | 1.00 | 1.16 |
| 20 | 100 | 7.93 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 | 1.00 | 1.01 | 1.68 | 1.02 | 1.00 | 1.00 | 1.12 |
| 20 | 200 | 7.87 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.06 |
| 30 | 30 | 7.96 | 7.95 | 1.00 | 1.07 | 1.00 | 1.00 | 1.00 | 1.08 | 8.00 | 2.37 | 1.10 | 8.00 | 3.04 | 1.00 | 1.00 | 1.28 |
| 30 | 60 | 7.94 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 5.13 | 1.03 | 1.02 | 7.52 | 1.08 | 1.00 | 1.00 | 1.16 |
| 30 | 100 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.15 | 1.00 | 1.00 | 2.14 | 1.00 | 1.00 | 1.00 | 1.13 |
| 30 | 200 | 7.65 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 |
| 100 | 30 | 8.00 | 5.94 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.22 | 4.32 | 1.10 | 1.18 | 4.90 | 1.07 | 1.00 | 1.00 | 1.23 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 5.48 | 1.00 | 1.01 | 6.51 | 1.00 | 1.00 | 1.00 | 1.12 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 7.74 | 1.00 | 1.00 | 7.99 | 1.00 | 1.00 | 1.00 | 1.10 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.14 | 1.00 | 1.00 | 1.00 | 1.09 |
| 200 | 30 | 8.00 | 5.26 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.33 | 2.59 | 1.01 | 1.19 | 2.43 | 1.00 | 1.00 | 1.00 | 1.23 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.41 | 1.00 | 1.00 | 2.42 | 1.00 | 1.00 | 1.00 | 1.08 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.80 | 1.00 | 1.00 | 3.11 | 1.00 | 1.00 | 1.00 | 1.07 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 6.30 | 1.00 | 1.00 | 7.61 | 1.00 | 1.00 | 1.00 | 1.05 |
| | $r_{mean}$ | 7.94 | 7.70 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.06 | 3.96 | 1.32 | 1.05 | 4.58 | 1.43 | 1.00 | 1.00 | 1.14 |
| | $RMSE$ | 6.94 | 6.74 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.11 | 3.92 | 0.87 | 0.09 | 4.59 | 1.09 | 0.00 | 0.00 | 0.16 |
| Part I.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 8.00 | 1.00 | 2.40 | 1.05 | 1.00 | 1.00 | 3.10 | 8.00 | 4.72 | 2.90 | 8.00 | 5.00 | 2.13 | 2.38 | 2.32 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.97 | 1.02 | 1.00 | 1.00 | 2.83 | 7.40 | 2.97 | 2.59 | 7.84 | 2.94 | 2.03 | 2.27 | 2.52 |
| 20 | 100 | 7.90 | 8.00 | 1.00 | 1.99 | 1.01 | 1.00 | 1.00 | 2.39 | 3.13 | 2.17 | 2.13 | 4.06 | 2.07 | 1.73 | 1.94 | 2.53 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 2.98 | 1.03 | 1.00 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.06 | 2.98 | 3.00 | 3.00 | 3.04 |
| 30 | 30 | 7.98 | 7.97 | 1.01 | 2.64 | 1.10 | 1.00 | 1.00 | 2.89 | 8.00 | 3.56 | 2.68 | 8.00 | 3.59 | 2.40 | 2.60 | 2.64 |
| 30 | 60 | 7.97 | 8.00 | 1.03 | 2.85 | 1.34 | 1.01 | 1.00 | 3.01 | 7.08 | 2.98 | 2.97 | 7.88 | 2.94 | 2.64 | 2.82 | 3.04 |
| 30 | 100 | 7.96 | 8.00 | 1.00 | 2.36 | 1.08 | 1.00 | 1.00 | 2.89 | 3.25 | 2.40 | 2.77 | 3.39 | 2.18 | 2.38 | 2.60 | 2.95 |
| 30 | 200 | 7.85 | 8.00 | 1.08 | 3.00 | 2.52 | 1.08 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.02 | 2.99 | 3.00 | 3.00 | 3.05 |
| 100 | 30 | 8.00 | 6.54 | 1.17 | 3.00 | 2.00 | 1.11 | 1.00 | 3.26 | 6.36 | 2.98 | 3.20 | 6.54 | 2.94 | 2.58 | 2.71 | 3.02 |
| 100 | 60 | 8.00 | 8.00 | 1.80 | 3.01 | 2.86 | 1.84 | 1.07 | 3.00 | 7.10 | 2.99 | 3.00 | 7.37 | 2.97 | 2.94 | 2.98 | 3.06 |
| 100 | 100 | 8.00 | 8.00 | 2.76 | 3.00 | 3.00 | 2.83 | 1.94 | 3.00 | 7.98 | 3.00 | 3.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.04 |
| 100 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.95 | 3.00 | 3.05 | 3.00 | 3.00 | 3.14 | 3.00 | 3.00 | 3.00 | 3.04 |
| 200 | 30 | 8.00 | 5.93 | 1.13 | 2.98 | 2.15 | 1.14 | 1.00 | 3.40 | 4.74 | 2.94 | 3.30 | 4.62 | 2.88 | 2.63 | 2.75 | 3.06 |
| 200 | 60 | 8.00 | 8.00 | 2.77 | 3.01 | 3.00 | 2.87 | 2.00 | 3.02 | 4.47 | 3.00 | 3.01 | 4.50 | 3.00 | 3.00 | 3.00 | 3.06 |
| 200 | 100 | 8.00 | 8.00 | 2.99 | 3.00 | 3.00 | 2.99 | 2.90 | 3.00 | 4.90 | 3.00 | 3.00 | 5.19 | 3.00 | 3.00 | 3.00 | 3.04 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 7.48 | 3.00 | 3.00 | 7.90 | 3.00 | 3.00 | 3.00 | 3.03 |
| | $r_{mean}$ | 7.97 | 7.78 | 1.67 | 2.76 | 2.01 | 1.68 | 1.49 | 2.99 | 5.56 | 3.04 | 2.91 | 5.78 | 3.03 | 2.65 | 2.75 | 2.90 |
| | $RMSE$ | 4.97 | 4.81 | 1.58 | 0.43 | 1.31 | 1.58 | 1.69 | 0.20 | 3.24 | 0.52 | 0.28 | 3.44 | 0.61 | 0.53 | 0.40 | 0.26 |
| Part I.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.00 | 2.32 | 1.01 | 1.00 | 1.00 | 3.95 | 8.00 | 5.35 | 3.45 | 8.00 | 5.36 | 2.31 | 2.62 | 1.73 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 1.53 | 1.00 | 1.00 | 1.00 | 3.20 | 7.88 | 3.40 | 2.58 | 7.96 | 3.29 | 2.38 | 2.67 | 1.32 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 1.68 | 1.00 | 1.00 | 1.00 | 3.91 | 5.74 | 3.07 | 2.88 | 6.46 | 2.67 | 2.92 | 3.35 | 2.19 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 2.41 | 1.00 | 1.00 | 1.00 | 5.00 | 5.01 | 3.63 | 4.95 | 5.23 | 3.32 | 4.68 | 4.89 | 4.99 |
| 30 | 30 | 7.99 | 7.98 | 1.00 | 2.57 | 1.02 | 1.00 | 1.00 | 3.31 | 8.00 | 4.39 | 2.49 | 8.00 | 4.32 | 2.52 | 2.99 | 1.89 |
| 30 | 60 | 7.99 | 8.00 | 1.00 | 3.04 | 1.01 | 1.00 | 1.00 | 4.03 | 7.81 | 3.64 | 3.64 | 7.94 | 3.39 | 3.08 | 3.46 | 2.63 |
| 30 | 100 | 7.99 | 8.00 | 1.00 | 2.40 | 1.00 | 1.00 | 1.00 | 4.38 | 5.42 | 2.90 | 3.88 | 5.66 | 2.41 | 3.82 | 4.23 | 3.31 |
| 30 | 200 | 7.94 | 8.00 | 1.00 | 3.02 | 1.01 | 1.00 | 1.00 | 4.31 | 4.81 | 2.80 | 3.84 | 4.47 | 2.21 | 3.87 | 4.38 | 4.64 |
| 100 | 30 | 8.00 | 7.07 | 1.00 | 3.51 | 1.08 | 1.00 | 1.00 | 4.76 | 7.55 | 3.58 | 4.49 | 7.53 | 3.23 | 2.76 | 3.25 | 2.04 |
| 100 | 60 | 8.00 | 8.00 | 1.04 | 4.95 | 2.69 | 1.05 | 1.00 | 4.98 | 7.86 | 4.23 | 4.94 | 7.91 | 3.76 | 4.55 | 4.74 | 4.63 |
| 100 | 100 | 8.00 | 8.00 | 1.42 | 5.00 | 4.58 | 1.73 | 1.01 | 5.00 | 8.00 | 4.71 | 4.99 | 8.00 | 4.31 | 4.99 | 5.00 | 5.05 |
| 100 | 200 | 8.00 | 8.00 | 3.29 | 5.00 | 5.00 | 4.43 | 1.13 | 5.00 | 5.05 | 4.99 | 5.00 | 5.15 | 4.89 | 5.00 | 5.00 | 5.02 |
| 200 | 30 | 8.00 | 6.61 | 1.00 | 4.17 | 1.26 | 1.00 | 1.00 | 5.21 | 6.59 | 3.59 | 5.04 | 6.56 | 3.24 | 3.17 | 3.67 | 2.56 |
| 200 | 60 | 8.00 | 8.00 | 1.12 | 5.01 | 3.91 | 1.27 | 1.00 | 5.02 | 6.51 | 4.34 | 5.01 | 6.45 | 3.91 | 4.79 | 4.89 | 4.87 |
| 200 | 100 | 8.00 | 8.00 | 3.03 | 5.00 | 4.97 | 3.94 | 1.35 | 5.00 | 6.82 | 4.92 | 5.00 | 6.87 | 4.76 | 5.00 | 5.00 | 5.04 |
| 200 | 200 | 8.00 | 8.00 | 5.00 | 5.00 | 5.00 | 5.00 | 4.96 | 5.00 | 7.96 | 5.00 | 5.00 | 7.99 | 5.00 | 5.00 | 5.00 | 5.02 |
| | $r_{mean}$ | 7.99 | 7.85 | 1.56 | 3.54 | 2.28 | 1.71 | 1.28 | 4.50 | 6.81 | 4.03 | 4.20 | 6.89 | 3.75 | 3.80 | 4.07 | 3.56 |
| | $RMSE$ | 2.99 | 2.88 | 3.63 | 1.94 | 3.20 | 3.55 | 3.84 | 0.80 | 2.17 | 1.25 | 1.22 | 2.22 | 1.55 | 1.57 | 1.28 | 2.02 |

| N | T | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part II.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.00 | 1.60 | 1.01 | 1.00 | 1.00 | 2.11 | 8.00 | 4.70 | 4.29 | 8.00 | 5.91 | 1.01 | 1.03 | 1.81 |
| 20 | 60 | 7.94 | 8.00 | 1.00 | 1.44 | 1.00 | 1.00 | 1.00 | 1.89 | 7.42 | 2.24 | 3.50 | 8.00 | 3.70 | 1.00 | 1.00 | 1.82 |
| 20 | 100 | 7.95 | 8.00 | 1.00 | 1.68 | 1.00 | 1.00 | 1.00 | 2.00 | 2.40 | 1.99 | 2.27 | 7.74 | 2.21 | 1.00 | 1.04 | 2.05 |
| 20 | 200 | 7.91 | 8.00 | 1.00 | 1.61 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.94 | 4.19 | 7.78 | 2.06 | 1.00 | 1.00 | 2.04 |
| 30 | 30 | 7.99 | 7.97 | 1.12 | 2.28 | 1.34 | 1.04 | 1.00 | 2.33 | 8.00 | 3.71 | 2.34 | 8.00 | 4.38 | 1.13 | 1.28 | 2.14 |
| 30 | 60 | 7.99 | 8.00 | 1.24 | 2.33 | 1.75 | 1.11 | 1.00 | 2.59 | 7.89 | 2.57 | 2.73 | 8.00 | 2.89 | 1.11 | 1.33 | 2.45 |
| 30 | 100 | 7.98 | 8.00 | 1.05 | 2.36 | 1.69 | 1.03 | 1.00 | 2.69 | 4.08 | 2.22 | 2.82 | 7.97 | 2.36 | 1.01 | 1.10 | 2.73 |
| 30 | 200 | 7.97 | 8.00 | 1.21 | 2.48 | 1.99 | 1.23 | 1.00 | 2.82 | 3.02 | 2.01 | 2.28 | 5.38 | 2.00 | 1.03 | 1.27 | 2.95 |
| 100 | 30 | 8.00 | 7.73 | 1.00 | 3.55 | 1.08 | 1.00 | 1.00 | 5.91 | 8.00 | 3.68 | 5.52 | 8.00 | 3.37 | 1.00 | 1.01 | 1.66 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 5.90 | 1.58 | 1.01 | 1.00 | 6.17 | 8.00 | 3.79 | 5.84 | 8.00 | 3.39 | 1.00 | 1.00 | 1.64 |
| 100 | 100 | 8.00 | 8.00 | 1.02 | 6.93 | 2.85 | 1.05 | 1.00 | 6.59 | 8.00 | 4.23 | 6.33 | 8.00 | 4.00 | 1.00 | 1.00 | 1.75 |
| 100 | 200 | 8.00 | 8.00 | 1.10 | 7.74 | 5.09 | 1.47 | 1.00 | 7.63 | 8.00 | 4.80 | 7.54 | 8.00 | 4.59 | 1.00 | 1.00 | 2.10 |
| 200 | 30 | 8.00 | 7.04 | 1.00 | 2.03 | 1.01 | 1.00 | 1.00 | 5.14 | 7.74 | 2.23 | 4.41 | 7.64 | 1.88 | 1.00 | 1.00 | 1.34 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 4.72 | 1.03 | 1.00 | 1.00 | 6.16 | 8.00 | 1.85 | 5.53 | 8.00 | 1.53 | 1.00 | 1.00 | 1.20 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.73 | 1.22 | 1.00 | 1.00 | 7.59 | 8.00 | 1.96 | 7.60 | 8.00 | 1.73 | 1.00 | 1.00 | 1.09 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 3.54 | 1.01 | 1.00 | 8.00 | 8.00 | 2.54 | 8.00 | 8.00 | 2.32 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 7.98 | 7.92 | 1.05 | 3.90 | 1.76 | 1.06 | 1.00 | 4.48 | 6.66 | 2.90 | 4.70 | 7.78 | 3.02 | 1.02 | 1.07 | 1.86 |
| | $RMSE$ | 6.98 | 6.92 | 0.09 | 3.78 | 1.35 | 0.13 | 0.00 | 4.16 | 6.08 | 2.16 | 4.17 | 6.81 | 2.35 | 0.04 | 0.13 | 1.02 |
| Part II.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.10 | 3.99 | 1.42 | 1.01 | 1.00 | 4.59 | 8.00 | 5.34 | 5.11 | 8.00 | 5.83 | 2.53 | 2.93 | 3.07 |
| 20 | 60 | 7.96 | 8.00 | 1.37 | 4.04 | 2.12 | 1.14 | 1.00 | 4.15 | 7.93 | 4.00 | 4.80 | 8.00 | 4.18 | 2.88 | 3.33 | 3.77 |
| 20 | 100 | 7.97 | 8.00 | 1.03 | 3.59 | 1.64 | 1.01 | 1.00 | 4.03 | 5.38 | 3.87 | 4.64 | 7.87 | 3.96 | 3.08 | 3.56 | 3.94 |
| 20 | 200 | 7.95 | 8.00 | 1.01 | 3.79 | 1.74 | 1.01 | 1.00 | 4.00 | 4.02 | 3.47 | 4.35 | 6.11 | 3.38 | 3.49 | 3.87 | 4.01 |
| 30 | 30 | 8.00 | 7.99 | 1.18 | 4.42 | 1.98 | 1.04 | 1.00 | 4.54 | 8.00 | 4.90 | 4.53 | 8.00 | 4.99 | 3.28 | 3.68 | 3.55 |
| 30 | 60 | 8.00 | 8.00 | 1.13 | 4.36 | 2.68 | 1.04 | 1.00 | 4.61 | 7.99 | 4.18 | 4.65 | 8.00 | 4.17 | 3.16 | 3.65 | 4.05 |
| 30 | 100 | 8.00 | 8.00 | 1.15 | 4.27 | 2.81 | 1.08 | 1.00 | 4.70 | 6.58 | 4.03 | 4.75 | 7.97 | 4.00 | 3.60 | 3.98 | 4.37 |
| 30 | 200 | 7.99 | 8.00 | 1.97 | 4.19 | 3.92 | 2.03 | 1.01 | 4.78 | 5.04 | 4.00 | 4.71 | 7.29 | 3.99 | 4.00 | 4.06 | 4.80 |
| 100 | 30 | 8.00 | 7.92 | 1.31 | 5.48 | 2.34 | 1.21 | 1.01 | 7.41 | 8.00 | 4.68 | 7.36 | 8.00 | 4.49 | 2.52 | 2.97 | 2.43 |
| 100 | 60 | 8.00 | 8.00 | 2.55 | 7.51 | 3.45 | 2.56 | 1.41 | 7.74 | 8.00 | 4.84 | 7.71 | 8.00 | 4.63 | 2.71 | 3.00 | 2.61 |
| 100 | 100 | 8.00 | 8.00 | 2.94 | 7.98 | 4.84 | 3.00 | 2.19 | 7.94 | 8.00 | 5.41 | 7.84 | 8.00 | 4.93 | 3.01 | 3.23 | 2.59 |
| 100 | 200 | 8.00 | 8.00 | 3.08 | 8.00 | 7.01 | 3.37 | 3.00 | 8.00 | 8.00 | 5.90 | 8.00 | 8.00 | 5.78 | 3.01 | 3.05 | 2.94 |
| 200 | 30 | 8.00 | 7.44 | 1.17 | 4.04 | 2.24 | 1.18 | 1.01 | 6.89 | 7.96 | 3.42 | 6.53 | 7.96 | 3.21 | 2.36 | 2.59 | 2.65 |
| 200 | 60 | 8.00 | 8.00 | 2.67 | 6.60 | 3.03 | 2.80 | 1.79 | 7.49 | 8.00 | 3.27 | 7.34 | 8.00 | 3.13 | 2.88 | 2.93 | 3.05 |
| 200 | 100 | 8.00 | 8.00 | 3.00 | 7.98 | 3.23 | 3.00 | 2.99 | 7.97 | 8.00 | 3.30 | 7.97 | 8.00 | 3.20 | 3.00 | 3.00 | 3.04 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 5.19 | 3.00 | 3.00 | 8.00 | 8.00 | 3.54 | 8.00 | 8.00 | 3.29 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.85 | 5.52 | 3.10 | 1.84 | 1.53 | 6.05 | 7.31 | 4.26 | 6.14 | 7.83 | 4.20 | 3.03 | 3.30 | 3.37 |
| | $RMSE$ | 4.99 | 4.96 | 1.41 | 3.06 | 1.46 | 1.47 | 1.67 | 3.48 | 4.49 | 1.49 | 3.48 | 4.85 | 1.46 | 0.41 | 0.52 | 0.78 |
| Part II.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 8.00 | 8.00 | 1.00 | 4.41 | 1.04 | 1.00 | 1.00 | 6.23 | 8.00 | 6.07 | 6.53 | 8.00 | 6.26 | 3.33 | 3.72 | 2.09 |
| 20 | 60 | 7.98 | 8.00 | 1.00 | 4.47 | 1.01 | 1.00 | 1.00 | 6.18 | 8.00 | 5.17 | 6.71 | 8.00 | 5.17 | 2.62 | 3.34 | 2.86 |
| 20 | 100 | 7.99 | 8.00 | 1.00 | 4.91 | 1.11 | 1.00 | 1.00 | 6.15 | 7.56 | 5.10 | 6.71 | 7.94 | 5.33 | 4.78 | 5.34 | 4.75 |
| 20 | 200 | 7.98 | 8.00 | 1.00 | 5.16 | 1.00 | 1.00 | 1.00 | 5.68 | 6.00 | 4.91 | 5.70 | 6.85 | 4.85 | 4.98 | 5.13 | 5.42 |
| 30 | 30 | 8.00 | 8.00 | 1.02 | 5.06 | 1.20 | 1.00 | 1.00 | 5.80 | 8.00 | 5.75 | 5.75 | 8.00 | 5.82 | 3.25 | 3.89 | 2.40 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 5.85 | 1.20 | 1.00 | 1.00 | 6.55 | 8.00 | 5.56 | 6.71 | 8.00 | 5.50 | 4.18 | 4.91 | 3.05 |
| 30 | 100 | 8.00 | 8.00 | 1.01 | 5.63 | 1.69 | 1.00 | 1.00 | 6.75 | 7.83 | 4.80 | 6.81 | 8.00 | 4.43 | 3.79 | 4.88 | 3.34 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 6.11 | 1.85 | 1.01 | 1.00 | 6.38 | 6.93 | 5.40 | 6.50 | 7.25 | 5.02 | 5.95 | 6.06 | 6.09 |
| 100 | 30 | 8.00 | 7.99 | 1.00 | 6.52 | 1.27 | 1.00 | 1.00 | 7.88 | 8.00 | 5.17 | 7.82 | 8.00 | 4.80 | 3.20 | 3.80 | 1.04 |
| 100 | 60 | 8.00 | 8.00 | 1.18 | 7.95 | 4.24 | 1.21 | 1.00 | 7.99 | 8.00 | 5.65 | 7.97 | 8.00 | 5.35 | 3.54 | 4.15 | 0.82 |
| 100 | 100 | 8.00 | 8.00 | 1.59 | 8.00 | 5.82 | 1.95 | 1.02 | 8.00 | 8.00 | 5.90 | 8.00 | 8.00 | 5.58 | 3.14 | 3.70 | 0.58 |
| 100 | 200 | 8.00 | 8.00 | 4.61 | 8.00 | 7.89 | 5.05 | 2.57 | 8.00 | 8.00 | 6.55 | 8.00 | 8.00 | 6.26 | 3.95 | 4.47 | 0.94 |
| 200 | 30 | 8.00 | 7.77 | 1.00 | 5.36 | 1.34 | 1.00 | 1.00 | 7.70 | 8.00 | 4.24 | 7.57 | 8.00 | 3.84 | 2.97 | 3.41 | 1.58 |
| 200 | 60 | 8.00 | 8.00 | 1.20 | 7.61 | 4.42 | 1.43 | 1.00 | 7.92 | 8.00 | 4.82 | 7.89 | 8.00 | 4.59 | 3.93 | 4.33 | 2.00 |
| 200 | 100 | 8.00 | 8.00 | 4.10 | 8.00 | 5.18 | 4.62 | 2.21 | 8.00 | 8.00 | 5.02 | 8.00 | 8.00 | 4.99 | 4.75 | 4.86 | 4.05 |
| 200 | 200 | 8.00 | 8.00 | 5.00 | 8.00 | 6.99 | 5.00 | 5.00 | 8.00 | 8.00 | 5.06 | 8.00 | 8.00 | 5.03 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 8.00 | 7.99 | 1.73 | 6.32 | 2.95 | 1.83 | 1.43 | 7.08 | 7.77 | 5.32 | 7.17 | 7.88 | 5.18 | 3.96 | 4.44 | 2.88 |
| | $RMSE$ | 3.00 | 2.99 | 3.55 | 1.89 | 3.10 | 3.50 | 3.72 | 2.26 | 2.82 | 0.64 | 2.31 | 2.90 | 0.64 | 1.37 | 0.94 | 2.72 |

Part III.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.94 | 8.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.00 | 1.17 | 8.00 | 4.15 | 1.25 | 8.00 | 4.59 | 1.02 | 1.03 | 1.25 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.02 | 5.07 | 1.33 | 1.20 | 7.70 | 2.11 | 1.00 | 1.00 | 1.14 |
| 20 | 100 | 7.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 | 1.00 | 1.02 | 3.14 | 1.09 | 1.00 | 1.00 | 1.11 |
| 20 | 200 | 7.89 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.14 | 1.00 | 1.00 | 1.00 | 1.06 |
| 30 | 30 | 7.97 | 7.96 | 1.00 | 1.06 | 1.00 | 1.00 | 1.00 | 1.05 | 8.00 | 2.41 | 1.03 | 8.00 | 2.46 | 1.01 | 1.01 | 1.24 |
| 30 | 60 | 7.94 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 5.12 | 1.03 | 1.01 | 6.05 | 1.03 | 1.00 | 1.00 | 1.14 |
| 30 | 100 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.13 | 1.00 | 1.00 | 1.16 | 1.00 | 1.00 | 1.00 | 1.12 |
| 30 | 200 | 7.72 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.07 |
| 100 | 30 | 8.00 | 5.76 | 1.00 | 1.03 | 1.00 | 1.00 | 1.00 | 1.20 | 3.83 | 1.06 | 1.09 | 3.80 | 1.02 | 1.00 | 1.00 | 1.26 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 5.29 | 1.00 | 1.01 | 6.04 | 1.00 | 1.00 | 1.00 | 1.15 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 7.74 | 1.00 | 1.00 | 7.98 | 1.00 | 1.00 | 1.00 | 1.10 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.23 | 1.00 | 1.00 | 1.00 | 1.09 |
| 200 | 30 | 8.00 | 5.14 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.30 | 2.39 | 1.00 | 1.14 | 2.21 | 1.00 | 1.00 | 1.00 | 1.22 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 2.29 | 1.00 | 1.01 | 2.28 | 1.00 | 1.00 | 1.00 | 1.12 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.77 | 1.00 | 1.00 | 3.03 | 1.00 | 1.00 | 1.00 | 1.09 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 6.13 | 1.00 | 1.00 | 7.59 | 1.00 | 1.00 | 1.00 | 1.05 |
| | $r_{mean}$ | 7.95 | 7.68 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.05 | 3.87 | 1.31 | 1.05 | 4.40 | 1.39 | 1.00 | 1.00 | 1.14 |
| | $RMSE$ | 6.95 | 6.73 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.10 | 3.84 | 0.87 | 0.09 | 4.37 | 1.01 | 0.01 | 0.01 | 0.15 |

Part III.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.96 | 8.00 | 1.02 | 1.88 | 1.07 | 1.00 | 1.00 | 2.50 | 8.00 | 4.62 | 2.43 | 8.00 | 4.83 | 1.63 | 1.82 | 2.04 |
| 20 | 60 | 7.95 | 8.00 | 1.02 | 2.04 | 1.14 | 1.00 | 1.00 | 2.74 | 7.43 | 2.88 | 2.79 | 7.85 | 3.00 | 1.82 | 2.02 | 2.57 |
| 20 | 100 | 7.95 | 8.00 | 1.00 | 1.34 | 1.00 | 1.00 | 1.00 | 2.37 | 3.07 | 2.17 | 2.02 | 3.63 | 1.97 | 1.70 | 1.87 | 2.40 |
| 20 | 200 | 7.93 | 8.00 | 1.00 | 1.79 | 1.00 | 1.00 | 1.00 | 2.97 | 3.00 | 2.38 | 2.91 | 2.98 | 2.22 | 2.52 | 2.75 | 3.02 |
| 30 | 30 | 7.96 | 7.94 | 1.00 | 1.49 | 1.01 | 1.00 | 1.00 | 1.71 | 8.00 | 3.06 | 1.44 | 8.00 | 2.92 | 1.50 | 1.65 | 1.64 |
| 30 | 60 | 7.96 | 8.00 | 1.03 | 2.46 | 1.20 | 1.01 | 1.00 | 2.83 | 7.08 | 2.79 | 2.54 | 7.69 | 2.54 | 2.15 | 2.36 | 2.84 |
| 30 | 100 | 7.95 | 8.00 | 1.00 | 1.93 | 1.05 | 1.00 | 1.00 | 2.63 | 3.16 | 2.09 | 2.48 | 3.21 | 1.89 | 2.10 | 2.29 | 2.76 |
| 30 | 200 | 7.84 | 8.00 | 1.01 | 2.86 | 1.50 | 1.02 | 1.00 | 3.00 | 3.00 | 2.69 | 2.97 | 3.00 | 2.21 | 2.87 | 2.95 | 3.04 |
| 100 | 30 | 8.00 | 6.17 | 1.04 | 2.39 | 1.32 | 1.03 | 1.00 | 2.86 | 5.50 | 2.49 | 2.68 | 5.61 | 2.28 | 1.69 | 1.90 | 2.42 |
| 100 | 60 | 8.00 | 8.00 | 1.36 | 2.91 | 2.11 | 1.37 | 1.07 | 2.95 | 6.74 | 2.62 | 2.88 | 7.08 | 2.40 | 2.21 | 2.41 | 2.97 |
| 100 | 100 | 8.00 | 8.00 | 2.26 | 3.00 | 2.92 | 2.39 | 1.60 | 3.00 | 7.95 | 2.96 | 3.00 | 7.99 | 2.92 | 2.91 | 2.97 | 3.06 |
| 100 | 200 | 8.00 | 8.00 | 2.85 | 3.00 | 3.00 | 2.94 | 2.51 | 3.00 | 3.04 | 3.00 | 3.00 | 3.13 | 2.99 | 3.00 | 3.00 | 3.04 |
| 200 | 30 | 8.00 | 5.49 | 1.04 | 2.39 | 1.31 | 1.04 | 1.00 | 2.99 | 3.90 | 2.30 | 2.78 | 3.78 | 2.00 | 1.73 | 1.90 | 2.52 |
| 200 | 60 | 8.00 | 8.00 | 1.62 | 3.00 | 2.56 | 1.74 | 1.25 | 3.01 | 4.05 | 2.77 | 3.00 | 4.07 | 2.56 | 2.46 | 2.65 | 3.05 |
| 200 | 100 | 8.00 | 8.00 | 2.73 | 3.00 | 2.99 | 2.85 | 2.35 | 3.00 | 4.60 | 2.98 | 3.00 | 4.92 | 2.96 | 2.96 | 2.98 | 3.03 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 7.41 | 3.00 | 3.00 | 7.89 | 3.00 | 3.00 | 3.00 | 3.03 |
| | $r_{mean}$ | 7.97 | 7.73 | 1.50 | 2.41 | 1.76 | 1.52 | 1.36 | 2.79 | 5.37 | 2.80 | 2.68 | 5.55 | 2.67 | 2.27 | 2.41 | 2.71 |
| | $RMSE$ | 4.97 | 4.78 | 1.67 | 0.82 | 1.48 | 1.66 | 1.76 | 0.40 | 3.11 | 0.59 | 0.53 | 3.29 | 0.76 | 0.91 | 0.76 | 0.50 |

Part III.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.96 | 7.99 | 1.00 | 1.49 | 1.01 | 1.00 | 1.00 | 2.25 | 8.00 | 4.75 | 1.73 | 8.00 | 4.70 | 1.75 | 1.95 | 1.49 |
| 20 | 60 | 7.96 | 8.00 | 1.00 | 1.77 | 1.01 | 1.00 | 1.00 | 2.93 | 7.76 | 3.22 | 2.56 | 7.92 | 3.10 | 1.81 | 2.04 | 2.01 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.80 | 1.00 | 1.00 | 1.00 | 3.92 | 5.59 | 3.13 | 3.65 | 6.43 | 2.98 | 2.65 | 3.06 | 2.58 |
| 20 | 200 | 7.96 | 8.00 | 1.00 | 1.85 | 1.00 | 1.00 | 1.00 | 3.56 | 3.85 | 2.63 | 3.36 | 3.77 | 2.44 | 2.79 | 3.11 | 3.51 |
| 30 | 30 | 7.97 | 7.96 | 1.00 | 1.91 | 1.03 | 1.00 | 1.00 | 2.35 | 8.00 | 3.77 | 1.82 | 8.00 | 3.63 | 1.83 | 2.05 | 1.79 |
| 30 | 60 | 7.98 | 8.00 | 1.00 | 2.17 | 1.03 | 1.00 | 1.00 | 3.11 | 7.66 | 2.94 | 2.65 | 7.83 | 2.66 | 2.07 | 2.34 | 2.18 |
| 30 | 100 | 7.99 | 8.00 | 1.00 | 2.43 | 1.04 | 1.00 | 1.00 | 3.70 | 5.06 | 2.69 | 3.18 | 5.46 | 2.31 | 2.34 | 2.72 | 2.85 |
| 30 | 200 | 7.95 | 8.00 | 1.00 | 3.18 | 1.08 | 1.00 | 1.00 | 4.32 | 4.74 | 2.79 | 4.27 | 4.77 | 2.42 | 3.58 | 4.05 | 4.68 |
| 100 | 30 | 8.00 | 6.45 | 1.00 | 2.43 | 1.13 | 1.00 | 1.00 | 3.33 | 6.47 | 2.61 | 2.96 | 6.39 | 2.31 | 1.81 | 2.04 | 2.05 |
| 100 | 60 | 8.00 | 8.00 | 1.19 | 4.07 | 2.09 | 1.19 | 1.02 | 4.24 | 7.62 | 3.11 | 4.02 | 7.74 | 2.81 | 2.27 | 2.72 | 3.44 |
| 100 | 100 | 8.00 | 8.00 | 1.50 | 4.79 | 3.18 | 1.67 | 1.09 | 4.73 | 7.99 | 3.59 | 4.64 | 8.00 | 3.28 | 3.80 | 4.25 | 4.85 |
| 100 | 200 | 8.00 | 8.00 | 2.81 | 5.00 | 4.81 | 3.41 | 1.70 | 5.00 | 5.04 | 4.58 | 5.00 | 5.15 | 4.18 | 4.99 | 5.00 | 5.03 |
| 200 | 30 | 8.00 | 5.85 | 1.00 | 2.63 | 1.18 | 1.00 | 1.00 | 3.71 | 4.97 | 2.44 | 3.37 | 4.74 | 2.09 | 1.78 | 2.04 | 2.36 |
| 200 | 60 | 8.00 | 8.00 | 1.27 | 4.35 | 2.49 | 1.38 | 1.04 | 4.58 | 5.79 | 3.04 | 4.47 | 5.81 | 2.73 | 2.42 | 2.91 | 3.58 |
| 200 | 100 | 8.00 | 8.00 | 2.09 | 4.97 | 4.03 | 2.52 | 1.40 | 4.97 | 6.40 | 3.82 | 4.94 | 6.55 | 3.48 | 4.27 | 4.61 | 4.96 |
| 200 | 200 | 8.00 | 8.00 | 4.33 | 5.00 | 5.00 | 4.74 | 3.75 | 5.00 | 7.92 | 4.91 | 5.00 | 7.97 | 4.80 | 5.00 | 5.00 | 5.02 |
| | $r_{mean}$ | 7.98 | 7.77 | 1.45 | 3.12 | 2.01 | 1.56 | 1.25 | 3.86 | 6.43 | 3.38 | 3.60 | 6.53 | 3.12 | 2.82 | 3.12 | 3.27 |
| | $RMSE$ | 2.98 | 2.83 | 3.66 | 2.29 | 3.31 | 3.60 | 3.81 | 1.44 | 1.99 | 1.80 | 1.75 | 2.07 | 2.05 | 2.44 | 2.17 | 2.13 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part IV.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.00 | 1.50 | 1.01 | 1.00 | 1.00 | 2.00 | 8.00 | 4.62 | 2.11 | 8.00 | 5.21 | 1.07 | 1.12 | 1.74 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.62 | 1.00 | 1.00 | 1.00 | 1.96 | 7.30 | 2.24 | 5.06 | 8.00 | 4.37 | 1.00 | 1.03 | 1.93 |
| 20 | 100 | 7.92 | 8.00 | 1.00 | 1.59 | 1.00 | 1.00 | 1.00 | 1.99 | 2.47 | 1.99 | 6.69 | 8.00 | 3.72 | 1.00 | 1.00 | 2.04 |
| 20 | 200 | 7.91 | 8.00 | 1.00 | 1.61 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.95 | 2.00 | 2.27 | 1.96 | 1.18 | 1.43 | 2.04 |
| 30 | 30 | 7.98 | 7.97 | 1.10 | 2.30 | 1.38 | 1.02 | 1.00 | 2.38 | 8.00 | 3.73 | 2.43 | 8.00 | 4.22 | 1.32 | 1.51 | 2.22 |
| 30 | 60 | 7.99 | 8.00 | 1.26 | 2.19 | 1.77 | 1.11 | 1.01 | 2.39 | 7.87 | 2.40 | 2.41 | 8.00 | 2.54 | 1.32 | 1.54 | 2.36 |
| 30 | 100 | 7.98 | 8.00 | 1.26 | 2.36 | 1.91 | 1.17 | 1.00 | 2.69 | 4.10 | 2.21 | 2.64 | 7.49 | 2.25 | 1.32 | 1.59 | 2.72 |
| 30 | 200 | 7.95 | 8.00 | 1.41 | 2.50 | 2.00 | 1.44 | 1.00 | 2.85 | 3.02 | 2.01 | 2.18 | 3.49 | 2.00 | 1.25 | 1.61 | 2.99 |
| 100 | 30 | 8.00 | 7.69 | 1.00 | 3.46 | 1.06 | 1.00 | 1.00 | 5.81 | 7.99 | 3.66 | 5.62 | 7.99 | 3.45 | 1.05 | 1.08 | 1.67 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 5.82 | 1.53 | 1.01 | 1.00 | 6.17 | 8.00 | 3.78 | 5.87 | 8.00 | 3.37 | 1.00 | 1.00 | 1.62 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 6.98 | 2.83 | 1.05 | 1.00 | 6.60 | 8.00 | 4.19 | 6.43 | 8.00 | 3.87 | 1.00 | 1.00 | 1.77 |
| 100 | 200 | 8.00 | 8.00 | 1.07 | 7.76 | 5.12 | 1.39 | 1.00 | 7.67 | 8.00 | 4.84 | 7.44 | 8.00 | 4.26 | 1.00 | 1.00 | 2.13 |
| 200 | 30 | 8.00 | 6.98 | 1.00 | 1.90 | 1.00 | 1.00 | 1.00 | 5.04 | 7.75 | 2.14 | 4.34 | 7.64 | 1.81 | 1.01 | 1.02 | 1.34 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 4.66 | 1.03 | 1.00 | 1.00 | 6.17 | 8.00 | 1.86 | 5.46 | 8.00 | 1.51 | 1.00 | 1.00 | 1.17 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.73 | 1.23 | 1.00 | 1.00 | 7.65 | 8.00 | 1.96 | 7.41 | 8.00 | 1.59 | 1.00 | 1.00 | 1.11 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 3.49 | 1.00 | 1.00 | 8.00 | 8.00 | 2.52 | 8.00 | 8.00 | 2.35 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 7.98 | 7.92 | 1.07 | 3.87 | 1.77 | 1.07 | 1.00 | 4.46 | 6.66 | 2.88 | 4.76 | 7.31 | 3.03 | 1.10 | 1.18 | 1.87 |
| | $RMSE$ | 6.98 | 6.92 | 0.14 | 3.77 | 1.36 | 0.16 | 0.00 | 4.15 | 6.07 | 2.14 | 4.30 | 6.53 | 2.32 | 0.16 | 0.30 | 1.02 |
| Part IV.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.02 | 2.72 | 1.10 | 1.00 | 1.00 | 3.87 | 8.00 | 5.20 | 3.94 | 8.00 | 5.60 | 1.95 | 2.30 | 2.31 |
| 20 | 60 | 7.97 | 8.00 | 1.10 | 3.40 | 1.38 | 1.04 | 1.00 | 4.05 | 7.92 | 3.94 | 4.37 | 8.00 | 4.03 | 2.24 | 2.73 | 3.38 |
| 20 | 100 | 7.96 | 8.00 | 1.16 | 3.39 | 1.68 | 1.08 | 1.00 | 3.91 | 5.35 | 3.53 | 4.52 | 7.92 | 3.38 | 2.32 | 2.79 | 3.73 |
| 20 | 200 | 7.94 | 8.00 | 1.08 | 4.04 | 2.23 | 1.06 | 1.00 | 4.00 | 4.01 | 3.40 | 4.01 | 4.95 | 3.06 | 3.37 | 3.62 | 4.01 |
| 30 | 30 | 8.00 | 7.99 | 1.10 | 3.72 | 1.45 | 1.03 | 1.00 | 4.07 | 8.00 | 4.67 | 4.03 | 8.00 | 4.92 | 2.12 | 2.58 | 2.92 |
| 30 | 60 | 7.99 | 8.00 | 1.20 | 4.14 | 1.94 | 1.09 | 1.00 | 4.50 | 7.97 | 4.08 | 4.58 | 8.00 | 4.07 | 2.42 | 2.98 | 3.61 |
| 30 | 100 | 8.00 | 8.00 | 1.32 | 4.30 | 2.59 | 1.22 | 1.01 | 4.75 | 6.42 | 4.01 | 4.78 | 7.88 | 3.96 | 2.78 | 3.49 | 4.36 |
| 30 | 200 | 7.99 | 8.00 | 2.06 | 4.30 | 2.98 | 2.09 | 1.08 | 4.93 | 5.04 | 3.54 | 4.90 | 6.05 | 3.70 | 3.06 | 3.49 | 4.88 |
| 100 | 30 | 8.00 | 7.88 | 1.11 | 5.01 | 1.62 | 1.07 | 1.01 | 7.21 | 8.00 | 4.43 | 7.08 | 8.00 | 4.16 | 1.74 | 2.17 | 2.01 |
| 100 | 60 | 8.00 | 8.00 | 1.84 | 7.41 | 3.12 | 1.86 | 1.22 | 7.68 | 8.00 | 4.72 | 7.55 | 8.00 | 4.42 | 1.90 | 2.26 | 2.13 |
| 100 | 100 | 8.00 | 8.00 | 2.59 | 7.98 | 4.62 | 2.71 | 1.91 | 7.95 | 8.00 | 5.33 | 7.89 | 8.00 | 5.02 | 2.16 | 2.48 | 2.31 |
| 100 | 200 | 8.00 | 8.00 | 3.02 | 8.00 | 6.98 | 3.30 | 2.87 | 8.00 | 8.00 | 6.01 | 8.00 | 8.00 | 5.88 | 2.50 | 2.69 | 2.83 |
| 200 | 30 | 8.00 | 7.27 | 1.05 | 3.33 | 1.44 | 1.06 | 1.00 | 6.43 | 7.93 | 3.01 | 5.87 | 7.90 | 2.73 | 1.54 | 1.76 | 1.96 |
| 200 | 60 | 8.00 | 8.00 | 1.68 | 6.29 | 2.65 | 1.82 | 1.25 | 7.25 | 8.00 | 3.12 | 6.97 | 8.00 | 2.96 | 1.95 | 2.13 | 2.46 |
| 200 | 100 | 8.00 | 8.00 | 2.74 | 7.96 | 3.20 | 2.86 | 2.36 | 7.95 | 8.00 | 3.28 | 7.91 | 8.00 | 3.15 | 2.52 | 2.66 | 2.94 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 5.42 | 3.01 | 3.00 | 8.00 | 8.00 | 3.57 | 8.00 | 8.00 | 3.40 | 2.96 | 2.98 | 3.02 |
| | $r_{mean}$ | 7.99 | 7.95 | 1.69 | 5.25 | 2.78 | 1.71 | 1.42 | 5.91 | 7.29 | 4.12 | 5.90 | 7.67 | 4.03 | 2.35 | 2.69 | 3.05 |
| | $RMSE$ | 4.99 | 4.95 | 1.50 | 2.96 | 1.60 | 1.52 | 1.72 | 3.38 | 4.48 | 1.40 | 3.31 | 4.74 | 1.37 | 0.82 | 0.59 | 0.85 |
| Part IV.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.01 | 2.87 | 1.05 | 1.00 | 1.00 | 4.35 | 8.00 | 5.49 | 3.51 | 8.00 | 5.62 | 1.95 | 2.28 | 1.97 |
| 20 | 60 | 7.97 | 8.00 | 1.03 | 4.19 | 1.21 | 1.01 | 1.00 | 5.21 | 7.98 | 4.52 | 5.69 | 8.00 | 4.57 | 2.06 | 2.55 | 2.65 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 3.62 | 1.10 | 1.00 | 1.00 | 5.76 | 7.42 | 4.40 | 6.28 | 7.96 | 4.28 | 2.76 | 3.42 | 3.48 |
| 20 | 200 | 7.99 | 8.00 | 1.00 | 4.38 | 1.05 | 1.00 | 1.00 | 5.98 | 6.13 | 4.35 | 6.44 | 7.46 | 4.30 | 4.32 | 5.13 | 5.49 |
| 30 | 30 | 8.00 | 7.99 | 1.02 | 3.35 | 1.17 | 1.01 | 1.00 | 4.01 | 8.00 | 4.89 | 3.40 | 8.00 | 4.85 | 2.12 | 2.53 | 2.13 |
| 30 | 60 | 8.00 | 8.00 | 1.06 | 4.85 | 1.62 | 1.02 | 1.00 | 5.78 | 8.00 | 4.80 | 5.81 | 8.00 | 4.74 | 2.71 | 3.39 | 2.71 |
| 30 | 100 | 8.00 | 8.00 | 1.07 | 5.10 | 1.86 | 1.05 | 1.00 | 6.34 | 7.77 | 4.58 | 6.52 | 8.00 | 4.36 | 3.00 | 3.95 | 3.28 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 6.11 | 1.58 | 1.00 | 1.00 | 6.70 | 7.04 | 4.93 | 6.57 | 7.42 | 4.49 | 5.39 | 6.11 | 5.66 |
| 100 | 30 | 8.00 | 7.92 | 1.02 | 4.69 | 1.28 | 1.02 | 1.00 | 7.33 | 8.00 | 4.29 | 6.97 | 8.00 | 3.90 | 1.84 | 2.21 | 1.48 |
| 100 | 60 | 8.00 | 8.00 | 1.24 | 7.73 | 2.59 | 1.24 | 1.02 | 7.91 | 8.00 | 4.86 | 7.81 | 8.00 | 4.47 | 2.02 | 2.36 | 1.24 |
| 100 | 100 | 8.00 | 8.00 | 2.05 | 8.00 | 5.29 | 2.29 | 1.27 | 8.00 | 8.00 | 5.68 | 8.00 | 8.00 | 5.40 | 2.12 | 2.53 | 1.23 |
| 100 | 200 | 8.00 | 8.00 | 2.93 | 8.00 | 7.79 | 3.79 | 1.70 | 8.00 | 8.00 | 6.34 | 8.00 | 8.00 | 5.95 | 2.53 | 2.90 | 0.72 |
| 200 | 30 | 8.00 | 7.36 | 1.01 | 3.37 | 1.22 | 1.01 | 1.00 | 6.70 | 7.96 | 3.10 | 6.16 | 7.93 | 2.73 | 1.65 | 1.87 | 1.59 |
| 200 | 60 | 8.00 | 8.00 | 1.39 | 7.21 | 3.05 | 1.52 | 1.09 | 7.76 | 8.00 | 3.87 | 7.65 | 8.00 | 3.58 | 1.96 | 2.28 | 1.92 |
| 200 | 100 | 8.00 | 8.00 | 2.77 | 8.00 | 4.83 | 3.26 | 1.86 | 8.00 | 8.00 | 4.71 | 8.00 | 8.00 | 4.49 | 2.57 | 3.01 | 2.37 |
| 200 | 200 | 8.00 | 8.00 | 4.65 | 8.00 | 6.85 | 4.91 | 4.20 | 8.00 | 8.00 | 5.04 | 8.00 | 8.00 | 5.00 | 4.16 | 4.51 | 4.35 |
| | $r_{mean}$ | 7.99 | 7.95 | 1.58 | 5.59 | 2.72 | 1.70 | 1.32 | 6.61 | 7.77 | 4.74 | 6.55 | 7.92 | 4.55 | 2.70 | 3.19 | 2.64 |
| | $RMSE$ | 2.99 | 2.96 | 3.57 | 1.98 | 3.14 | 3.51 | 3.76 | 2.07 | 2.81 | 0.76 | 2.10 | 2.93 | 0.88 | 2.52 | 2.14 | 2.76 |

| N | T | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part V.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.20 | 8.00 | 4.16 | 2.92 | 8.00 | 6.01 | 1.00 | 1.00 | 1.26 |
| 20 | 60 | 7.93 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.04 | 5.12 | 1.31 | 1.05 | 7.49 | 1.68 | 1.00 | 1.00 | 1.10 |
| 20 | 100 | 7.91 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.10 | 1.01 | 1.01 | 1.33 | 1.01 | 1.00 | 1.00 | 1.12 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.06 |
| 30 | 30 | 7.97 | 7.95 | 1.00 | 1.07 | 1.00 | 1.00 | 1.00 | 1.06 | 8.00 | 2.40 | 1.04 | 8.00 | 2.70 | 1.00 | 1.00 | 1.23 |
| 30 | 60 | 7.94 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 5.17 | 1.03 | 1.02 | 7.35 | 1.09 | 1.00 | 1.00 | 1.16 |
| 30 | 100 | 7.89 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.15 | 1.00 | 1.00 | 1.47 | 1.00 | 1.00 | 1.00 | 1.14 |
| 30 | 200 | 7.65 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.09 |
| 100 | 30 | 8.00 | 5.94 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.22 | 4.34 | 1.10 | 1.14 | 4.55 | 1.05 | 1.00 | 1.00 | 1.26 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 5.50 | 1.00 | 1.00 | 6.49 | 1.00 | 1.00 | 1.00 | 1.15 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 7.80 | 1.00 | 1.00 | 7.99 | 1.00 | 1.00 | 1.00 | 1.11 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.19 | 1.00 | 1.00 | 1.00 | 1.08 |
| 200 | 30 | 8.00 | 5.24 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.31 | 2.52 | 1.00 | 1.16 | 2.31 | 1.00 | 1.00 | 1.00 | 1.18 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 2.40 | 1.00 | 1.01 | 2.30 | 1.00 | 1.00 | 1.00 | 1.11 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.82 | 1.00 | 1.00 | 3.19 | 1.00 | 1.00 | 1.00 | 1.09 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 6.20 | 1.00 | 1.00 | 7.67 | 1.00 | 1.00 | 1.00 | 1.05 |
| | $r_{mean}$ | 7.95 | 7.70 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.05 | 3.95 | 1.31 | 1.15 | 4.46 | 1.47 | 1.00 | 1.00 | 1.14 |
| | $RMSE$ | 6.95 | 6.74 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.11 | 3.91 | 0.87 | 0.48 | 4.51 | 1.33 | 0.00 | 0.00 | 0.15 |
| Part V.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 8.00 | 1.01 | 2.32 | 1.09 | 1.00 | 1.00 | 3.08 | 8.00 | 4.74 | 3.16 | 8.00 | 5.06 | 2.00 | 2.25 | 2.32 |
| 20 | 60 | 7.96 | 8.00 | 1.00 | 2.52 | 1.07 | 1.00 | 1.00 | 3.04 | 7.45 | 3.07 | 3.02 | 7.95 | 3.16 | 2.48 | 2.69 | 2.89 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 2.84 | 1.12 | 1.00 | 1.00 | 3.01 | 3.39 | 3.00 | 3.04 | 5.59 | 3.00 | 2.89 | 2.95 | 3.04 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 2.53 | 1.01 | 1.00 | 1.00 | 3.00 | 3.00 | 2.85 | 3.00 | 3.02 | 2.73 | 2.96 | 2.99 | 3.02 |
| 30 | 30 | 7.99 | 7.97 | 1.00 | 2.27 | 1.04 | 1.00 | 1.00 | 2.69 | 8.00 | 3.56 | 2.11 | 8.00 | 3.53 | 2.13 | 2.36 | 2.24 |
| 30 | 60 | 7.97 | 8.00 | 1.03 | 2.95 | 1.54 | 1.01 | 1.00 | 3.02 | 7.16 | 3.00 | 3.03 | 7.82 | 3.00 | 2.86 | 2.93 | 3.04 |
| 30 | 100 | 7.95 | 8.00 | 1.10 | 2.99 | 2.04 | 1.06 | 1.00 | 3.00 | 3.24 | 3.00 | 3.01 | 3.69 | 2.98 | 2.98 | 3.00 | 3.05 |
| 30 | 200 | 7.86 | 8.00 | 1.01 | 3.00 | 2.13 | 1.01 | 1.00 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 2.92 | 3.00 | 3.00 | 3.05 |
| 100 | 30 | 8.00 | 6.54 | 1.09 | 2.94 | 1.73 | 1.06 | 1.00 | 3.23 | 6.37 | 2.94 | 3.15 | 6.53 | 2.87 | 2.39 | 2.59 | 2.96 |
| 100 | 60 | 8.00 | 8.00 | 1.97 | 3.01 | 2.92 | 2.00 | 1.09 | 3.01 | 7.15 | 3.00 | 3.00 | 7.49 | 2.99 | 2.97 | 2.98 | 3.08 |
| 100 | 100 | 8.00 | 8.00 | 2.97 | 3.00 | 3.00 | 2.98 | 2.48 | 3.00 | 7.98 | 3.00 | 3.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.05 |
| 100 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.05 | 3.00 | 3.00 | 3.14 | 3.00 | 3.00 | 3.00 | 3.04 |
| 200 | 30 | 8.00 | 5.91 | 1.11 | 2.99 | 2.08 | 1.11 | 1.01 | 3.35 | 4.60 | 2.93 | 3.25 | 4.50 | 2.83 | 2.59 | 2.73 | 3.01 |
| 200 | 60 | 8.00 | 8.00 | 2.70 | 3.01 | 2.99 | 2.80 | 1.95 | 3.02 | 4.46 | 3.00 | 3.01 | 4.46 | 3.00 | 2.99 | 2.99 | 3.05 |
| 200 | 100 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.99 | 3.00 | 4.92 | 3.00 | 3.00 | 5.27 | 3.00 | 3.00 | 3.00 | 3.04 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 7.51 | 3.00 | 3.00 | 7.91 | 3.00 | 3.00 | 3.00 | 3.02 |
| | $r_{mean}$ | 7.97 | 7.78 | 1.69 | 2.84 | 2.05 | 1.69 | 1.53 | 3.03 | 5.58 | 3.13 | 2.99 | 5.90 | 3.13 | 2.77 | 2.84 | 2.93 |
| | $RMSE$ | 4.97 | 4.81 | 1.58 | 0.30 | 1.25 | 1.58 | 1.68 | 0.13 | 3.25 | 0.46 | 0.24 | 3.50 | 0.54 | 0.40 | 0.29 | 0.26 |
| Part V.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.00 | 1.91 | 1.00 | 1.00 | 1.00 | 3.32 | 8.00 | 5.13 | 2.69 | 8.00 | 5.13 | 2.48 | 2.77 | 1.60 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 1.62 | 1.00 | 1.00 | 1.00 | 3.99 | 7.91 | 3.87 | 3.35 | 7.98 | 3.71 | 2.88 | 3.26 | 1.58 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.48 | 1.00 | 1.00 | 1.00 | 4.08 | 5.83 | 3.16 | 3.53 | 6.94 | 2.88 | 2.71 | 3.18 | 2.19 |
| 20 | 200 | 7.95 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 3.16 | 3.89 | 1.65 | 2.25 | 3.59 | 1.23 | 3.02 | 3.36 | 2.36 |
| 30 | 30 | 7.99 | 7.97 | 1.00 | 2.15 | 1.01 | 1.00 | 1.00 | 2.90 | 8.00 | 4.33 | 2.25 | 8.00 | 4.24 | 2.58 | 2.98 | 1.54 |
| 30 | 60 | 7.99 | 8.00 | 1.00 | 3.54 | 1.08 | 1.00 | 1.00 | 4.65 | 7.79 | 4.04 | 4.50 | 7.96 | 3.89 | 3.58 | 4.04 | 3.30 |
| 30 | 100 | 7.99 | 8.00 | 1.00 | 4.41 | 1.02 | 1.00 | 1.00 | 4.98 | 5.41 | 4.15 | 4.97 | 5.96 | 3.88 | 4.84 | 4.92 | 4.85 |
| 30 | 200 | 7.96 | 8.00 | 1.00 | 3.25 | 1.00 | 1.00 | 1.00 | 4.74 | 4.97 | 2.85 | 4.50 | 4.93 | 2.35 | 4.75 | 4.85 | 4.89 |
| 100 | 30 | 8.00 | 7.09 | 1.00 | 3.46 | 1.07 | 1.00 | 1.00 | 4.75 | 7.53 | 3.54 | 4.41 | 7.53 | 3.14 | 2.78 | 3.25 | 1.92 |
| 100 | 60 | 8.00 | 8.00 | 1.03 | 4.89 | 2.45 | 1.03 | 1.00 | 4.96 | 7.85 | 4.00 | 4.93 | 7.87 | 3.69 | 4.33 | 4.61 | 4.42 |
| 100 | 100 | 8.00 | 8.00 | 1.56 | 5.00 | 4.47 | 1.83 | 1.01 | 5.00 | 8.00 | 4.61 | 5.00 | 8.00 | 4.36 | 4.97 | 4.98 | 5.03 |
| 100 | 200 | 8.00 | 8.00 | 2.50 | 5.00 | 4.99 | 3.65 | 1.14 | 5.00 | 5.05 | 4.93 | 5.00 | 5.09 | 4.78 | 5.00 | 5.00 | 5.03 |
| 200 | 30 | 8.00 | 6.57 | 1.00 | 4.08 | 1.25 | 1.00 | 1.00 | 5.22 | 6.54 | 3.57 | 5.04 | 6.48 | 3.18 | 3.12 | 3.55 | 2.33 |
| 200 | 60 | 8.00 | 8.00 | 1.04 | 5.00 | 3.61 | 1.13 | 1.00 | 5.01 | 6.46 | 4.17 | 5.00 | 6.37 | 3.73 | 4.77 | 4.88 | 4.82 |
| 200 | 100 | 8.00 | 8.00 | 2.97 | 5.00 | 4.96 | 3.85 | 1.43 | 5.00 | 6.75 | 4.87 | 5.00 | 6.92 | 4.64 | 5.00 | 5.00 | 5.01 |
| 200 | 200 | 8.00 | 8.00 | 4.99 | 5.00 | 5.00 | 5.00 | 4.87 | 5.00 | 7.96 | 5.00 | 5.00 | 7.99 | 5.00 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 7.99 | 7.85 | 1.51 | 3.55 | 2.24 | 1.66 | 1.28 | 4.49 | 6.75 | 3.99 | 4.21 | 6.85 | 3.74 | 3.86 | 4.10 | 3.49 |
| | $RMSE$ | 2.99 | 2.88 | 3.65 | 2.03 | 3.21 | 3.57 | 3.84 | 0.89 | 2.18 | 1.34 | 1.28 | 2.27 | 1.60 | 1.52 | 1.23 | 2.09 |

Part VI.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.95 | 7.99 | 1.00 | 1.39 | 1.00 | 1.00 | 1.00 | 1.84 | 8.00 | 4.63 | 5.46 | 8.00 | 6.67 | 1.00 | 1.00 | 1.60 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.54 | 1.00 | 1.00 | 1.00 | 1.99 | 7.37 | 2.24 | 2.66 | 8.00 | 3.23 | 1.01 | 1.05 | 1.93 |
| 20 | 100 | 7.93 | 8.00 | 1.00 | 1.58 | 1.00 | 1.00 | 1.00 | 1.99 | 2.40 | 1.98 | 1.98 | 6.70 | 2.07 | 1.01 | 1.04 | 2.03 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 1.65 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.95 | 3.10 | 7.53 | 1.84 | 1.00 | 1.00 | 2.04 |
| 30 | 30 | 7.98 | 7.98 | 1.11 | 2.31 | 1.39 | 1.03 | 1.00 | 2.43 | 8.00 | 3.73 | 2.33 | 8.00 | 4.34 | 1.11 | 1.25 | 2.27 |
| 30 | 60 | 7.98 | 8.00 | 1.12 | 2.12 | 1.63 | 1.05 | 1.00 | 2.29 | 7.88 | 2.31 | 2.30 | 8.00 | 2.50 | 1.12 | 1.33 | 2.27 |
| 30 | 100 | 7.98 | 8.00 | 1.24 | 2.32 | 1.90 | 1.15 | 1.00 | 2.71 | 4.20 | 2.23 | 2.91 | 7.95 | 2.49 | 1.14 | 1.41 | 2.73 |
| 30 | 200 | 7.97 | 8.00 | 1.19 | 2.26 | 1.99 | 1.21 | 1.00 | 2.86 | 3.02 | 2.01 | 3.00 | 4.04 | 2.23 | 1.00 | 1.15 | 2.95 |
| 100 | 30 | 8.00 | 7.74 | 1.00 | 3.64 | 1.08 | 1.00 | 1.00 | 5.92 | 7.99 | 3.76 | 5.66 | 8.00 | 3.54 | 1.00 | 1.01 | 1.61 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 5.88 | 1.56 | 1.01 | 1.00 | 6.19 | 8.00 | 3.83 | 5.96 | 8.00 | 3.51 | 1.00 | 1.00 | 1.62 |
| 100 | 100 | 8.00 | 8.00 | 1.03 | 7.04 | 2.86 | 1.05 | 1.00 | 6.65 | 8.00 | 4.23 | 6.42 | 8.00 | 3.95 | 1.00 | 1.00 | 2.00 |
| 100 | 200 | 8.00 | 8.00 | 1.08 | 7.76 | 5.14 | 1.36 | 1.00 | 7.66 | 8.00 | 4.83 | 7.52 | 8.00 | 4.47 | 1.00 | 1.00 | 2.06 |
| 200 | 30 | 8.00 | 7.02 | 1.00 | 1.99 | 1.00 | 1.00 | 1.00 | 5.09 | 7.80 | 2.20 | 4.34 | 7.68 | 1.85 | 1.00 | 1.00 | 1.34 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 4.84 | 1.03 | 1.00 | 1.00 | 6.25 | 8.00 | 1.92 | 5.72 | 8.00 | 1.57 | 1.00 | 1.00 | 1.17 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.74 | 1.23 | 1.00 | 1.00 | 7.63 | 8.00 | 1.96 | 7.47 | 8.00 | 1.67 | 1.00 | 1.00 | 1.08 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 3.45 | 1.00 | 1.00 | 8.00 | 8.00 | 2.50 | 8.00 | 8.00 | 2.18 | 1.00 | 1.00 | 1.03 |
| | $r_{mean}$ | 7.98 | 7.92 | 1.05 | 3.88 | 1.77 | 1.05 | 1.00 | 4.47 | 6.67 | 2.89 | 4.68 | 7.62 | 3.01 | 1.02 | 1.08 | 1.86 |
| | $RMSE$ | 6.98 | 6.92 | 0.09 | 3.80 | 1.35 | 0.11 | 0.00 | 4.17 | 6.08 | 2.16 | 4.20 | 6.69 | 2.40 | 0.05 | 0.15 | 1.01 |

Part VI.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.97 | 8.00 | 1.19 | 3.35 | 1.50 | 1.03 | 1.00 | 4.13 | 8.00 | 5.25 | 5.14 | 8.00 | 5.89 | 2.33 | 2.65 | 2.92 |
| 20 | 60 | 7.97 | 8.00 | 1.07 | 3.73 | 1.60 | 1.01 | 1.00 | 4.17 | 7.93 | 4.02 | 5.67 | 8.00 | 4.38 | 2.96 | 3.40 | 3.75 |
| 20 | 100 | 7.97 | 8.00 | 1.09 | 3.56 | 2.17 | 1.03 | 1.00 | 4.01 | 5.44 | 3.79 | 5.45 | 7.96 | 3.98 | 3.14 | 3.39 | 3.92 |
| 20 | 200 | 7.94 | 8.00 | 1.00 | 3.71 | 1.39 | 1.00 | 1.00 | 4.00 | 4.01 | 3.67 | 4.34 | 6.41 | 3.40 | 3.81 | 3.95 | 4.01 |
| 30 | 30 | 7.99 | 7.99 | 1.04 | 4.29 | 1.36 | 1.00 | 1.00 | 4.45 | 8.00 | 4.86 | 4.19 | 8.00 | 4.94 | 2.89 | 3.39 | 3.10 |
| 30 | 60 | 8.00 | 8.00 | 1.53 | 4.18 | 3.11 | 1.26 | 1.01 | 4.34 | 7.97 | 4.06 | 4.58 | 8.00 | 4.11 | 3.59 | 3.84 | 4.09 |
| 30 | 100 | 8.00 | 8.00 | 2.12 | 4.38 | 2.99 | 1.88 | 1.08 | 4.74 | 6.50 | 3.99 | 4.54 | 7.84 | 3.97 | 3.21 | 3.49 | 4.34 |
| 30 | 200 | 7.99 | 8.00 | 1.71 | 4.22 | 3.94 | 1.77 | 1.00 | 4.61 | 4.98 | 4.00 | 4.23 | 5.26 | 4.00 | 4.01 | 4.02 | 4.72 |
| 100 | 30 | 8.00 | 7.91 | 1.25 | 5.29 | 2.20 | 1.17 | 1.01 | 7.30 | 8.00 | 4.54 | 7.26 | 8.00 | 4.40 | 2.23 | 2.68 | 2.41 |
| 100 | 60 | 8.00 | 8.00 | 2.42 | 7.50 | 3.45 | 2.44 | 1.26 | 7.72 | 8.00 | 4.88 | 7.54 | 8.00 | 4.55 | 2.75 | 3.04 | 2.51 |
| 100 | 100 | 8.00 | 8.00 | 3.01 | 7.98 | 4.63 | 3.03 | 2.88 | 7.95 | 8.00 | 5.33 | 7.95 | 8.00 | 5.21 | 2.98 | 3.04 | 2.97 |
| 100 | 200 | 8.00 | 8.00 | 3.06 | 8.00 | 7.14 | 3.32 | 3.00 | 8.00 | 8.00 | 6.02 | 8.00 | 8.00 | 5.74 | 2.99 | 3.00 | 3.00 |
| 200 | 30 | 8.00 | 7.44 | 1.18 | 3.96 | 2.29 | 1.19 | 1.01 | 6.79 | 7.97 | 3.39 | 6.48 | 7.96 | 3.19 | 2.37 | 2.60 | 2.70 |
| 200 | 60 | 8.00 | 8.00 | 2.81 | 6.59 | 3.02 | 2.89 | 2.18 | 7.47 | 8.00 | 3.27 | 7.36 | 8.00 | 3.13 | 2.90 | 2.96 | 3.08 |
| 200 | 100 | 8.00 | 8.00 | 3.00 | 7.97 | 3.19 | 3.00 | 3.00 | 7.96 | 8.00 | 3.29 | 7.93 | 8.00 | 3.15 | 3.00 | 3.00 | 3.04 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 5.52 | 3.01 | 3.00 | 8.00 | 8.00 | 3.57 | 8.00 | 8.00 | 3.58 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.91 | 5.42 | 3.09 | 1.88 | 1.59 | 5.98 | 7.30 | 4.25 | 6.17 | 7.71 | 4.23 | 3.01 | 3.22 | 3.35 |
| | $RMSE$ | 4.99 | 4.96 | 1.37 | 3.03 | 1.55 | 1.43 | 1.64 | 3.43 | 4.49 | 1.48 | 3.50 | 4.77 | 1.49 | 0.48 | 0.48 | 0.75 |

Part VI.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.99 | 8.00 | 1.00 | 3.83 | 1.05 | 1.00 | 1.00 | 5.49 | 8.00 | 5.80 | 5.51 | 8.00 | 5.98 | 2.92 | 3.31 | 2.17 |
| 20 | 60 | 7.98 | 8.00 | 1.00 | 4.35 | 1.01 | 1.00 | 1.00 | 6.17 | 7.99 | 5.11 | 6.52 | 8.00 | 4.97 | 3.60 | 4.16 | 2.42 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 4.43 | 1.01 | 1.00 | 1.00 | 6.10 | 7.55 | 4.90 | 6.91 | 7.99 | 4.47 | 4.27 | 4.92 | 4.13 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 2.37 | 1.00 | 1.00 | 1.00 | 5.50 | 5.92 | 3.87 | 5.36 | 6.47 | 3.52 | 3.95 | 4.51 | 3.68 |
| 30 | 30 | 8.00 | 8.00 | 1.00 | 5.17 | 1.09 | 1.00 | 1.00 | 5.87 | 8.00 | 5.86 | 5.86 | 8.00 | 5.89 | 3.52 | 4.05 | 2.04 |
| 30 | 60 | 8.00 | 8.00 | 1.02 | 6.21 | 1.79 | 1.00 | 1.00 | 6.62 | 8.00 | 5.81 | 6.76 | 8.00 | 5.69 | 5.03 | 5.57 | 4.11 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 6.58 | 1.88 | 1.00 | 1.00 | 6.90 | 7.85 | 5.89 | 6.98 | 7.98 | 5.90 | 5.95 | 6.29 | 5.62 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 6.23 | 1.56 | 1.00 | 1.00 | 6.77 | 7.11 | 5.23 | 6.63 | 7.54 | 5.01 | 5.98 | 6.28 | 5.94 |
| 100 | 30 | 8.00 | 7.98 | 1.00 | 6.37 | 1.22 | 1.00 | 1.00 | 7.86 | 8.00 | 5.06 | 7.78 | 8.00 | 4.72 | 3.04 | 3.52 | 1.04 |
| 100 | 60 | 8.00 | 8.00 | 1.14 | 7.90 | 3.99 | 1.15 | 1.00 | 7.98 | 8.00 | 5.46 | 7.96 | 8.00 | 5.23 | 3.51 | 4.06 | 0.79 |
| 100 | 100 | 8.00 | 8.00 | 2.27 | 8.00 | 6.24 | 2.76 | 1.05 | 8.00 | 8.00 | 6.05 | 8.00 | 8.00 | 5.82 | 3.88 | 4.43 | 0.59 |
| 100 | 200 | 8.00 | 8.00 | 4.11 | 8.00 | 7.91 | 4.92 | 1.53 | 8.00 | 8.00 | 6.57 | 8.00 | 8.00 | 6.43 | 4.37 | 4.83 | 0.26 |
| 200 | 30 | 8.00 | 7.77 | 1.00 | 5.41 | 1.41 | 1.00 | 1.00 | 7.66 | 8.00 | 4.24 | 7.50 | 8.00 | 3.89 | 2.88 | 3.41 | 1.51 |
| 200 | 60 | 8.00 | 8.00 | 1.13 | 7.55 | 4.21 | 1.31 | 1.00 | 7.91 | 8.00 | 4.75 | 7.85 | 8.00 | 4.52 | 3.85 | 4.27 | 1.86 |
| 200 | 100 | 8.00 | 8.00 | 3.62 | 8.00 | 5.16 | 4.29 | 1.73 | 8.00 | 8.00 | 5.02 | 8.00 | 8.00 | 4.99 | 4.56 | 4.78 | 3.73 |
| 200 | 200 | 8.00 | 8.00 | 5.00 | 8.00 | 7.00 | 5.00 | 4.95 | 8.00 | 8.00 | 5.07 | 8.00 | 8.00 | 5.05 | 5.00 | 5.00 | 4.99 |
| | $r_{mean}$ | 8.00 | 7.98 | 1.71 | 6.15 | 2.97 | 1.84 | 1.33 | 7.05 | 7.78 | 5.29 | 7.10 | 7.87 | 5.13 | 4.14 | 4.59 | 2.81 |
| | $RMSE$ | 3.00 | 2.98 | 3.53 | 2.05 | 3.10 | 3.48 | 3.79 | 2.26 | 2.83 | 0.73 | 2.29 | 2.90 | 0.78 | 1.27 | 0.96 | 2.82 |

| | Part VII.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
| 20 | 30 | 7.93 | 7.99 | 1.00 | 1.08 | 1.00 | 1.00 | 1.00 | 1.16 | 8.00 | 4.15 | 1.24 | 8.00 | 4.60 | 1.00 | 1.01 | 1.24 |
| 20 | 60 | 7.92 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.03 | 5.25 | 1.32 | 1.02 | 6.58 | 1.47 | 1.00 | 1.00 | 1.16 |
| 20 | 100 | 7.92 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.08 | 1.01 | 1.00 | 1.46 | 1.02 | 1.00 | 1.00 | 1.10 |
| 20 | 200 | 7.88 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.07 |
| 30 | 30 | 7.96 | 7.94 | 1.00 | 1.06 | 1.00 | 1.00 | 1.00 | 1.06 | 8.00 | 2.40 | 1.03 | 8.00 | 2.58 | 1.01 | 1.01 | 1.24 |
| 30 | 60 | 7.95 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 5.03 | 1.03 | 1.01 | 6.97 | 1.07 | 1.00 | 1.00 | 1.17 |
| 30 | 100 | 7.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.15 | 1.00 | 1.00 | 1.43 | 1.00 | 1.00 | 1.00 | 1.13 |
| 30 | 200 | 7.65 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.10 |
| 100 | 30 | 8.00 | 5.84 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.22 | 4.05 | 1.08 | 1.13 | 4.15 | 1.05 | 1.00 | 1.00 | 1.24 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 5.28 | 1.00 | 1.01 | 6.24 | 1.00 | 1.00 | 1.00 | 1.16 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 7.75 | 1.00 | 1.00 | 7.97 | 1.00 | 1.00 | 1.00 | 1.11 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 | 1.00 | 1.00 | 1.28 | 1.00 | 1.00 | 1.00 | 1.08 |
| 200 | 30 | 8.00 | 5.12 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.29 | 2.30 | 1.00 | 1.15 | 2.13 | 1.00 | 1.00 | 1.00 | 1.24 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 2.32 | 1.00 | 1.00 | 2.27 | 1.00 | 1.00 | 1.00 | 1.10 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.74 | 1.00 | 1.00 | 3.18 | 1.00 | 1.00 | 1.00 | 1.07 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 6.15 | 1.00 | 1.00 | 7.48 | 1.00 | 1.00 | 1.00 | 1.07 |
| | $r_{mean}$ | 7.94 | 7.68 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.05 | 3.88 | 1.31 | 1.04 | 4.32 | 1.36 | 1.00 | 1.00 | 1.14 |
| | $RMSE$ | 6.94 | 6.73 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.10 | 3.86 | 0.87 | 0.08 | 4.34 | 0.99 | 0.00 | 0.00 | 0.16 |
| | Part VII.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.00 | 1.66 | 1.02 | 1.00 | 1.00 | 2.16 | 8.00 | 4.52 | 1.81 | 8.00 | 4.60 | 1.61 | 1.75 | 1.73 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 1.84 | 1.03 | 1.00 | 1.00 | 2.68 | 7.31 | 2.89 | 2.47 | 7.85 | 2.89 | 1.85 | 2.07 | 2.45 |
| 20 | 100 | 7.92 | 8.00 | 1.00 | 1.73 | 1.01 | 1.00 | 1.00 | 2.70 | 3.20 | 2.50 | 2.53 | 4.09 | 2.38 | 2.00 | 2.22 | 2.75 |
| 20 | 200 | 7.93 | 8.00 | 1.00 | 2.31 | 1.06 | 1.00 | 1.00 | 2.99 | 3.00 | 2.69 | 2.99 | 3.03 | 2.76 | 2.49 | 2.77 | 3.03 |
| 30 | 30 | 7.97 | 7.96 | 1.01 | 1.93 | 1.08 | 1.00 | 1.00 | 2.26 | 8.00 | 3.30 | 1.81 | 8.00 | 3.26 | 1.68 | 1.89 | 2.11 |
| 30 | 60 | 7.98 | 8.00 | 1.02 | 2.18 | 1.15 | 1.01 | 1.00 | 2.65 | 7.00 | 2.58 | 2.47 | 7.64 | 2.48 | 1.76 | 1.96 | 2.62 |
| 30 | 100 | 7.95 | 8.00 | 1.00 | 2.33 | 1.12 | 1.00 | 1.00 | 2.86 | 3.20 | 2.44 | 2.72 | 3.37 | 2.18 | 2.36 | 2.55 | 2.96 |
| 30 | 200 | 7.85 | 8.00 | 1.09 | 2.99 | 2.17 | 1.09 | 1.00 | 3.00 | 3.00 | 2.96 | 3.00 | 3.00 | 2.82 | 2.99 | 3.00 | 3.04 |
| 100 | 30 | 8.00 | 6.20 | 1.05 | 2.40 | 1.33 | 1.03 | 1.00 | 2.90 | 5.63 | 2.50 | 2.74 | 5.68 | 2.30 | 1.70 | 1.89 | 2.45 |
| 100 | 60 | 8.00 | 8.00 | 1.47 | 2.97 | 2.28 | 1.48 | 1.08 | 2.98 | 6.75 | 2.76 | 2.97 | 7.07 | 2.63 | 2.40 | 2.60 | 3.03 |
| 100 | 100 | 8.00 | 8.00 | 2.32 | 3.00 | 2.93 | 2.43 | 1.70 | 3.00 | 7.95 | 2.97 | 3.00 | 7.99 | 2.92 | 2.91 | 2.96 | 3.05 |
| 100 | 200 | 8.00 | 8.00 | 2.94 | 3.00 | 3.00 | 2.97 | 2.74 | 3.00 | 3.04 | 3.00 | 3.00 | 3.13 | 3.00 | 3.00 | 3.00 | 3.03 |
| 200 | 30 | 8.00 | 5.51 | 1.04 | 2.46 | 1.33 | 1.04 | 1.00 | 3.00 | 3.93 | 2.30 | 2.87 | 3.81 | 2.07 | 1.71 | 1.91 | 2.47 |
| 200 | 60 | 8.00 | 8.00 | 1.84 | 3.00 | 2.70 | 2.01 | 1.36 | 3.01 | 4.14 | 2.85 | 3.00 | 4.18 | 2.73 | 2.56 | 2.73 | 3.03 |
| 200 | 100 | 8.00 | 8.00 | 2.80 | 3.00 | 2.99 | 2.88 | 2.47 | 3.00 | 4.64 | 2.99 | 3.00 | 4.89 | 2.97 | 2.97 | 2.99 | 3.03 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.99 | 3.00 | 7.41 | 3.00 | 3.00 | 7.88 | 3.00 | 3.00 | 3.00 | 3.02 |
| | $r_{mean}$ | 7.97 | 7.73 | 1.54 | 2.49 | 1.83 | 1.56 | 1.40 | 2.82 | 5.39 | 2.89 | 2.71 | 5.60 | 2.81 | 2.31 | 2.46 | 2.74 |
| | $RMSE$ | 4.97 | 4.78 | 1.65 | 0.71 | 1.44 | 1.64 | 1.74 | 0.32 | 3.11 | 0.50 | 0.49 | 3.30 | 0.59 | 0.87 | 0.72 | 0.47 |
| | Part VII.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.00 | 1.72 | 1.01 | 1.00 | 1.00 | 2.64 | 8.00 | 4.86 | 2.24 | 8.00 | 4.91 | 1.80 | 2.00 | 1.71 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 1.63 | 1.00 | 1.00 | 1.00 | 2.99 | 7.82 | 3.27 | 2.81 | 7.89 | 3.27 | 1.92 | 2.17 | 1.92 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.68 | 1.00 | 1.00 | 1.00 | 3.07 | 4.93 | 2.70 | 2.83 | 5.82 | 2.56 | 2.03 | 2.30 | 2.22 |
| 20 | 200 | 7.96 | 8.00 | 1.00 | 2.00 | 1.00 | 1.00 | 1.00 | 4.11 | 4.50 | 2.82 | 3.86 | 4.39 | 2.56 | 2.82 | 3.33 | 3.97 |
| 30 | 30 | 7.98 | 7.96 | 1.00 | 1.79 | 1.02 | 1.00 | 1.00 | 2.13 | 8.00 | 3.61 | 1.68 | 8.00 | 3.44 | 1.71 | 1.94 | 1.65 |
| 30 | 60 | 7.98 | 8.00 | 1.00 | 2.33 | 1.07 | 1.00 | 1.00 | 3.15 | 7.68 | 2.98 | 2.89 | 7.87 | 2.87 | 1.90 | 2.24 | 2.30 |
| 30 | 100 | 7.97 | 8.00 | 1.00 | 2.02 | 1.03 | 1.00 | 1.00 | 3.06 | 4.63 | 2.34 | 2.59 | 4.66 | 1.94 | 2.18 | 2.50 | 2.54 |
| 30 | 200 | 7.96 | 8.00 | 1.00 | 2.26 | 1.01 | 1.00 | 1.00 | 3.85 | 4.52 | 2.14 | 3.56 | 4.32 | 1.86 | 2.90 | 3.43 | 4.19 |
| 100 | 30 | 8.00 | 6.45 | 1.01 | 2.57 | 1.18 | 1.00 | 1.00 | 3.44 | 6.49 | 2.69 | 3.18 | 6.53 | 2.47 | 1.77 | 2.06 | 2.19 |
| 100 | 60 | 8.00 | 8.00 | 1.16 | 3.94 | 2.08 | 1.17 | 1.01 | 4.11 | 7.59 | 3.10 | 3.81 | 7.72 | 2.73 | 2.39 | 2.78 | 3.39 |
| 100 | 100 | 8.00 | 8.00 | 1.42 | 4.66 | 2.95 | 1.56 | 1.08 | 4.57 | 8.00 | 3.38 | 4.50 | 8.00 | 3.14 | 3.43 | 3.94 | 4.68 |
| 100 | 200 | 8.00 | 8.00 | 2.74 | 5.00 | 4.81 | 3.37 | 1.66 | 5.00 | 5.04 | 4.55 | 5.00 | 5.13 | 4.26 | 5.00 | 5.00 | 5.01 |
| 200 | 30 | 8.00 | 5.84 | 1.00 | 2.66 | 1.17 | 1.00 | 1.00 | 3.73 | 4.99 | 2.42 | 3.44 | 4.76 | 2.12 | 1.80 | 2.04 | 2.15 |
| 200 | 60 | 8.00 | 8.00 | 1.26 | 4.35 | 2.47 | 1.35 | 1.06 | 4.60 | 5.79 | 3.07 | 4.47 | 5.79 | 2.69 | 2.44 | 2.92 | 3.61 |
| 200 | 100 | 8.00 | 8.00 | 2.02 | 4.97 | 3.96 | 2.43 | 1.37 | 4.96 | 6.42 | 3.74 | 4.94 | 6.53 | 3.48 | 4.27 | 4.60 | 4.94 |
| 200 | 200 | 8.00 | 8.00 | 4.49 | 5.00 | 5.00 | 4.82 | 4.00 | 5.00 | 7.92 | 4.93 | 5.00 | 7.99 | 4.87 | 5.00 | 5.00 | 5.02 |
| | $r_{mean}$ | 7.98 | 7.77 | 1.44 | 3.04 | 1.99 | 1.54 | 1.26 | 3.78 | 6.40 | 3.29 | 3.55 | 6.46 | 3.07 | 2.71 | 3.02 | 3.22 |
| | $RMSE$ | 2.98 | 2.83 | 3.67 | 2.36 | 3.32 | 3.62 | 3.81 | 1.50 | 1.98 | 1.91 | 1.76 | 2.05 | 2.13 | 2.54 | 2.25 | 2.16 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part VIII.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.00 | 1.40 | 1.00 | 1.00 | 1.00 | 1.81 | 8.00 | 4.61 | 3.74 | 8.00 | 6.24 | 1.00 | 1.01 | 1.58 |
| 20 | 60 | 7.94 | 8.00 | 1.00 | 1.53 | 1.00 | 1.00 | 1.00 | 1.98 | 7.31 | 2.25 | 2.43 | 7.97 | 2.98 | 1.06 | 1.13 | 1.95 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 1.58 | 1.00 | 1.00 | 1.00 | 1.98 | 2.38 | 1.97 | 2.01 | 6.04 | 2.05 | 1.04 | 1.12 | 2.03 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 1.66 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 1.95 | 3.20 | 6.68 | 1.85 | 1.00 | 1.01 | 2.04 |
| 30 | 30 | 7.99 | 7.98 | 1.09 | 2.30 | 1.37 | 1.02 | 1.00 | 2.42 | 8.00 | 3.72 | 2.24 | 8.00 | 4.12 | 1.27 | 1.43 | 2.28 |
| 30 | 60 | 7.98 | 8.00 | 1.11 | 2.13 | 1.61 | 1.04 | 1.00 | 2.29 | 7.89 | 2.29 | 2.25 | 8.00 | 2.40 | 1.23 | 1.45 | 2.27 |
| 30 | 100 | 7.98 | 8.00 | 1.24 | 2.31 | 1.91 | 1.15 | 1.00 | 2.71 | 4.15 | 2.24 | 2.86 | 7.82 | 2.44 | 1.25 | 1.51 | 2.72 |
| 30 | 200 | 7.97 | 8.00 | 1.18 | 2.26 | 1.98 | 1.20 | 1.00 | 2.86 | 3.02 | 2.01 | 3.00 | 4.06 | 2.21 | 1.02 | 1.20 | 2.97 |
| 100 | 30 | 8.00 | 7.71 | 1.00 | 3.47 | 1.09 | 1.00 | 1.00 | 5.80 | 7.99 | 3.68 | 5.45 | 7.99 | 3.41 | 1.12 | 1.16 | 1.63 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 5.87 | 1.53 | 1.01 | 1.00 | 6.20 | 8.00 | 3.79 | 5.83 | 8.00 | 3.47 | 1.00 | 1.00 | 1.59 |
| 100 | 100 | 8.00 | 8.00 | 1.03 | 7.02 | 2.86 | 1.06 | 1.00 | 6.66 | 8.00 | 4.22 | 6.39 | 8.00 | 3.93 | 1.00 | 1.00 | 1.91 |
| 100 | 200 | 8.00 | 8.00 | 1.08 | 7.77 | 5.13 | 1.36 | 1.00 | 7.66 | 8.00 | 4.84 | 7.51 | 8.00 | 4.46 | 1.00 | 1.00 | 2.08 |
| 200 | 30 | 8.00 | 6.99 | 1.00 | 1.94 | 1.01 | 1.00 | 1.00 | 4.91 | 7.76 | 2.15 | 4.08 | 7.56 | 1.78 | 1.02 | 1.02 | 1.29 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 4.76 | 1.03 | 1.00 | 1.00 | 6.18 | 8.00 | 1.88 | 5.56 | 8.00 | 1.51 | 1.00 | 1.00 | 1.16 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.71 | 1.23 | 1.00 | 1.00 | 7.63 | 8.00 | 1.94 | 7.39 | 8.00 | 1.65 | 1.00 | 1.00 | 1.10 |
| 200 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 3.45 | 1.00 | 1.00 | 8.00 | 8.00 | 2.49 | 8.00 | 8.00 | 2.18 | 1.00 | 1.00 | 1.03 |
| | $r_{mean}$ | 7.98 | 7.92 | 1.05 | 3.86 | 1.76 | 1.05 | 1.00 | 4.44 | 6.66 | 2.88 | 4.50 | 7.51 | 2.92 | 1.06 | 1.13 | 1.85 |
| | $RMSE$ | 6.98 | 6.92 | 0.09 | 3.78 | 1.35 | 0.11 | 0.00 | 4.15 | 6.08 | 2.14 | 4.04 | 6.59 | 2.29 | 0.11 | 0.22 | 1.01 |
| Part VIII.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.05 | 2.69 | 1.18 | 1.01 | 1.00 | 3.61 | 8.00 | 5.13 | 4.06 | 8.00 | 5.59 | 1.81 | 2.08 | 2.39 |
| 20 | 60 | 7.97 | 8.00 | 1.06 | 3.25 | 1.30 | 1.01 | 1.00 | 4.05 | 7.93 | 3.96 | 5.16 | 8.00 | 4.22 | 2.29 | 2.76 | 3.46 |
| 20 | 100 | 7.97 | 8.00 | 1.11 | 3.36 | 1.70 | 1.04 | 1.00 | 3.99 | 5.33 | 3.71 | 5.13 | 7.94 | 3.92 | 2.63 | 3.12 | 3.85 |
| 20 | 200 | 7.95 | 8.00 | 1.02 | 3.57 | 1.42 | 1.01 | 1.00 | 4.00 | 4.02 | 3.60 | 4.36 | 6.25 | 3.33 | 3.41 | 3.81 | 4.01 |
| 30 | 30 | 7.99 | 7.98 | 1.05 | 3.40 | 1.24 | 1.01 | 1.00 | 3.79 | 8.00 | 4.64 | 3.11 | 8.00 | 4.58 | 2.12 | 2.47 | 2.51 |
| 30 | 60 | 8.00 | 8.00 | 1.31 | 4.03 | 2.28 | 1.16 | 1.01 | 4.27 | 7.97 | 4.01 | 4.44 | 8.00 | 4.03 | 2.80 | 3.27 | 3.83 |
| 30 | 100 | 8.00 | 8.00 | 1.66 | 4.27 | 2.65 | 1.50 | 1.06 | 4.71 | 6.44 | 3.88 | 4.52 | 7.82 | 3.86 | 2.75 | 3.31 | 4.25 |
| 30 | 200 | 7.99 | 8.00 | 1.44 | 4.21 | 3.81 | 1.46 | 1.01 | 4.58 | 4.96 | 4.00 | 4.24 | 5.33 | 3.99 | 3.89 | 4.03 | 4.70 |
| 100 | 30 | 8.00 | 7.85 | 1.09 | 4.61 | 1.52 | 1.07 | 1.01 | 7.04 | 8.00 | 4.23 | 6.79 | 8.00 | 4.00 | 1.77 | 2.02 | 1.83 |
| 100 | 60 | 8.00 | 8.00 | 1.65 | 7.38 | 3.00 | 1.67 | 1.14 | 7.64 | 8.00 | 4.75 | 7.38 | 8.00 | 4.36 | 1.88 | 2.21 | 1.89 |
| 100 | 100 | 8.00 | 8.00 | 2.73 | 7.98 | 4.50 | 2.82 | 2.07 | 7.95 | 8.00 | 5.25 | 7.94 | 8.00 | 5.09 | 2.26 | 2.54 | 2.42 |
| 100 | 200 | 8.00 | 8.00 | 3.05 | 8.00 | 7.11 | 3.28 | 2.92 | 8.00 | 8.00 | 6.00 | 8.00 | 8.00 | 5.71 | 2.73 | 2.86 | 2.87 |
| 200 | 30 | 8.00 | 7.23 | 1.07 | 3.32 | 1.50 | 1.08 | 1.01 | 6.33 | 7.93 | 3.03 | 5.77 | 7.86 | 2.73 | 1.61 | 1.79 | 2.02 |
| 200 | 60 | 8.00 | 8.00 | 2.02 | 6.38 | 2.86 | 2.17 | 1.50 | 7.34 | 8.00 | 3.18 | 7.16 | 8.00 | 3.06 | 2.06 | 2.31 | 2.80 |
| 200 | 100 | 8.00 | 8.00 | 2.89 | 7.97 | 3.16 | 2.95 | 2.66 | 7.96 | 8.00 | 3.24 | 7.92 | 8.00 | 3.12 | 2.69 | 2.82 | 3.00 |
| 200 | 200 | 8.00 | 8.00 | 3.00 | 8.00 | 5.48 | 3.01 | 3.00 | 8.00 | 8.00 | 3.56 | 8.00 | 8.00 | 3.53 | 2.96 | 2.98 | 3.01 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.70 | 5.15 | 2.79 | 1.70 | 1.46 | 5.83 | 7.29 | 4.14 | 5.87 | 7.70 | 4.07 | 2.48 | 2.77 | 3.05 |
| | $RMSE$ | 4.99 | 4.94 | 1.50 | 2.93 | 1.67 | 1.53 | 1.70 | 3.34 | 4.48 | 1.39 | 3.31 | 4.76 | 1.35 | 0.80 | 0.66 | 0.85 |
| Part VIII.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.01 | 2.59 | 1.06 | 1.00 | 1.00 | 4.09 | 8.00 | 5.41 | 3.78 | 8.00 | 5.61 | 1.99 | 2.32 | 1.92 |
| 20 | 60 | 7.98 | 8.00 | 1.00 | 3.09 | 1.06 | 1.00 | 1.00 | 5.08 | 7.99 | 4.39 | 5.30 | 8.00 | 4.35 | 2.26 | 2.66 | 2.20 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 3.60 | 1.04 | 1.00 | 1.00 | 5.71 | 7.49 | 4.29 | 6.24 | 7.99 | 3.96 | 2.72 | 3.38 | 3.02 |
| 20 | 200 | 7.98 | 8.00 | 1.00 | 2.28 | 1.00 | 1.00 | 1.00 | 5.16 | 5.71 | 3.48 | 5.09 | 6.34 | 3.16 | 3.27 | 3.81 | 3.39 |
| 30 | 30 | 8.00 | 7.99 | 1.02 | 3.47 | 1.14 | 1.00 | 1.00 | 4.24 | 8.00 | 5.07 | 3.86 | 8.00 | 5.09 | 2.27 | 2.75 | 2.04 |
| 30 | 60 | 8.00 | 8.00 | 1.07 | 5.35 | 1.70 | 1.03 | 1.00 | 6.09 | 8.00 | 5.08 | 6.20 | 8.00 | 5.00 | 2.95 | 3.78 | 3.13 |
| 30 | 100 | 8.00 | 8.00 | 1.05 | 6.21 | 1.94 | 1.03 | 1.00 | 6.80 | 7.82 | 5.29 | 6.88 | 7.98 | 5.23 | 4.12 | 5.18 | 4.43 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 5.86 | 1.77 | 1.00 | 1.00 | 6.72 | 7.06 | 4.88 | 6.58 | 7.55 | 4.65 | 5.01 | 5.83 | 5.31 |
| 100 | 30 | 8.00 | 7.93 | 1.02 | 4.64 | 1.27 | 1.01 | 1.00 | 7.34 | 8.00 | 4.30 | 7.06 | 8.00 | 3.96 | 2.00 | 2.32 | 1.43 |
| 100 | 60 | 8.00 | 8.00 | 1.32 | 7.71 | 2.88 | 1.34 | 1.03 | 7.89 | 8.00 | 4.85 | 7.80 | 8.00 | 4.57 | 2.02 | 2.48 | 1.33 |
| 100 | 100 | 8.00 | 8.00 | 2.07 | 8.00 | 5.21 | 2.31 | 1.24 | 8.00 | 8.00 | 5.61 | 8.00 | 8.00 | 5.35 | 2.26 | 2.63 | 1.17 |
| 100 | 200 | 8.00 | 8.00 | 3.13 | 8.00 | 7.81 | 3.95 | 1.76 | 8.00 | 8.00 | 6.40 | 8.00 | 8.00 | 6.23 | 2.58 | 3.04 | 0.58 |
| 200 | 30 | 8.00 | 7.41 | 1.02 | 3.66 | 1.32 | 1.02 | 1.00 | 6.79 | 7.97 | 3.25 | 6.31 | 7.95 | 2.88 | 1.75 | 2.00 | 1.74 |
| 200 | 60 | 8.00 | 8.00 | 1.33 | 7.14 | 2.90 | 1.47 | 1.06 | 7.76 | 8.00 | 3.81 | 7.64 | 8.00 | 3.47 | 2.01 | 2.27 | 1.75 |
| 200 | 100 | 8.00 | 8.00 | 2.59 | 8.00 | 4.69 | 3.08 | 1.78 | 8.00 | 8.00 | 4.58 | 8.00 | 8.00 | 4.35 | 2.49 | 2.87 | 2.29 |
| 200 | 200 | 8.00 | 8.00 | 4.67 | 8.00 | 6.89 | 4.92 | 4.22 | 8.00 | 8.00 | 5.05 | 8.00 | 8.00 | 5.02 | 4.13 | 4.44 | 4.20 |
| | $r_{mean}$ | 8.00 | 7.96 | 1.58 | 5.48 | 2.73 | 1.70 | 1.32 | 6.60 | 7.75 | 4.73 | 6.55 | 7.86 | 4.56 | 2.74 | 3.24 | 2.50 |
| | $RMSE$ | 3.00 | 2.96 | 3.57 | 2.14 | 3.13 | 3.51 | 3.77 | 2.09 | 2.81 | 0.83 | 2.07 | 2.89 | 0.99 | 2.44 | 2.07 | 2.81 |

Table 3: Fat-tailed, heteroskedastic idiosyncratic errors

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.98 | 1.04 | 2.72 | 1.09 | 1.01 | 1.00 | 2.18 | 8.00 | 5.02 | 1.08 | 8.00 | 3.90 | 1.61 | 1.74 | 1.33 |
| 20 | 60 | 7.94 | 8.00 | 1.12 | 3.19 | 1.25 | 1.08 | 1.02 | 1.74 | 7.61 | 2.40 | 1.02 | 5.47 | 1.30 | 1.29 | 1.36 | 1.41 |
| 20 | 100 | 7.95 | 8.00 | 1.13 | 3.34 | 1.33 | 1.10 | 1.03 | 1.38 | 2.71 | 1.41 | 1.00 | 1.06 | 1.00 | 1.21 | 1.25 | 1.41 |
| 20 | 200 | 7.95 | 8.00 | 1.24 | 4.35 | 1.69 | 1.22 | 1.10 | 1.29 | 1.38 | 1.17 | 1.00 | 1.00 | 1.00 | 1.14 | 1.17 | 1.44 |
| 30 | 30 | 7.96 | 7.94 | 1.03 | 1.34 | 1.04 | 1.01 | 1.00 | 1.65 | 8.00 | 3.48 | 1.03 | 8.00 | 2.35 | 1.27 | 1.38 | 1.45 |
| 30 | 60 | 7.95 | 8.00 | 1.06 | 1.49 | 1.12 | 1.04 | 1.01 | 1.50 | 7.52 | 1.63 | 1.01 | 5.92 | 1.02 | 1.13 | 1.16 | 1.48 |
| 30 | 100 | 7.93 | 8.00 | 1.09 | 1.75 | 1.19 | 1.07 | 1.02 | 1.40 | 2.66 | 1.27 | 1.00 | 1.21 | 1.00 | 1.09 | 1.11 | 1.50 |
| 30 | 200 | 7.93 | 8.00 | 1.13 | 2.16 | 1.33 | 1.13 | 1.05 | 1.27 | 1.41 | 1.10 | 1.00 | 1.00 | 1.00 | 1.12 | 1.15 | 1.51 |
| 100 | 30 | 8.00 | 5.96 | 1.01 | 1.06 | 1.02 | 1.01 | 1.00 | 1.98 | 6.58 | 1.66 | 1.11 | 4.16 | 1.04 | 1.17 | 1.21 | 1.46 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 1.03 | 1.01 | 1.01 | 1.01 | 1.42 | 7.38 | 1.20 | 1.00 | 5.64 | 1.00 | 1.08 | 1.10 | 1.45 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 1.11 | 1.02 | 1.01 | 1.01 | 1.30 | 8.00 | 1.15 | 1.00 | 7.90 | 1.00 | 1.07 | 1.09 | 1.48 |
| 100 | 200 | 8.00 | 8.00 | 1.02 | 1.32 | 1.10 | 1.02 | 1.01 | 1.31 | 2.28 | 1.12 | 1.00 | 1.03 | 1.00 | 1.09 | 1.11 | 1.57 |
| 200 | 30 | 8.00 | 5.20 | 1.01 | 1.03 | 1.02 | 1.01 | 1.00 | 2.06 | 3.80 | 1.30 | 1.15 | 2.29 | 1.00 | 1.09 | 1.10 | 1.42 |
| 200 | 60 | 8.00 | 7.99 | 1.01 | 1.02 | 1.02 | 1.01 | 1.01 | 1.52 | 3.99 | 1.18 | 1.00 | 2.13 | 1.00 | 1.11 | 1.12 | 1.42 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 1.02 | 1.01 | 1.01 | 1.01 | 1.36 | 4.92 | 1.12 | 1.00 | 2.68 | 1.00 | 1.07 | 1.09 | 1.43 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 1.10 | 1.01 | 1.01 | 1.01 | 1.27 | 7.85 | 1.10 | 1.00 | 6.67 | 1.00 | 1.06 | 1.07 | 1.54 |
| | $r_{mean}$ | 7.97 | 7.69 | 1.06 | 1.81 | 1.14 | 1.05 | 1.02 | 1.54 | 5.26 | 1.71 | 1.03 | 4.01 | 1.29 | 1.16 | 1.20 | 1.46 |
| | $RMSE$ | 6.97 | 6.74 | 0.09 | 1.30 | 0.23 | 0.07 | 0.03 | 0.61 | 4.95 | 1.27 | 0.05 | 4.03 | 0.80 | 0.21 | 0.26 | 0.46 |
| Part I.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.98 | 1.01 | 2.61 | 1.03 | 1.00 | 1.00 | 2.28 | 8.00 | 5.19 | 1.14 | 8.00 | 4.17 | 2.42 | 2.52 | 1.06 |
| 20 | 60 | 7.96 | 8.00 | 1.02 | 3.24 | 1.09 | 1.02 | 1.00 | 2.48 | 7.93 | 3.12 | 1.38 | 7.31 | 2.16 | 1.94 | 2.12 | 1.17 |
| 20 | 100 | 7.97 | 8.00 | 1.01 | 3.32 | 1.07 | 1.01 | 1.00 | 1.94 | 4.89 | 2.01 | 1.28 | 2.27 | 1.31 | 2.15 | 2.27 | 1.08 |
| 20 | 200 | 7.95 | 8.00 | 1.04 | 3.77 | 1.15 | 1.04 | 1.01 | 1.37 | 1.81 | 1.12 | 1.03 | 1.16 | 1.00 | 1.35 | 1.44 | 1.43 |
| 30 | 30 | 7.98 | 7.97 | 1.00 | 1.58 | 1.02 | 1.00 | 1.00 | 1.79 | 8.00 | 3.93 | 1.21 | 8.00 | 2.90 | 2.15 | 2.41 | 1.12 |
| 30 | 60 | 7.97 | 8.00 | 1.01 | 2.15 | 1.05 | 1.01 | 1.00 | 2.42 | 7.88 | 2.54 | 1.77 | 7.06 | 1.89 | 2.12 | 2.35 | 1.46 |
| 30 | 100 | 7.95 | 8.00 | 1.01 | 2.22 | 1.06 | 1.00 | 1.00 | 2.30 | 4.71 | 1.75 | 1.33 | 2.84 | 1.08 | 2.03 | 2.24 | 1.43 |
| 30 | 200 | 7.96 | 8.00 | 1.02 | 2.09 | 1.10 | 1.02 | 1.00 | 1.46 | 2.23 | 1.04 | 1.05 | 1.46 | 1.00 | 1.53 | 1.69 | 1.68 |
| 100 | 30 | 7.99 | 6.38 | 1.00 | 1.82 | 1.02 | 1.00 | 1.00 | 3.11 | 7.38 | 2.38 | 2.22 | 5.86 | 1.81 | 1.99 | 2.22 | 1.23 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 2.66 | 1.10 | 1.00 | 1.00 | 2.96 | 7.88 | 1.92 | 2.43 | 7.04 | 1.42 | 2.27 | 2.52 | 1.64 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 3.12 | 1.51 | 1.02 | 1.00 | 3.18 | 8.00 | 2.04 | 2.85 | 7.97 | 1.57 | 2.63 | 2.80 | 2.76 |
| 100 | 200 | 8.00 | 8.00 | 1.07 | 3.30 | 2.63 | 1.24 | 1.01 | 3.28 | 4.32 | 2.30 | 3.00 | 3.04 | 1.95 | 3.13 | 3.19 | 3.37 |
| 200 | 30 | 8.00 | 5.72 | 1.00 | 2.15 | 1.03 | 1.00 | 1.00 | 3.52 | 5.61 | 2.06 | 2.80 | 4.19 | 1.70 | 1.94 | 2.20 | 1.41 |
| 200 | 60 | 8.00 | 7.99 | 1.00 | 2.94 | 1.48 | 1.01 | 1.00 | 3.38 | 5.86 | 2.00 | 2.96 | 4.18 | 1.69 | 2.47 | 2.68 | 2.12 |
| 200 | 100 | 8.00 | 8.00 | 1.08 | 3.07 | 2.67 | 1.27 | 1.01 | 3.36 | 6.79 | 2.46 | 3.00 | 4.75 | 2.12 | 2.98 | 3.09 | 3.02 |
| 200 | 200 | 8.00 | 8.00 | 2.28 | 3.13 | 3.06 | 2.82 | 1.57 | 3.29 | 7.99 | 3.01 | 3.00 | 7.63 | 2.86 | 3.16 | 3.19 | 3.47 |
| | $r_{mean}$ | 7.98 | 7.75 | 1.10 | 2.70 | 1.44 | 1.15 | 1.04 | 2.63 | 6.21 | 2.43 | 2.03 | 5.17 | 1.91 | 2.27 | 2.43 | 1.84 |
| | $RMSE$ | 4.98 | 4.80 | 1.93 | 0.68 | 1.69 | 1.90 | 1.97 | 0.79 | 3.80 | 1.14 | 1.26 | 3.24 | 1.35 | 0.89 | 0.74 | 1.42 |
| Part I.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.01 | 2.44 | 1.02 | 1.00 | 1.00 | 2.21 | 8.00 | 5.22 | 1.11 | 8.00 | 4.14 | 2.50 | 2.56 | 0.87 |
| 20 | 60 | 7.97 | 8.00 | 1.01 | 2.33 | 1.03 | 1.00 | 1.00 | 1.73 | 7.92 | 2.89 | 1.08 | 7.26 | 1.88 | 2.35 | 2.52 | 0.65 |
| 20 | 100 | 7.96 | 8.00 | 1.01 | 2.74 | 1.08 | 1.01 | 1.00 | 1.67 | 5.40 | 1.84 | 1.10 | 2.41 | 1.18 | 2.20 | 2.34 | 0.75 |
| 20 | 200 | 7.97 | 8.00 | 1.01 | 3.20 | 1.08 | 1.01 | 1.00 | 1.12 | 1.51 | 1.02 | 1.00 | 1.05 | 1.00 | 2.12 | 2.27 | 0.60 |
| 30 | 30 | 7.97 | 7.96 | 1.00 | 1.29 | 1.01 | 1.00 | 1.00 | 1.37 | 8.00 | 3.75 | 1.03 | 8.00 | 2.63 | 2.37 | 2.59 | 0.78 |
| 30 | 60 | 7.98 | 8.00 | 1.00 | 1.46 | 1.01 | 1.00 | 1.00 | 1.51 | 7.94 | 1.97 | 1.06 | 7.22 | 1.23 | 2.14 | 2.31 | 0.66 |
| 30 | 100 | 7.98 | 8.00 | 1.01 | 1.54 | 1.03 | 1.00 | 1.00 | 1.55 | 5.02 | 1.27 | 1.18 | 2.95 | 1.03 | 2.13 | 2.33 | 0.69 |
| 30 | 200 | 7.98 | 8.00 | 1.01 | 1.68 | 1.03 | 1.01 | 1.00 | 1.44 | 3.13 | 1.02 | 1.06 | 2.03 | 1.00 | 2.86 | 3.13 | 0.86 |
| 100 | 30 | 8.00 | 6.62 | 1.00 | 1.47 | 1.00 | 1.00 | 1.00 | 2.87 | 7.65 | 2.17 | 1.95 | 6.65 | 1.58 | 2.32 | 2.59 | 0.75 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 2.48 | 1.00 | 1.00 | 1.00 | 2.72 | 7.97 | 1.55 | 2.17 | 7.71 | 1.19 | 2.60 | 2.85 | 0.67 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 3.11 | 1.04 | 1.00 | 1.00 | 2.66 | 8.00 | 1.37 | 2.17 | 8.00 | 1.06 | 2.73 | 3.07 | 0.95 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 5.15 | 1.23 | 1.00 | 1.00 | 4.97 | 6.24 | 1.47 | 4.63 | 5.04 | 1.04 | 4.93 | 5.05 | 4.01 |
| 200 | 30 | 8.00 | 6.06 | 1.00 | 1.56 | 1.00 | 1.00 | 1.00 | 3.46 | 6.08 | 1.67 | 2.67 | 5.00 | 1.31 | 2.43 | 2.73 | 0.71 |
| 200 | 60 | 8.00 | 7.99 | 1.00 | 3.58 | 1.04 | 1.00 | 1.00 | 4.07 | 7.12 | 1.49 | 3.77 | 6.01 | 1.24 | 2.71 | 3.08 | 0.71 |
| 200 | 100 | 7.99 | 8.00 | 1.00 | 4.69 | 1.26 | 1.00 | 1.00 | 4.59 | 7.67 | 1.45 | 4.31 | 6.57 | 1.17 | 3.83 | 4.10 | 1.78 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 5.15 | 3.14 | 1.11 | 1.00 | 5.02 | 8.00 | 2.15 | 4.74 | 7.96 | 1.69 | 4.79 | 4.95 | 4.13 |
| | $r_{mean}$ | 7.98 | 7.79 | 1.00 | 2.74 | 1.19 | 1.01 | 1.00 | 2.69 | 6.60 | 2.02 | 2.19 | 5.74 | 1.52 | 2.81 | 3.03 | 1.22 |
| | $RMSE$ | 2.98 | 2.84 | 4.00 | 2.59 | 3.85 | 3.99 | 4.00 | 2.66 | 2.49 | 3.17 | 3.12 | 2.43 | 3.57 | 2.35 | 2.15 | 3.94 |

Part II.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.97 | 7.99 | 1.04 | 3.18 | 1.08 | 1.02 | 1.00 | 3.58 | 8.00 | 5.39 | 3.02 | 8.00 | 5.47 | 1.20 | 1.30 | 2.02 |
| 20 | 60 | 7.96 | 8.00 | 1.06 | 3.37 | 1.18 | 1.03 | 1.00 | 2.77 | 7.92 | 3.09 | 1.98 | 7.86 | 2.44 | 1.46 | 1.62 | 1.99 |
| 20 | 100 | 7.96 | 8.00 | 1.06 | 3.84 | 1.24 | 1.03 | 1.00 | 2.46 | 5.08 | 2.33 | 1.91 | 4.76 | 1.96 | 1.49 | 1.64 | 2.14 |
| 20 | 200 | 7.95 | 8.00 | 1.05 | 4.15 | 1.36 | 1.04 | 1.00 | 2.26 | 2.42 | 2.06 | 1.99 | 2.00 | 1.79 | 1.48 | 1.70 | 2.28 |
| 30 | 30 | 7.99 | 7.99 | 1.13 | 2.63 | 1.36 | 1.04 | 1.01 | 3.05 | 8.00 | 4.49 | 2.36 | 8.00 | 4.13 | 1.35 | 1.55 | 2.25 |
| 30 | 60 | 7.99 | 8.00 | 1.26 | 2.94 | 1.81 | 1.13 | 1.01 | 3.24 | 7.99 | 3.13 | 2.44 | 8.00 | 2.56 | 1.77 | 1.97 | 2.47 |
| 30 | 100 | 7.98 | 8.00 | 1.06 | 3.08 | 1.64 | 1.04 | 1.00 | 3.17 | 6.22 | 2.60 | 2.19 | 6.15 | 2.04 | 1.71 | 1.95 | 2.66 |
| 30 | 200 | 7.98 | 8.00 | 1.24 | 2.88 | 2.08 | 1.26 | 1.01 | 2.92 | 3.51 | 2.13 | 2.16 | 2.75 | 2.00 | 1.84 | 2.00 | 2.78 |
| 100 | 30 | 8.00 | 7.74 | 1.02 | 3.95 | 1.17 | 1.02 | 1.00 | 6.65 | 8.00 | 3.97 | 5.92 | 8.00 | 3.57 | 1.29 | 1.48 | 1.79 |
| 100 | 60 | 8.00 | 8.00 | 1.04 | 6.19 | 1.78 | 1.05 | 1.01 | 6.72 | 8.00 | 4.03 | 6.17 | 8.00 | 3.65 | 1.20 | 1.31 | 1.62 |
| 100 | 100 | 8.00 | 8.00 | 1.11 | 7.12 | 3.29 | 1.17 | 1.03 | 6.91 | 8.00 | 4.44 | 6.51 | 8.00 | 4.13 | 1.13 | 1.19 | 1.74 |
| 100 | 200 | 8.00 | 8.00 | 1.21 | 7.78 | 5.40 | 1.65 | 1.03 | 7.77 | 8.00 | 4.96 | 7.48 | 8.00 | 4.58 | 1.05 | 1.08 | 1.94 |
| 200 | 30 | 8.00 | 7.22 | 1.00 | 2.27 | 1.02 | 1.00 | 1.00 | 6.09 | 7.93 | 2.58 | 4.92 | 7.84 | 1.99 | 1.12 | 1.18 | 1.41 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 5.47 | 1.08 | 1.01 | 1.01 | 6.84 | 8.00 | 2.31 | 6.27 | 8.00 | 1.73 | 1.06 | 1.07 | 1.34 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 7.86 | 1.41 | 1.01 | 1.01 | 7.83 | 8.00 | 2.44 | 7.69 | 8.00 | 1.82 | 1.04 | 1.06 | 1.25 |
| 200 | 200 | 8.00 | 8.00 | 1.03 | 8.00 | 4.10 | 1.07 | 1.01 | 8.00 | 8.00 | 2.90 | 8.00 | 8.00 | 2.27 | 1.05 | 1.05 | 1.18 |
| | $r_{mean}$ | 7.99 | 7.93 | 1.08 | 4.67 | 1.94 | 1.10 | 1.01 | 5.02 | 7.07 | 3.30 | 4.44 | 6.96 | 2.88 | 1.33 | 1.45 | 1.93 |
| | $RMSE$ | 6.99 | 6.94 | 0.12 | 4.18 | 1.53 | 0.19 | 0.01 | 4.56 | 6.31 | 2.53 | 4.14 | 6.27 | 2.21 | 0.42 | 0.56 | 1.04 |

Part II.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.98 | 7.99 | 1.00 | 3.45 | 1.03 | 1.00 | 1.00 | 4.49 | 8.00 | 5.79 | 2.16 | 8.00 | 5.20 | 2.51 | 2.77 | 1.27 |
| 20 | 60 | 7.98 | 8.00 | 1.01 | 4.22 | 1.06 | 1.00 | 1.00 | 4.40 | 8.00 | 4.03 | 3.39 | 7.99 | 3.54 | 2.36 | 2.70 | 1.83 |
| 20 | 100 | 7.97 | 8.00 | 1.00 | 4.64 | 1.13 | 1.00 | 1.00 | 3.90 | 7.08 | 3.25 | 3.12 | 7.48 | 2.88 | 1.99 | 2.36 | 2.39 |
| 20 | 200 | 7.97 | 8.00 | 1.00 | 4.99 | 1.08 | 1.00 | 1.00 | 4.22 | 4.76 | 2.96 | 3.39 | 3.95 | 2.14 | 3.37 | 3.65 | 2.78 |
| 30 | 30 | 7.99 | 7.99 | 1.01 | 3.14 | 1.09 | 1.00 | 1.00 | 4.02 | 8.00 | 5.08 | 2.61 | 8.00 | 4.57 | 2.17 | 2.63 | 1.76 |
| 30 | 60 | 8.00 | 8.00 | 1.01 | 3.99 | 1.07 | 1.00 | 1.00 | 4.84 | 8.00 | 4.13 | 3.92 | 8.00 | 3.59 | 2.47 | 2.96 | 1.75 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 3.96 | 1.21 | 1.00 | 1.00 | 4.54 | 7.57 | 3.34 | 3.64 | 7.50 | 2.95 | 2.62 | 3.01 | 2.29 |
| 30 | 200 | 7.99 | 8.00 | 1.01 | 3.77 | 1.11 | 1.01 | 1.00 | 4.53 | 5.56 | 2.94 | 3.52 | 4.71 | 2.26 | 2.07 | 2.58 | 2.39 |
| 100 | 30 | 8.00 | 7.91 | 1.00 | 4.97 | 1.10 | 1.00 | 1.00 | 7.43 | 8.00 | 4.45 | 7.03 | 8.00 | 4.01 | 2.69 | 3.16 | 1.00 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.55 | 1.72 | 1.00 | 1.00 | 7.77 | 8.00 | 4.72 | 7.54 | 8.00 | 4.19 | 3.58 | 4.21 | 0.58 |
| 100 | 100 | 8.00 | 8.00 | 1.03 | 7.97 | 4.19 | 1.07 | 1.00 | 7.95 | 8.00 | 5.30 | 7.83 | 8.00 | 4.68 | 4.19 | 4.84 | 0.44 |
| 100 | 200 | 8.00 | 8.00 | 1.12 | 8.00 | 7.20 | 1.64 | 1.00 | 8.00 | 8.00 | 5.95 | 8.00 | 8.00 | 5.55 | 4.78 | 5.26 | 0.24 |
| 200 | 30 | 8.00 | 7.49 | 1.00 | 3.17 | 1.05 | 1.00 | 1.00 | 7.03 | 7.97 | 3.06 | 6.34 | 7.95 | 2.56 | 2.20 | 2.49 | 1.00 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.65 | 1.51 | 1.00 | 1.00 | 7.63 | 8.00 | 3.05 | 7.20 | 8.00 | 2.50 | 2.24 | 2.56 | 0.75 |
| 200 | 100 | 8.00 | 8.00 | 1.16 | 7.98 | 3.20 | 1.46 | 1.00 | 7.97 | 8.00 | 3.53 | 7.93 | 8.00 | 3.07 | 2.52 | 2.79 | 1.07 |
| 200 | 200 | 8.00 | 8.00 | 2.73 | 8.00 | 5.75 | 3.04 | 1.91 | 8.00 | 8.00 | 3.92 | 8.00 | 8.00 | 3.36 | 2.91 | 3.00 | 2.11 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.13 | 5.40 | 2.16 | 1.20 | 1.06 | 6.05 | 7.56 | 4.09 | 5.35 | 7.47 | 3.57 | 2.79 | 3.19 | 1.48 |
| | $RMSE$ | 4.99 | 4.96 | 1.92 | 3.04 | 2.04 | 1.87 | 1.96 | 3.49 | 4.66 | 1.48 | 3.22 | 4.63 | 1.16 | 0.80 | 0.86 | 1.70 |

Part II.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 1, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.98 | 8.00 | 1.01 | 3.49 | 1.03 | 1.00 | 1.00 | 4.79 | 8.00 | 5.89 | 2.63 | 8.00 | 5.47 | 2.50 | 2.77 | 1.41 |
| 20 | 60 | 7.98 | 8.00 | 1.00 | 3.62 | 1.03 | 1.00 | 1.00 | 4.96 | 8.00 | 4.26 | 2.72 | 8.00 | 3.56 | 2.71 | 3.06 | 1.28 |
| 20 | 100 | 7.99 | 8.00 | 1.01 | 3.10 | 1.05 | 1.00 | 1.00 | 4.23 | 7.72 | 3.16 | 2.51 | 7.64 | 2.49 | 2.25 | 2.53 | 1.14 |
| 20 | 200 | 7.99 | 8.00 | 1.01 | 2.98 | 1.04 | 1.01 | 1.00 | 4.10 | 5.53 | 2.48 | 2.92 | 4.35 | 1.79 | 2.23 | 2.61 | 1.24 |
| 30 | 30 | 8.00 | 7.99 | 1.00 | 2.20 | 1.01 | 1.00 | 1.00 | 3.17 | 8.00 | 4.98 | 1.73 | 8.00 | 4.44 | 1.76 | 1.99 | 1.35 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 3.38 | 1.01 | 1.00 | 1.00 | 5.25 | 8.00 | 4.20 | 3.81 | 8.00 | 3.60 | 2.36 | 2.80 | 1.17 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 2.98 | 1.01 | 1.00 | 1.00 | 5.14 | 7.92 | 3.19 | 3.91 | 7.84 | 2.65 | 1.39 | 1.65 | 1.26 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 3.33 | 1.02 | 1.00 | 1.00 | 6.56 | 7.46 | 2.99 | 5.33 | 6.98 | 1.74 | 1.98 | 2.69 | 2.11 |
| 100 | 30 | 8.00 | 7.93 | 1.00 | 4.00 | 1.01 | 1.00 | 1.00 | 7.35 | 8.00 | 4.05 | 6.95 | 8.00 | 3.65 | 2.73 | 3.09 | 0.75 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.21 | 1.09 | 1.00 | 1.00 | 7.63 | 8.00 | 3.77 | 7.23 | 8.00 | 3.20 | 2.79 | 3.10 | 0.43 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 7.99 | 1.52 | 1.00 | 1.00 | 7.97 | 8.00 | 4.26 | 7.90 | 8.00 | 3.48 | 3.19 | 3.50 | 0.28 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 4.93 | 1.01 | 1.00 | 8.00 | 8.00 | 4.72 | 8.00 | 8.00 | 4.10 | 3.61 | 3.88 | 0.16 |
| 200 | 30 | 8.00 | 7.53 | 1.00 | 2.63 | 1.00 | 1.00 | 1.00 | 7.02 | 7.99 | 2.80 | 6.35 | 7.98 | 2.26 | 2.63 | 2.90 | 0.66 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.67 | 1.04 | 1.00 | 1.00 | 7.67 | 8.00 | 2.48 | 7.21 | 8.00 | 1.82 | 2.68 | 2.99 | 0.35 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.98 | 1.57 | 1.00 | 1.00 | 7.99 | 8.00 | 2.76 | 7.97 | 8.00 | 2.09 | 2.81 | 3.06 | 0.27 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 8.00 | 6.51 | 1.17 | 1.00 | 8.00 | 8.00 | 3.99 | 8.00 | 8.00 | 3.30 | 2.89 | 3.23 | 0.21 |
| | $r_{mean}$ | 8.00 | 7.97 | 1.00 | 4.85 | 1.68 | 1.01 | 1.00 | 6.24 | 7.79 | 3.75 | 5.32 | 7.67 | 3.10 | 2.53 | 2.87 | 0.88 |
| | $RMSE$ | 3.00 | 2.97 | 4.00 | 2.23 | 3.67 | 3.99 | 4.00 | 2.04 | 2.85 | 1.57 | 2.31 | 2.82 | 2.16 | 2.52 | 2.20 | 4.16 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part III.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.05 | 2.71 | 1.12 | 1.02 | 1.01 | 2.19 | 8.00 | 5.05 | 1.12 | 8.00 | 4.19 | 1.45 | 1.55 | 1.48 |
| 20 | 60 | 7.93 | 8.00 | 1.08 | 3.02 | 1.19 | 1.05 | 1.01 | 1.57 | 7.57 | 2.29 | 1.01 | 4.98 | 1.25 | 1.30 | 1.37 | 1.31 |
| 20 | 100 | 7.93 | 8.00 | 1.12 | 3.57 | 1.33 | 1.08 | 1.01 | 1.38 | 2.68 | 1.41 | 1.00 | 1.04 | 1.00 | 1.31 | 1.37 | 1.30 |
| 20 | 200 | 7.95 | 8.00 | 1.22 | 4.46 | 1.60 | 1.21 | 1.08 | 1.25 | 1.36 | 1.16 | 1.00 | 1.00 | 1.00 | 1.12 | 1.14 | 1.45 |
| 30 | 30 | 7.97 | 7.96 | 1.02 | 1.36 | 1.03 | 1.01 | 1.00 | 1.49 | 8.00 | 3.43 | 1.02 | 8.00 | 2.12 | 1.53 | 1.70 | 1.27 |
| 30 | 60 | 7.94 | 8.00 | 1.03 | 1.48 | 1.09 | 1.02 | 1.01 | 1.44 | 7.54 | 1.60 | 1.00 | 5.66 | 1.02 | 1.15 | 1.19 | 1.44 |
| 30 | 100 | 7.93 | 8.00 | 1.07 | 1.83 | 1.20 | 1.07 | 1.03 | 1.38 | 2.53 | 1.25 | 1.00 | 1.11 | 1.00 | 1.16 | 1.19 | 1.48 |
| 30 | 200 | 7.91 | 8.00 | 1.13 | 2.17 | 1.37 | 1.13 | 1.03 | 1.25 | 1.42 | 1.08 | 1.00 | 1.00 | 1.00 | 1.20 | 1.23 | 1.48 |
| 100 | 30 | 7.99 | 5.81 | 1.01 | 1.05 | 1.02 | 1.01 | 1.00 | 1.89 | 6.34 | 1.59 | 1.10 | 3.77 | 1.02 | 1.13 | 1.18 | 1.43 |
| 100 | 60 | 7.99 | 8.00 | 1.02 | 1.04 | 1.02 | 1.02 | 1.02 | 1.42 | 7.38 | 1.22 | 1.00 | 5.57 | 1.00 | 1.10 | 1.11 | 1.44 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 1.11 | 1.03 | 1.01 | 1.01 | 1.32 | 7.99 | 1.14 | 1.00 | 7.87 | 1.00 | 1.09 | 1.10 | 1.51 |
| 100 | 200 | 8.00 | 8.00 | 1.01 | 1.30 | 1.09 | 1.02 | 1.01 | 1.29 | 2.26 | 1.11 | 1.00 | 1.05 | 1.00 | 1.09 | 1.09 | 1.57 |
| 200 | 30 | 8.00 | 5.04 | 1.00 | 1.04 | 1.01 | 1.00 | 1.00 | 1.92 | 3.51 | 1.25 | 1.13 | 2.05 | 1.00 | 1.20 | 1.27 | 1.27 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 1.02 | 1.01 | 1.01 | 1.01 | 1.48 | 3.76 | 1.14 | 1.00 | 2.10 | 1.00 | 1.09 | 1.11 | 1.38 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 1.02 | 1.01 | 1.01 | 1.01 | 1.36 | 4.91 | 1.11 | 1.00 | 2.71 | 1.00 | 1.07 | 1.08 | 1.43 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 1.12 | 1.02 | 1.01 | 1.01 | 1.30 | 7.85 | 1.11 | 1.00 | 6.69 | 1.00 | 1.07 | 1.08 | 1.55 |
| | $r_{mean}$ | 7.97 | 7.68 | 1.05 | 1.83 | 1.13 | 1.04 | 1.02 | 1.50 | 5.19 | 1.68 | 1.02 | 3.91 | 1.29 | 1.19 | 1.24 | 1.42 |
| | $RMSE$ | 6.97 | 6.73 | 0.08 | 1.33 | 0.21 | 0.07 | 0.02 | 0.56 | 4.91 | 1.25 | 0.05 | 3.94 | 0.85 | 0.23 | 0.29 | 0.43 |
| Part III.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.98 | 1.03 | 2.86 | 1.05 | 1.01 | 1.00 | 2.53 | 8.00 | 5.21 | 1.26 | 8.00 | 4.19 | 2.12 | 2.29 | 1.22 |
| 20 | 60 | 7.94 | 8.00 | 1.03 | 2.93 | 1.11 | 1.01 | 1.00 | 1.73 | 7.85 | 2.57 | 1.06 | 5.81 | 1.48 | 1.64 | 1.73 | 1.10 |
| 20 | 100 | 7.95 | 8.00 | 1.02 | 3.20 | 1.10 | 1.01 | 1.00 | 1.57 | 3.97 | 1.66 | 1.11 | 1.65 | 1.12 | 1.76 | 1.88 | 1.02 |
| 20 | 200 | 7.96 | 8.00 | 1.06 | 4.09 | 1.25 | 1.05 | 1.01 | 1.53 | 2.01 | 1.17 | 1.04 | 1.19 | 1.00 | 1.50 | 1.61 | 1.55 |
| 30 | 30 | 7.97 | 7.96 | 1.00 | 1.54 | 1.03 | 1.00 | 1.00 | 1.69 | 8.00 | 3.72 | 1.10 | 8.00 | 2.53 | 2.00 | 2.22 | 1.14 |
| 30 | 60 | 7.96 | 8.00 | 1.02 | 1.83 | 1.06 | 1.01 | 1.00 | 1.99 | 7.82 | 2.22 | 1.36 | 6.59 | 1.49 | 1.77 | 1.95 | 1.34 |
| 30 | 100 | 7.96 | 8.00 | 1.01 | 1.90 | 1.08 | 1.01 | 1.00 | 1.77 | 4.12 | 1.42 | 1.26 | 2.45 | 1.06 | 1.83 | 1.99 | 1.35 |
| 30 | 200 | 7.95 | 8.00 | 1.02 | 2.35 | 1.12 | 1.03 | 1.01 | 1.91 | 2.67 | 1.12 | 1.40 | 2.02 | 1.00 | 1.87 | 2.03 | 1.92 |
| 100 | 30 | 8.00 | 6.01 | 1.00 | 1.52 | 1.02 | 1.00 | 1.00 | 2.53 | 6.93 | 2.03 | 1.75 | 4.78 | 1.47 | 1.68 | 1.84 | 1.25 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 2.04 | 1.12 | 1.01 | 1.00 | 2.35 | 7.78 | 1.60 | 1.89 | 6.57 | 1.29 | 1.75 | 1.92 | 1.52 |
| 100 | 100 | 8.00 | 8.00 | 1.03 | 2.77 | 1.39 | 1.04 | 1.00 | 2.68 | 8.00 | 1.75 | 2.32 | 7.95 | 1.37 | 2.08 | 2.26 | 2.38 |
| 100 | 200 | 8.00 | 8.00 | 1.05 | 3.27 | 1.98 | 1.11 | 1.01 | 3.22 | 4.28 | 1.87 | 2.90 | 3.03 | 1.44 | 2.78 | 2.91 | 3.23 |
| 200 | 30 | 8.00 | 5.28 | 1.00 | 1.65 | 1.02 | 1.00 | 1.00 | 2.81 | 4.45 | 1.69 | 2.14 | 3.24 | 1.34 | 1.65 | 1.83 | 1.37 |
| 200 | 60 | 8.00 | 7.99 | 1.02 | 2.48 | 1.27 | 1.03 | 1.00 | 2.92 | 5.28 | 1.63 | 2.50 | 3.80 | 1.33 | 1.82 | 1.96 | 1.85 |
| 200 | 100 | 8.00 | 8.00 | 1.26 | 3.03 | 2.39 | 1.46 | 1.06 | 3.28 | 6.45 | 2.29 | 2.96 | 4.57 | 2.00 | 2.43 | 2.63 | 2.98 |
| 200 | 200 | 8.00 | 8.00 | 2.15 | 3.12 | 3.04 | 2.57 | 1.71 | 3.31 | 7.98 | 2.88 | 3.00 | 7.59 | 2.64 | 3.07 | 3.13 | 3.50 |
| | $r_{mean}$ | 7.98 | 7.70 | 1.11 | 2.54 | 1.38 | 1.15 | 1.05 | 2.36 | 5.97 | 2.18 | 1.82 | 4.83 | 1.67 | 1.98 | 2.14 | 1.80 |
| | $RMSE$ | 4.98 | 4.77 | 1.91 | 0.86 | 1.72 | 1.89 | 1.96 | 0.88 | 3.61 | 1.30 | 1.38 | 2.98 | 1.55 | 1.10 | 0.96 | 1.43 |
| Part III.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 1, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.94 | 7.99 | 1.02 | 2.58 | 1.04 | 1.00 | 1.00 | 2.29 | 8.00 | 5.14 | 1.21 | 8.00 | 4.11 | 2.33 | 2.46 | 1.18 |
| 20 | 60 | 7.94 | 8.00 | 1.03 | 2.72 | 1.08 | 1.01 | 1.00 | 1.75 | 7.88 | 2.63 | 1.06 | 6.32 | 1.52 | 1.92 | 2.08 | 0.93 |
| 20 | 100 | 7.96 | 8.00 | 1.02 | 3.07 | 1.07 | 1.01 | 1.00 | 1.62 | 4.97 | 1.75 | 1.07 | 1.94 | 1.11 | 2.09 | 2.20 | 0.81 |
| 20 | 200 | 7.96 | 8.00 | 1.03 | 3.49 | 1.11 | 1.03 | 1.01 | 1.44 | 2.07 | 1.14 | 1.06 | 1.26 | 1.00 | 1.96 | 2.09 | 0.90 |
| 30 | 30 | 7.98 | 7.96 | 1.01 | 1.43 | 1.01 | 1.00 | 1.00 | 1.56 | 8.00 | 3.60 | 1.09 | 8.00 | 2.52 | 2.06 | 2.25 | 1.12 |
| 30 | 60 | 7.97 | 8.00 | 1.01 | 1.61 | 1.03 | 1.00 | 1.00 | 1.79 | 7.90 | 2.09 | 1.19 | 6.98 | 1.37 | 1.92 | 2.10 | 1.04 |
| 30 | 100 | 7.97 | 8.00 | 1.00 | 1.57 | 1.03 | 1.00 | 1.00 | 1.49 | 4.42 | 1.27 | 1.10 | 2.39 | 1.02 | 1.95 | 2.09 | 0.90 |
| 30 | 200 | 7.97 | 8.00 | 1.01 | 1.76 | 1.03 | 1.01 | 1.00 | 1.68 | 2.87 | 1.05 | 1.20 | 1.91 | 1.00 | 2.21 | 2.49 | 0.92 |
| 100 | 30 | 7.99 | 6.17 | 1.00 | 1.48 | 1.01 | 1.00 | 1.00 | 2.55 | 7.14 | 2.02 | 1.70 | 5.20 | 1.44 | 1.82 | 2.01 | 1.13 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 2.19 | 1.05 | 1.00 | 1.00 | 2.48 | 7.92 | 1.63 | 1.92 | 7.20 | 1.26 | 1.87 | 2.09 | 1.25 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 2.83 | 1.13 | 1.01 | 1.00 | 2.53 | 8.00 | 1.45 | 1.96 | 8.00 | 1.12 | 2.04 | 2.29 | 1.38 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 3.69 | 1.18 | 1.01 | 1.00 | 3.39 | 6.04 | 1.20 | 2.82 | 4.96 | 1.04 | 2.93 | 3.26 | 2.19 |
| 200 | 30 | 8.00 | 5.38 | 1.00 | 1.51 | 1.01 | 1.00 | 1.00 | 2.77 | 4.58 | 1.54 | 2.09 | 3.40 | 1.19 | 1.82 | 1.97 | 1.20 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 2.58 | 1.12 | 1.00 | 1.00 | 3.09 | 6.02 | 1.43 | 2.61 | 4.87 | 1.20 | 1.79 | 1.99 | 1.27 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 3.58 | 1.49 | 1.03 | 1.00 | 3.54 | 7.40 | 1.58 | 3.16 | 6.12 | 1.31 | 2.05 | 2.27 | 1.55 |
| 200 | 200 | 8.00 | 8.00 | 1.13 | 5.05 | 3.23 | 1.45 | 1.02 | 4.82 | 8.00 | 2.37 | 4.55 | 7.94 | 2.01 | 3.92 | 4.28 | 3.96 |
| | $r_{mean}$ | 7.98 | 7.72 | 1.02 | 2.57 | 1.23 | 1.04 | 1.00 | 2.42 | 6.33 | 1.99 | 1.86 | 5.28 | 1.51 | 2.17 | 2.37 | 1.36 |
| | $RMSE$ | 2.98 | 2.82 | 3.98 | 2.63 | 3.81 | 3.97 | 4.00 | 2.73 | 2.34 | 3.18 | 3.28 | 2.37 | 3.57 | 2.88 | 2.69 | 3.72 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part IV.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 8.00 | 1.02 | 3.06 | 1.07 | 1.01 | 1.00 | 3.10 | 8.00 | 5.32 | 1.60 | 8.00 | 5.08 | 1.40 | 1.50 | 1.80 |
| 20 | 60 | 7.95 | 8.00 | 1.04 | 3.80 | 1.22 | 1.00 | 1.00 | 2.90 | 7.91 | 3.18 | 1.55 | 7.86 | 2.32 | 1.66 | 1.80 | 2.00 |
| 20 | 100 | 7.97 | 8.00 | 1.06 | 3.63 | 1.22 | 1.04 | 1.00 | 2.41 | 4.96 | 2.28 | 1.88 | 4.58 | 1.95 | 1.41 | 1.50 | 2.06 |
| 20 | 200 | 7.95 | 8.00 | 1.04 | 4.21 | 1.42 | 1.03 | 1.00 | 2.24 | 2.44 | 2.07 | 2.00 | 2.00 | 1.96 | 1.61 | 1.78 | 2.26 |
| 30 | 30 | 7.99 | 7.98 | 1.15 | 2.78 | 1.42 | 1.07 | 1.01 | 3.10 | 8.00 | 4.45 | 2.39 | 8.00 | 3.92 | 1.61 | 1.90 | 2.22 |
| 30 | 60 | 7.99 | 8.00 | 1.15 | 2.86 | 1.63 | 1.08 | 1.01 | 3.19 | 8.00 | 3.10 | 2.22 | 8.00 | 2.37 | 1.53 | 1.73 | 2.44 |
| 30 | 100 | 7.98 | 8.00 | 1.13 | 3.03 | 1.68 | 1.10 | 1.00 | 3.13 | 6.31 | 2.57 | 2.46 | 5.71 | 2.10 | 2.00 | 2.14 | 2.60 |
| 30 | 200 | 7.98 | 8.00 | 1.22 | 3.25 | 2.07 | 1.23 | 1.01 | 3.10 | 3.60 | 2.26 | 2.58 | 3.00 | 2.01 | 2.11 | 2.22 | 2.94 |
| 100 | 30 | 8.00 | 7.63 | 1.02 | 3.86 | 1.14 | 1.02 | 1.00 | 6.48 | 8.00 | 3.89 | 5.51 | 8.00 | 3.46 | 1.67 | 1.98 | 1.46 |
| 100 | 60 | 8.00 | 8.00 | 1.02 | 6.19 | 1.74 | 1.02 | 1.01 | 6.65 | 8.00 | 4.06 | 5.96 | 8.00 | 3.58 | 1.37 | 1.63 | 1.70 |
| 100 | 100 | 8.00 | 8.00 | 1.06 | 7.24 | 3.14 | 1.10 | 1.01 | 6.99 | 8.00 | 4.42 | 6.52 | 8.00 | 4.04 | 1.14 | 1.26 | 1.73 |
| 100 | 200 | 8.00 | 8.00 | 1.18 | 7.81 | 5.34 | 1.56 | 1.02 | 7.77 | 8.00 | 4.87 | 7.43 | 8.00 | 4.42 | 1.04 | 1.04 | 2.03 |
| 200 | 30 | 8.00 | 7.18 | 1.01 | 2.15 | 1.03 | 1.01 | 1.00 | 5.96 | 7.87 | 2.53 | 4.94 | 7.77 | 1.91 | 1.26 | 1.35 | 1.34 |
| 200 | 60 | 8.00 | 8.00 | 1.03 | 5.34 | 1.10 | 1.04 | 1.03 | 6.82 | 8.00 | 2.26 | 6.16 | 8.00 | 1.73 | 1.10 | 1.11 | 1.34 |
| 200 | 100 | 8.00 | 8.00 | 1.02 | 7.84 | 1.36 | 1.03 | 1.02 | 7.80 | 8.00 | 2.44 | 7.53 | 8.00 | 1.73 | 1.06 | 1.08 | 1.20 |
| 200 | 200 | 8.00 | 8.00 | 1.03 | 8.00 | 4.10 | 1.08 | 1.02 | 8.00 | 8.00 | 2.96 | 8.00 | 8.00 | 2.34 | 1.06 | 1.07 | 1.20 |
| | $r_{mean}$ | 7.99 | 7.92 | 1.07 | 4.69 | 1.92 | 1.09 | 1.01 | 4.98 | 7.07 | 3.29 | 4.30 | 6.93 | 2.81 | 1.44 | 1.57 | 1.90 |
| | $RMSE$ | 6.99 | 6.93 | 0.10 | 4.19 | 1.50 | 0.16 | 0.01 | 4.52 | 6.31 | 2.51 | 4.04 | 6.24 | 2.09 | 0.54 | 0.68 | 1.03 |
| Part IV.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.02 | 3.32 | 1.09 | 1.01 | 1.00 | 4.04 | 8.00 | 5.57 | 2.17 | 8.00 | 5.14 | 1.97 | 2.22 | 1.64 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 3.93 | 1.07 | 1.00 | 1.00 | 4.18 | 7.98 | 3.94 | 2.98 | 7.98 | 3.42 | 2.04 | 2.36 | 1.66 |
| 20 | 100 | 7.98 | 8.00 | 1.01 | 3.96 | 1.12 | 1.01 | 1.00 | 3.62 | 7.01 | 2.99 | 2.28 | 6.44 | 2.34 | 2.04 | 2.36 | 1.73 |
| 20 | 200 | 7.98 | 8.00 | 1.02 | 4.46 | 1.18 | 1.02 | 1.00 | 3.27 | 3.75 | 2.65 | 2.86 | 3.26 | 2.17 | 2.33 | 2.54 | 2.72 |
| 30 | 30 | 7.99 | 7.99 | 1.01 | 3.18 | 1.13 | 1.00 | 1.00 | 3.74 | 8.00 | 4.86 | 2.80 | 8.00 | 4.44 | 2.03 | 2.28 | 1.84 |
| 30 | 60 | 8.00 | 8.00 | 1.01 | 3.43 | 1.13 | 1.00 | 1.00 | 4.24 | 7.99 | 3.76 | 3.10 | 8.00 | 3.03 | 1.78 | 2.09 | 1.81 |
| 30 | 100 | 8.00 | 8.00 | 1.02 | 4.20 | 1.17 | 1.01 | 1.00 | 4.84 | 7.56 | 3.54 | 4.07 | 7.43 | 2.99 | 2.16 | 2.71 | 2.22 |
| 30 | 200 | 7.99 | 8.00 | 1.01 | 4.27 | 1.50 | 1.01 | 1.00 | 4.69 | 5.62 | 3.20 | 4.07 | 4.91 | 2.89 | 3.03 | 3.32 | 3.07 |
| 100 | 30 | 8.00 | 7.84 | 1.00 | 4.04 | 1.09 | 1.00 | 1.00 | 6.94 | 8.00 | 4.02 | 6.20 | 8.00 | 3.49 | 2.26 | 2.61 | 1.08 |
| 100 | 60 | 8.00 | 8.00 | 1.03 | 6.96 | 1.85 | 1.03 | 1.00 | 7.27 | 8.00 | 4.26 | 6.97 | 8.00 | 3.84 | 2.40 | 2.80 | 0.81 |
| 100 | 100 | 8.00 | 8.00 | 1.08 | 7.94 | 3.40 | 1.13 | 1.00 | 7.86 | 8.00 | 4.71 | 7.78 | 8.00 | 4.34 | 2.52 | 3.15 | 0.59 |
| 100 | 200 | 8.00 | 8.00 | 1.30 | 8.00 | 6.59 | 1.86 | 1.02 | 8.00 | 8.00 | 5.52 | 8.00 | 8.00 | 5.06 | 3.64 | 4.27 | 0.40 |
| 200 | 30 | 8.00 | 7.38 | 1.00 | 2.74 | 1.07 | 1.00 | 1.00 | 6.33 | 7.94 | 2.81 | 5.26 | 7.89 | 2.25 | 1.69 | 1.98 | 1.18 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 6.24 | 1.58 | 1.03 | 1.00 | 7.30 | 8.00 | 2.83 | 6.71 | 8.00 | 2.30 | 1.74 | 1.92 | 1.06 |
| 200 | 100 | 8.00 | 8.00 | 1.16 | 7.94 | 2.62 | 1.31 | 1.04 | 7.95 | 8.00 | 3.20 | 7.92 | 8.00 | 2.66 | 1.88 | 1.99 | 1.16 |
| 200 | 200 | 8.00 | 8.00 | 2.05 | 8.00 | 5.80 | 2.46 | 1.62 | 8.00 | 8.00 | 3.84 | 8.00 | 8.00 | 3.29 | 1.96 | 2.04 | 1.66 |
| | $r_{mean}$ | 7.99 | 7.95 | 1.11 | 5.16 | 2.09 | 1.18 | 1.04 | 5.77 | 7.49 | 3.86 | 5.07 | 7.37 | 3.35 | 2.22 | 2.54 | 1.54 |
| | $RMSE$ | 4.99 | 4.95 | 1.91 | 2.89 | 1.91 | 1.86 | 1.96 | 3.29 | 4.63 | 1.24 | 3.02 | 4.57 | 1.01 | 0.92 | 0.76 | 1.62 |
| Part IV.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 1, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.98 | 1.01 | 2.85 | 1.03 | 1.00 | 1.00 | 3.36 | 8.00 | 5.47 | 1.46 | 8.00 | 4.83 | 1.93 | 2.15 | 1.30 |
| 20 | 60 | 7.97 | 8.00 | 1.02 | 3.29 | 1.07 | 1.01 | 1.00 | 3.84 | 8.00 | 3.73 | 2.54 | 7.99 | 3.14 | 2.02 | 2.31 | 1.55 |
| 20 | 100 | 7.95 | 8.00 | 1.00 | 3.41 | 1.05 | 1.00 | 1.00 | 3.60 | 7.35 | 2.98 | 2.02 | 6.34 | 2.18 | 1.89 | 2.13 | 1.38 |
| 20 | 200 | 8.00 | 8.00 | 1.01 | 2.74 | 1.12 | 1.01 | 1.00 | 3.62 | 5.30 | 2.17 | 1.86 | 3.51 | 1.22 | 2.26 | 2.37 | 1.40 |
| 30 | 30 | 7.99 | 7.98 | 1.02 | 2.55 | 1.07 | 1.01 | 1.00 | 3.28 | 8.00 | 4.77 | 2.14 | 8.00 | 4.12 | 1.82 | 2.07 | 1.66 |
| 30 | 60 | 8.00 | 8.00 | 1.01 | 3.36 | 1.11 | 1.00 | 1.00 | 4.34 | 8.00 | 3.80 | 3.10 | 8.00 | 3.08 | 2.04 | 2.36 | 1.56 |
| 30 | 100 | 8.00 | 8.00 | 1.01 | 3.42 | 1.08 | 1.01 | 1.00 | 4.68 | 7.84 | 3.29 | 3.83 | 7.68 | 2.76 | 1.76 | 2.18 | 1.52 |
| 30 | 200 | 8.00 | 8.00 | 1.01 | 3.70 | 1.11 | 1.01 | 1.00 | 5.83 | 7.24 | 2.96 | 5.02 | 6.67 | 2.60 | 2.09 | 2.68 | 1.78 |
| 100 | 30 | 8.00 | 7.80 | 1.00 | 3.84 | 1.02 | 1.00 | 1.00 | 6.89 | 8.00 | 3.98 | 6.23 | 8.00 | 3.47 | 2.32 | 2.83 | 0.99 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 7.10 | 1.62 | 1.01 | 1.00 | 7.44 | 8.00 | 4.14 | 7.16 | 8.00 | 3.74 | 2.40 | 2.72 | 0.67 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 7.89 | 2.31 | 1.02 | 1.00 | 7.71 | 8.00 | 4.21 | 7.49 | 8.00 | 3.71 | 2.78 | 3.22 | 0.58 |
| 100 | 200 | 8.00 | 8.00 | 1.02 | 8.00 | 5.10 | 1.12 | 1.00 | 8.00 | 8.00 | 4.69 | 8.00 | 8.00 | 4.23 | 3.02 | 3.55 | 0.28 |
| 200 | 30 | 8.00 | 7.17 | 1.00 | 2.52 | 1.03 | 1.00 | 1.00 | 6.20 | 7.87 | 2.73 | 4.96 | 7.77 | 2.12 | 1.86 | 2.22 | 1.15 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 6.10 | 1.24 | 1.01 | 1.00 | 7.25 | 8.00 | 2.58 | 6.66 | 8.00 | 2.04 | 1.77 | 2.05 | 0.83 |
| 200 | 100 | 8.00 | 8.00 | 1.02 | 7.97 | 2.36 | 1.06 | 1.00 | 7.96 | 8.00 | 3.04 | 7.89 | 8.00 | 2.54 | 1.94 | 2.20 | 0.60 |
| 200 | 200 | 8.00 | 8.00 | 1.05 | 8.00 | 5.62 | 1.43 | 1.00 | 8.00 | 8.00 | 3.66 | 8.00 | 8.00 | 3.03 | 1.99 | 2.18 | 0.37 |
| | $r_{mean}$ | 7.99 | 7.93 | 1.01 | 4.80 | 1.81 | 1.04 | 1.00 | 5.75 | 7.73 | 3.64 | 4.90 | 7.50 | 3.05 | 2.12 | 2.45 | 1.10 |
| | $RMSE$ | 2.99 | 2.94 | 3.99 | 2.18 | 3.49 | 3.96 | 4.00 | 1.97 | 2.81 | 1.61 | 2.40 | 2.75 | 2.15 | 2.90 | 2.58 | 3.93 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part V.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.07 | 2.65 | 1.13 | 1.03 | 1.01 | 2.30 | 8.00 | 5.08 | 1.15 | 8.00 | 4.53 | 1.22 | 1.31 | 1.59 |
| 20 | 60 | 7.95 | 8.00 | 1.09 | 2.90 | 1.21 | 1.06 | 1.02 | 1.62 | 7.60 | 2.30 | 1.02 | 5.44 | 1.27 | 1.25 | 1.29 | 1.40 |
| 20 | 100 | 7.93 | 8.00 | 1.11 | 3.56 | 1.32 | 1.08 | 1.02 | 1.41 | 2.71 | 1.43 | 1.00 | 1.04 | 1.00 | 1.25 | 1.30 | 1.36 |
| 20 | 200 | 7.94 | 8.00 | 1.23 | 4.40 | 1.60 | 1.21 | 1.09 | 1.25 | 1.36 | 1.16 | 1.00 | 1.00 | 1.00 | 1.12 | 1.14 | 1.45 |
| 30 | 30 | 7.97 | 7.96 | 1.03 | 1.43 | 1.05 | 1.02 | 1.00 | 1.63 | 8.00 | 3.53 | 1.03 | 8.00 | 2.28 | 1.34 | 1.46 | 1.40 |
| 30 | 60 | 7.94 | 8.00 | 1.04 | 1.47 | 1.09 | 1.03 | 1.01 | 1.45 | 7.55 | 1.61 | 1.01 | 6.12 | 1.03 | 1.12 | 1.15 | 1.48 |
| 30 | 100 | 7.95 | 8.00 | 1.08 | 1.81 | 1.19 | 1.07 | 1.03 | 1.37 | 2.58 | 1.26 | 1.00 | 1.12 | 1.00 | 1.13 | 1.16 | 1.49 |
| 30 | 200 | 7.91 | 8.00 | 1.13 | 2.18 | 1.37 | 1.13 | 1.04 | 1.26 | 1.42 | 1.09 | 1.00 | 1.00 | 1.00 | 1.19 | 1.22 | 1.48 |
| 100 | 30 | 7.99 | 5.96 | 1.01 | 1.04 | 1.02 | 1.01 | 1.01 | 1.98 | 6.53 | 1.67 | 1.13 | 4.26 | 1.04 | 1.07 | 1.09 | 1.49 |
| 100 | 60 | 7.99 | 8.00 | 1.01 | 1.04 | 1.02 | 1.01 | 1.01 | 1.43 | 7.48 | 1.23 | 1.00 | 5.85 | 1.00 | 1.08 | 1.10 | 1.46 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 1.11 | 1.03 | 1.01 | 1.01 | 1.33 | 7.99 | 1.15 | 1.00 | 7.91 | 1.00 | 1.08 | 1.09 | 1.52 |
| 100 | 200 | 7.99 | 8.00 | 1.01 | 1.30 | 1.09 | 1.02 | 1.01 | 1.29 | 2.26 | 1.11 | 1.00 | 1.05 | 1.00 | 1.08 | 1.09 | 1.58 |
| 200 | 30 | 8.00 | 5.20 | 1.00 | 1.02 | 1.01 | 1.00 | 1.00 | 2.02 | 3.83 | 1.29 | 1.14 | 2.25 | 1.00 | 1.12 | 1.14 | 1.38 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.50 | 3.92 | 1.15 | 1.00 | 2.22 | 1.00 | 1.07 | 1.09 | 1.37 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 1.02 | 1.01 | 1.01 | 1.01 | 1.36 | 5.01 | 1.11 | 1.00 | 2.78 | 1.00 | 1.06 | 1.07 | 1.42 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 1.12 | 1.02 | 1.01 | 1.01 | 1.30 | 7.86 | 1.11 | 1.00 | 6.74 | 1.00 | 1.07 | 1.08 | 1.57 |
| | $r_{mean}$ | 7.97 | 7.69 | 1.05 | 1.82 | 1.14 | 1.04 | 1.02 | 1.53 | 5.26 | 1.71 | 1.03 | 4.05 | 1.32 | 1.14 | 1.17 | 1.47 |
| | $RMSE$ | 6.97 | 6.74 | 0.08 | 1.30 | 0.21 | 0.07 | 0.03 | 0.61 | 4.96 | 1.28 | 0.06 | 4.07 | 0.94 | 0.16 | 0.20 | 0.47 |
| Part V.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.02 | 3.03 | 1.04 | 1.00 | 1.00 | 3.26 | 8.00 | 5.38 | 1.49 | 8.00 | 4.54 | 2.36 | 2.63 | 1.16 |
| 20 | 60 | 7.95 | 8.00 | 1.03 | 2.83 | 1.09 | 1.02 | 1.00 | 1.72 | 7.87 | 2.66 | 1.04 | 6.21 | 1.52 | 1.69 | 1.83 | 1.02 |
| 20 | 100 | 7.96 | 8.00 | 1.01 | 3.14 | 1.09 | 1.01 | 1.00 | 1.61 | 4.24 | 1.74 | 1.12 | 1.84 | 1.13 | 2.02 | 2.18 | 0.94 |
| 20 | 200 | 7.96 | 8.00 | 1.06 | 4.12 | 1.22 | 1.04 | 1.02 | 1.55 | 2.05 | 1.17 | 1.02 | 1.12 | 1.00 | 1.45 | 1.55 | 1.54 |
| 30 | 30 | 7.98 | 7.96 | 1.00 | 1.64 | 1.02 | 1.00 | 1.00 | 1.92 | 8.00 | 4.01 | 1.15 | 8.00 | 2.94 | 2.26 | 2.49 | 1.05 |
| 30 | 60 | 7.97 | 8.00 | 1.01 | 2.00 | 1.05 | 1.01 | 1.00 | 2.31 | 7.86 | 2.50 | 1.56 | 7.01 | 1.74 | 2.10 | 2.35 | 1.28 |
| 30 | 100 | 7.95 | 8.00 | 1.01 | 1.90 | 1.05 | 1.01 | 1.00 | 1.91 | 4.53 | 1.42 | 1.27 | 2.86 | 1.03 | 2.08 | 2.29 | 1.19 |
| 30 | 200 | 7.94 | 8.00 | 1.02 | 2.36 | 1.13 | 1.02 | 1.00 | 2.02 | 2.91 | 1.09 | 1.43 | 2.22 | 1.00 | 2.04 | 2.27 | 1.93 |
| 100 | 30 | 8.00 | 6.34 | 1.00 | 1.93 | 1.02 | 1.00 | 1.00 | 3.22 | 7.47 | 2.44 | 2.38 | 5.97 | 1.91 | 2.13 | 2.37 | 1.19 |
| 100 | 60 | 7.99 | 8.00 | 1.00 | 2.53 | 1.08 | 1.00 | 1.00 | 2.81 | 7.88 | 1.82 | 2.42 | 7.05 | 1.46 | 2.30 | 2.48 | 1.45 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 3.10 | 1.55 | 1.01 | 1.00 | 3.12 | 8.00 | 2.04 | 2.84 | 7.98 | 1.55 | 2.78 | 2.92 | 2.62 |
| 100 | 200 | 8.00 | 8.00 | 1.02 | 3.32 | 2.40 | 1.07 | 1.00 | 3.29 | 4.34 | 2.14 | 3.00 | 3.04 | 1.58 | 3.17 | 3.21 | 3.34 |
| 200 | 30 | 8.00 | 5.75 | 1.00 | 2.17 | 1.04 | 1.00 | 1.00 | 3.53 | 5.60 | 2.12 | 2.84 | 4.25 | 1.76 | 2.04 | 2.32 | 1.30 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 2.94 | 1.44 | 1.01 | 1.00 | 3.37 | 5.78 | 1.97 | 2.93 | 4.14 | 1.61 | 2.46 | 2.64 | 2.10 |
| 200 | 100 | 8.00 | 8.00 | 1.36 | 3.06 | 2.94 | 1.78 | 1.02 | 3.33 | 6.71 | 2.88 | 3.00 | 4.78 | 2.61 | 3.08 | 3.13 | 3.20 |
| 200 | 200 | 8.00 | 8.00 | 2.78 | 3.12 | 3.05 | 2.98 | 2.14 | 3.32 | 7.99 | 3.10 | 3.00 | 7.66 | 2.98 | 3.17 | 3.19 | 3.51 |
| | $r_{mean}$ | 7.98 | 7.75 | 1.15 | 2.70 | 1.45 | 1.19 | 1.07 | 2.64 | 6.20 | 2.41 | 2.03 | 5.13 | 1.90 | 2.32 | 2.49 | 1.80 |
| | $RMSE$ | 4.98 | 4.80 | 1.90 | 0.71 | 1.69 | 1.88 | 1.95 | 0.80 | 3.76 | 1.21 | 1.26 | 3.16 | 1.43 | 0.84 | 0.68 | 1.48 |
| Part V.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 3, 0.5, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.00 | 2.33 | 1.02 | 1.00 | 1.00 | 2.30 | 8.00 | 5.26 | 1.19 | 8.00 | 4.36 | 2.54 | 2.61 | 0.88 |
| 20 | 60 | 7.97 | 8.00 | 1.02 | 2.59 | 1.07 | 1.01 | 1.00 | 1.63 | 7.86 | 2.65 | 1.02 | 6.69 | 1.50 | 1.99 | 2.08 | 0.86 |
| 20 | 100 | 7.96 | 8.00 | 1.01 | 2.74 | 1.04 | 1.01 | 1.00 | 1.42 | 5.34 | 1.65 | 1.01 | 1.85 | 1.04 | 2.53 | 2.64 | 0.53 |
| 20 | 200 | 7.97 | 8.00 | 1.01 | 3.25 | 1.10 | 1.01 | 1.00 | 1.28 | 1.98 | 1.08 | 1.01 | 1.15 | 1.00 | 2.17 | 2.31 | 0.70 |
| 30 | 30 | 7.99 | 7.98 | 1.00 | 1.40 | 1.01 | 1.00 | 1.00 | 1.49 | 8.00 | 3.81 | 1.08 | 8.00 | 2.82 | 2.39 | 2.59 | 0.79 |
| 30 | 60 | 7.98 | 8.00 | 1.00 | 1.47 | 1.02 | 1.00 | 1.00 | 1.70 | 7.96 | 2.14 | 1.12 | 7.48 | 1.35 | 2.27 | 2.52 | 0.72 |
| 30 | 100 | 7.98 | 8.00 | 1.00 | 1.37 | 1.02 | 1.00 | 1.00 | 1.29 | 4.77 | 1.12 | 1.03 | 2.46 | 1.00 | 2.21 | 2.34 | 0.58 |
| 30 | 200 | 7.98 | 8.00 | 1.00 | 1.63 | 1.01 | 1.00 | 1.00 | 1.46 | 3.11 | 1.01 | 1.07 | 1.97 | 1.00 | 2.98 | 3.27 | 0.76 |
| 100 | 30 | 8.00 | 6.66 | 1.00 | 1.49 | 1.00 | 1.00 | 1.00 | 3.02 | 7.68 | 2.23 | 1.92 | 6.79 | 1.57 | 2.36 | 2.61 | 0.80 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 2.29 | 1.00 | 1.00 | 1.00 | 2.61 | 7.97 | 1.48 | 1.94 | 7.71 | 1.11 | 2.52 | 2.82 | 0.68 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 3.21 | 1.01 | 1.00 | 1.00 | 2.48 | 8.00 | 1.21 | 1.75 | 8.00 | 1.01 | 3.10 | 3.43 | 0.90 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 4.31 | 1.05 | 1.00 | 1.00 | 3.78 | 6.17 | 1.05 | 3.07 | 5.02 | 1.00 | 4.27 | 4.54 | 2.90 |
| 200 | 30 | 8.00 | 6.01 | 1.00 | 1.48 | 1.00 | 1.00 | 1.00 | 3.27 | 5.90 | 1.62 | 2.57 | 4.85 | 1.24 | 2.41 | 2.67 | 0.72 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 3.08 | 1.02 | 1.00 | 1.00 | 3.62 | 7.03 | 1.27 | 3.19 | 5.90 | 1.07 | 2.55 | 2.85 | 0.63 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 4.35 | 1.21 | 1.00 | 1.00 | 4.18 | 7.65 | 1.32 | 3.88 | 6.55 | 1.11 | 3.23 | 3.64 | 1.30 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 5.15 | 4.03 | 1.12 | 1.00 | 5.23 | 8.00 | 2.42 | 4.99 | 7.96 | 1.95 | 5.11 | 5.19 | 4.76 |
| | $r_{mean}$ | 7.99 | 7.79 | 1.00 | 2.63 | 1.23 | 1.01 | 1.00 | 2.55 | 6.59 | 1.96 | 1.99 | 5.65 | 1.51 | 2.79 | 3.01 | 1.16 |
| | $RMSE$ | 2.99 | 2.85 | 4.00 | 2.63 | 3.84 | 3.99 | 4.00 | 2.71 | 2.44 | 3.24 | 3.23 | 2.49 | 3.60 | 2.35 | 2.15 | 3.99 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part VI.1 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (1, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.05 | 3.31 | 1.10 | 1.02 | 1.00 | 3.46 | 8.00 | 5.42 | 2.55 | 8.00 | 5.52 | 1.26 | 1.40 | 2.03 |
| 20 | 60 | 7.95 | 8.00 | 1.05 | 3.34 | 1.18 | 1.02 | 1.00 | 2.74 | 7.94 | 3.06 | 1.92 | 7.96 | 2.53 | 1.43 | 1.58 | 2.05 |
| 20 | 100 | 7.95 | 8.00 | 1.05 | 3.90 | 1.24 | 1.02 | 1.00 | 2.48 | 5.04 | 2.34 | 1.90 | 3.49 | 1.95 | 1.78 | 1.95 | 2.17 |
| 20 | 200 | 7.96 | 8.00 | 1.05 | 4.03 | 1.31 | 1.04 | 1.01 | 2.25 | 2.45 | 2.01 | 1.97 | 2.04 | 1.66 | 1.20 | 1.37 | 2.23 |
| 30 | 30 | 7.99 | 7.98 | 1.09 | 2.61 | 1.33 | 1.03 | 1.00 | 3.06 | 8.00 | 4.50 | 2.20 | 8.00 | 3.97 | 1.63 | 1.88 | 2.11 |
| 30 | 60 | 7.99 | 8.00 | 1.20 | 2.76 | 1.72 | 1.10 | 1.00 | 3.15 | 7.99 | 3.06 | 2.36 | 8.00 | 2.51 | 1.61 | 1.82 | 2.46 |
| 30 | 100 | 7.98 | 8.00 | 1.25 | 3.18 | 1.98 | 1.17 | 1.01 | 3.17 | 6.31 | 2.60 | 2.19 | 6.06 | 2.04 | 2.01 | 2.14 | 2.67 |
| 30 | 200 | 7.98 | 8.00 | 1.01 | 3.13 | 1.55 | 1.01 | 1.00 | 3.10 | 3.55 | 2.20 | 2.41 | 2.89 | 2.00 | 2.08 | 2.25 | 2.91 |
| 100 | 30 | 8.00 | 7.75 | 1.02 | 4.01 | 1.15 | 1.01 | 1.01 | 6.60 | 8.00 | 4.03 | 6.11 | 8.00 | 3.69 | 1.13 | 1.21 | 1.77 |
| 100 | 60 | 8.00 | 8.00 | 1.05 | 6.28 | 1.82 | 1.05 | 1.02 | 6.72 | 8.00 | 4.05 | 6.46 | 8.00 | 3.91 | 1.10 | 1.16 | 1.76 |
| 100 | 100 | 8.00 | 8.00 | 1.06 | 7.17 | 3.20 | 1.11 | 1.02 | 6.95 | 8.00 | 4.38 | 6.59 | 8.00 | 3.98 | 1.06 | 1.08 | 1.63 |
| 100 | 200 | 8.00 | 8.00 | 1.19 | 7.82 | 5.42 | 1.61 | 1.02 | 7.79 | 8.00 | 4.97 | 7.42 | 8.00 | 4.50 | 1.05 | 1.08 | 1.93 |
| 200 | 30 | 8.00 | 7.24 | 1.00 | 2.31 | 1.02 | 1.01 | 1.00 | 6.22 | 7.94 | 2.63 | 5.05 | 7.86 | 2.06 | 1.21 | 1.30 | 1.42 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 5.47 | 1.08 | 1.01 | 1.01 | 6.86 | 8.00 | 2.26 | 6.07 | 8.00 | 1.63 | 1.06 | 1.08 | 1.32 |
| 200 | 100 | 8.00 | 8.00 | 1.02 | 7.87 | 1.39 | 1.02 | 1.01 | 7.82 | 8.00 | 2.40 | 7.48 | 8.00 | 1.70 | 1.04 | 1.05 | 1.23 |
| 200 | 200 | 8.00 | 8.00 | 1.02 | 8.00 | 4.19 | 1.06 | 1.02 | 8.00 | 8.00 | 2.96 | 8.00 | 8.00 | 2.48 | 1.04 | 1.05 | 1.17 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.07 | 4.70 | 1.92 | 1.08 | 1.01 | 5.02 | 7.08 | 3.30 | 4.42 | 6.89 | 2.88 | 1.36 | 1.46 | 1.93 |
| | $RMSE$ | 6.99 | 6.94 | 0.10 | 4.20 | 1.53 | 0.16 | 0.01 | 4.57 | 6.32 | 2.53 | 4.13 | 6.23 | 2.21 | 0.50 | 0.61 | 1.05 |
| Part VI.2 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (3, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 7.99 | 1.01 | 4.34 | 1.05 | 1.00 | 1.00 | 5.29 | 8.00 | 5.89 | 3.23 | 8.00 | 5.51 | 2.57 | 2.95 | 1.78 |
| 20 | 60 | 7.97 | 8.00 | 1.01 | 3.25 | 1.06 | 1.01 | 1.00 | 3.44 | 7.98 | 3.68 | 1.59 | 7.99 | 2.79 | 1.56 | 1.77 | 1.44 |
| 20 | 100 | 7.97 | 8.00 | 1.00 | 3.75 | 1.03 | 1.00 | 1.00 | 3.90 | 7.16 | 3.21 | 2.97 | 6.22 | 2.77 | 2.79 | 3.07 | 1.64 |
| 20 | 200 | 7.96 | 8.00 | 1.01 | 4.46 | 1.19 | 1.00 | 1.00 | 3.26 | 3.80 | 2.46 | 2.71 | 3.07 | 1.99 | 2.38 | 2.58 | 2.59 |
| 30 | 30 | 8.00 | 7.99 | 1.01 | 3.57 | 1.07 | 1.00 | 1.00 | 4.52 | 8.00 | 5.22 | 3.29 | 8.00 | 4.75 | 2.54 | 3.01 | 1.67 |
| 30 | 60 | 7.99 | 8.00 | 1.02 | 4.33 | 1.18 | 1.01 | 1.00 | 4.98 | 8.00 | 4.24 | 4.06 | 8.00 | 3.74 | 2.66 | 3.12 | 2.05 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 4.42 | 1.12 | 1.00 | 1.00 | 5.05 | 7.59 | 3.73 | 4.32 | 7.26 | 3.29 | 2.69 | 3.21 | 2.17 |
| 30 | 200 | 7.99 | 8.00 | 1.00 | 4.22 | 1.26 | 1.00 | 1.00 | 5.16 | 5.70 | 3.09 | 4.46 | 5.07 | 2.74 | 3.24 | 3.87 | 3.19 |
| 100 | 30 | 8.00 | 7.90 | 1.00 | 5.00 | 1.07 | 1.00 | 1.00 | 7.47 | 8.00 | 4.45 | 7.13 | 8.00 | 4.03 | 2.89 | 3.41 | 0.92 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.45 | 1.51 | 1.00 | 1.00 | 7.71 | 8.00 | 4.48 | 7.45 | 8.00 | 4.01 | 3.30 | 3.80 | 0.57 |
| 100 | 100 | 8.00 | 8.00 | 1.04 | 7.95 | 4.04 | 1.08 | 1.00 | 7.92 | 8.00 | 5.09 | 7.89 | 8.00 | 4.80 | 4.06 | 4.73 | 0.44 |
| 100 | 200 | 8.00 | 8.00 | 1.15 | 8.00 | 6.60 | 1.64 | 1.00 | 8.00 | 8.00 | 5.54 | 8.00 | 8.00 | 4.91 | 4.12 | 4.83 | 0.22 |
| 200 | 30 | 8.00 | 7.49 | 1.00 | 3.32 | 1.05 | 1.00 | 1.00 | 7.08 | 7.99 | 3.14 | 6.38 | 7.96 | 2.63 | 2.21 | 2.55 | 1.04 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.75 | 1.60 | 1.01 | 1.00 | 7.64 | 8.00 | 3.12 | 7.29 | 8.00 | 2.60 | 2.38 | 2.72 | 0.73 |
| 200 | 100 | 8.00 | 8.00 | 1.39 | 7.98 | 3.30 | 1.89 | 1.04 | 7.98 | 8.00 | 3.61 | 7.96 | 8.00 | 3.16 | 2.69 | 2.91 | 1.50 |
| 200 | 200 | 8.00 | 8.00 | 2.90 | 8.00 | 5.92 | 3.08 | 2.32 | 8.00 | 8.00 | 3.97 | 8.00 | 8.00 | 3.47 | 2.95 | 3.01 | 2.34 |
| | $r_{mean}$ | 7.99 | 7.96 | 1.16 | 5.42 | 2.13 | 1.23 | 1.09 | 6.09 | 7.51 | 4.06 | 5.42 | 7.35 | 3.57 | 2.81 | 3.22 | 1.52 |
| | $RMSE$ | 4.99 | 4.96 | 1.90 | 3.04 | 1.98 | 1.85 | 1.94 | 3.54 | 4.65 | 1.43 | 3.28 | 4.56 | 1.14 | 0.65 | 0.79 | 1.69 |
| Part VI.3 :$(r^o, \sigma^2_\lambda, \alpha, \beta) = (5, 3, 0.5, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.01 | 3.00 | 1.02 | 1.00 | 1.00 | 4.33 | 8.00 | 5.81 | 2.37 | 8.00 | 5.37 | 2.34 | 2.59 | 1.19 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 2.84 | 1.06 | 1.00 | 1.00 | 3.48 | 8.00 | 3.70 | 1.72 | 7.99 | 2.97 | 1.77 | 1.96 | 1.23 |
| 20 | 100 | 7.98 | 8.00 | 1.00 | 2.97 | 1.03 | 1.00 | 1.00 | 3.96 | 7.75 | 3.06 | 1.81 | 7.08 | 2.16 | 2.47 | 2.73 | 0.85 |
| 20 | 200 | 7.98 | 8.00 | 1.00 | 3.62 | 1.04 | 1.00 | 1.00 | 4.38 | 5.55 | 2.63 | 3.07 | 4.35 | 1.81 | 3.33 | 3.60 | 1.40 |
| 30 | 30 | 8.00 | 8.00 | 1.00 | 2.52 | 1.03 | 1.00 | 1.00 | 3.30 | 8.00 | 5.07 | 2.15 | 8.00 | 4.61 | 1.95 | 2.31 | 1.35 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 3.28 | 1.02 | 1.00 | 1.00 | 5.15 | 8.00 | 4.15 | 4.02 | 8.00 | 3.61 | 2.01 | 2.40 | 1.20 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 2.79 | 1.01 | 1.00 | 1.00 | 4.47 | 7.91 | 3.01 | 3.19 | 7.81 | 2.34 | 1.53 | 1.82 | 1.28 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 3.12 | 1.02 | 1.00 | 1.00 | 6.33 | 7.42 | 2.83 | 5.19 | 7.07 | 2.14 | 1.71 | 2.19 | 1.73 |
| 100 | 30 | 8.00 | 7.94 | 1.00 | 4.09 | 1.01 | 1.00 | 1.00 | 7.50 | 8.00 | 4.13 | 6.96 | 8.00 | 3.58 | 2.78 | 3.14 | 0.74 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.47 | 1.11 | 1.00 | 1.00 | 7.79 | 8.00 | 4.06 | 7.51 | 8.00 | 3.50 | 3.05 | 3.48 | 0.38 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 7.97 | 1.66 | 1.00 | 1.00 | 7.97 | 8.00 | 4.15 | 7.90 | 8.00 | 3.59 | 3.14 | 3.50 | 0.28 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 4.73 | 1.00 | 1.00 | 8.00 | 8.00 | 4.60 | 8.00 | 8.00 | 4.17 | 3.48 | 3.69 | 0.11 |
| 200 | 30 | 8.00 | 7.51 | 1.00 | 2.50 | 1.00 | 1.00 | 1.00 | 6.97 | 8.00 | 2.74 | 6.00 | 7.97 | 2.16 | 2.63 | 2.93 | 0.59 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.82 | 1.05 | 1.00 | 1.00 | 7.71 | 8.00 | 2.54 | 7.35 | 8.00 | 1.88 | 2.69 | 2.97 | 0.37 |
| 200 | 100 | 8.00 | 8.00 | 1.00 | 7.99 | 1.77 | 1.00 | 1.00 | 7.99 | 8.00 | 2.86 | 7.96 | 8.00 | 2.25 | 2.53 | 2.85 | 0.30 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 8.00 | 6.76 | 1.29 | 1.00 | 8.00 | 8.00 | 4.26 | 8.00 | 8.00 | 3.60 | 3.18 | 3.46 | 0.20 |
| | $r_{mean}$ | 7.99 | 7.97 | 1.00 | 4.81 | 1.71 | 1.02 | 1.00 | 6.08 | 7.79 | 3.73 | 5.20 | 7.64 | 3.11 | 2.54 | 2.85 | 0.82 |
| | $RMSE$ | 2.99 | 2.97 | 4.00 | 2.30 | 3.65 | 3.98 | 4.00 | 2.09 | 2.85 | 1.58 | 2.45 | 2.79 | 2.15 | 2.53 | 2.23 | 4.21 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part VII.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.05 | 2.68 | 1.11 | 1.03 | 1.01 | 2.14 | 8.00 | 5.01 | 1.09 | 8.00 | 4.10 | 1.46 | 1.57 | 1.42 |
| 20 | 60 | 7.95 | 8.00 | 1.11 | 3.05 | 1.24 | 1.07 | 1.01 | 1.67 | 7.72 | 2.36 | 1.01 | 5.65 | 1.29 | 1.24 | 1.33 | 1.41 |
| 20 | 100 | 7.93 | 8.00 | 1.15 | 3.48 | 1.34 | 1.12 | 1.04 | 1.39 | 2.67 | 1.41 | 1.00 | 1.28 | 1.01 | 1.10 | 1.13 | 1.42 |
| 20 | 200 | 7.96 | 8.00 | 1.25 | 4.32 | 1.57 | 1.23 | 1.08 | 1.26 | 1.38 | 1.14 | 1.00 | 1.00 | 1.00 | 1.11 | 1.14 | 1.47 |
| 30 | 30 | 7.98 | 7.96 | 1.01 | 1.34 | 1.05 | 1.01 | 1.00 | 1.50 | 8.00 | 3.43 | 1.01 | 8.00 | 2.08 | 1.56 | 1.71 | 1.27 |
| 30 | 60 | 7.95 | 8.00 | 1.06 | 1.54 | 1.11 | 1.04 | 1.02 | 1.47 | 7.57 | 1.64 | 1.01 | 5.70 | 1.02 | 1.15 | 1.20 | 1.47 |
| 30 | 100 | 7.93 | 8.00 | 1.09 | 1.73 | 1.19 | 1.07 | 1.03 | 1.35 | 2.53 | 1.26 | 1.00 | 1.12 | 1.00 | 1.14 | 1.17 | 1.47 |
| 30 | 200 | 7.92 | 8.00 | 1.13 | 2.17 | 1.36 | 1.14 | 1.05 | 1.25 | 1.41 | 1.11 | 1.00 | 1.00 | 1.00 | 1.13 | 1.15 | 1.49 |
| 100 | 30 | 7.99 | 5.81 | 1.01 | 1.08 | 1.02 | 1.01 | 1.00 | 1.88 | 6.22 | 1.59 | 1.10 | 3.61 | 1.02 | 1.24 | 1.31 | 1.34 |
| 100 | 60 | 7.99 | 8.00 | 1.02 | 1.06 | 1.03 | 1.02 | 1.01 | 1.44 | 7.28 | 1.22 | 1.00 | 5.34 | 1.00 | 1.13 | 1.17 | 1.44 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 1.11 | 1.02 | 1.02 | 1.01 | 1.29 | 8.00 | 1.14 | 1.00 | 7.84 | 1.00 | 1.10 | 1.11 | 1.47 |
| 100 | 200 | 8.00 | 8.00 | 1.02 | 1.30 | 1.09 | 1.03 | 1.01 | 1.28 | 2.36 | 1.11 | 1.00 | 1.03 | 1.00 | 1.10 | 1.11 | 1.56 |
| 200 | 30 | 7.99 | 5.04 | 1.01 | 1.04 | 1.02 | 1.01 | 1.00 | 1.95 | 3.53 | 1.28 | 1.13 | 2.07 | 1.00 | 1.21 | 1.26 | 1.35 |
| 200 | 60 | 8.00 | 7.99 | 1.01 | 1.03 | 1.02 | 1.01 | 1.01 | 1.46 | 3.75 | 1.12 | 1.00 | 2.11 | 1.00 | 1.07 | 1.09 | 1.37 |
| 200 | 100 | 8.00 | 8.00 | 1.02 | 1.03 | 1.02 | 1.02 | 1.02 | 1.36 | 4.87 | 1.12 | 1.00 | 2.75 | 1.00 | 1.08 | 1.09 | 1.42 |
| 200 | 200 | 8.00 | 8.00 | 1.01 | 1.14 | 1.02 | 1.01 | 1.01 | 1.31 | 7.85 | 1.12 | 1.00 | 6.72 | 1.00 | 1.08 | 1.10 | 1.58 |
| | $r_{mean}$ | 7.97 | 7.67 | 1.06 | 1.82 | 1.14 | 1.05 | 1.02 | 1.50 | 5.20 | 1.69 | 1.02 | 3.95 | 1.28 | 1.18 | 1.23 | 1.43 |
| | $RMSE$ | 6.97 | 6.73 | 0.09 | 1.29 | 0.21 | 0.08 | 0.03 | 0.56 | 4.91 | 1.25 | 0.05 | 3.97 | 0.82 | 0.23 | 0.29 | 0.44 |
| Part VII.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.02 | 2.84 | 1.05 | 1.01 | 1.00 | 2.28 | 8.00 | 5.13 | 1.14 | 8.00 | 3.94 | 2.15 | 2.27 | 1.07 |
| 20 | 60 | 7.96 | 8.00 | 1.05 | 3.23 | 1.14 | 1.02 | 1.00 | 2.02 | 7.83 | 2.76 | 1.12 | 6.31 | 1.66 | 1.68 | 1.82 | 1.24 |
| 20 | 100 | 7.97 | 8.00 | 1.02 | 3.88 | 1.14 | 1.01 | 1.00 | 1.99 | 4.96 | 2.03 | 1.23 | 2.09 | 1.25 | 1.88 | 2.03 | 1.18 |
| 20 | 200 | 7.95 | 8.00 | 1.03 | 4.16 | 1.27 | 1.03 | 1.00 | 1.67 | 2.23 | 1.19 | 1.15 | 1.42 | 1.00 | 1.85 | 2.00 | 1.52 |
| 30 | 30 | 7.97 | 7.96 | 1.01 | 1.38 | 1.02 | 1.00 | 1.00 | 1.49 | 8.00 | 3.56 | 1.05 | 8.00 | 2.33 | 1.97 | 2.16 | 1.02 |
| 30 | 60 | 7.96 | 8.00 | 1.02 | 1.67 | 1.05 | 1.01 | 1.00 | 1.85 | 7.87 | 2.06 | 1.16 | 6.43 | 1.28 | 1.61 | 1.75 | 1.32 |
| 30 | 100 | 7.96 | 8.00 | 1.02 | 1.93 | 1.08 | 1.01 | 1.00 | 1.91 | 4.25 | 1.52 | 1.28 | 2.52 | 1.07 | 1.76 | 1.92 | 1.36 |
| 30 | 200 | 7.95 | 8.00 | 1.01 | 2.84 | 1.18 | 1.02 | 1.00 | 2.42 | 3.04 | 1.34 | 1.96 | 2.53 | 1.04 | 2.10 | 2.29 | 2.32 |
| 100 | 30 | 8.00 | 6.03 | 1.00 | 1.50 | 1.02 | 1.00 | 1.00 | 2.44 | 6.89 | 1.96 | 1.72 | 4.72 | 1.43 | 1.67 | 1.82 | 1.32 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 2.17 | 1.16 | 1.01 | 1.00 | 2.44 | 7.78 | 1.72 | 2.04 | 6.59 | 1.37 | 1.73 | 1.84 | 1.74 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 2.56 | 1.22 | 1.01 | 1.00 | 2.42 | 8.00 | 1.50 | 2.04 | 7.96 | 1.20 | 1.98 | 2.13 | 2.04 |
| 100 | 200 | 8.00 | 8.00 | 1.07 | 3.29 | 2.16 | 1.18 | 1.02 | 3.20 | 4.26 | 1.99 | 2.94 | 3.04 | 1.65 | 2.73 | 2.84 | 3.20 |
| 200 | 30 | 8.00 | 5.28 | 1.00 | 1.56 | 1.02 | 1.00 | 1.00 | 2.69 | 4.48 | 1.61 | 2.03 | 3.20 | 1.26 | 1.67 | 1.82 | 1.37 |
| 200 | 60 | 8.00 | 8.00 | 1.02 | 2.51 | 1.29 | 1.04 | 1.00 | 2.94 | 5.31 | 1.68 | 2.52 | 3.83 | 1.36 | 1.82 | 1.94 | 1.98 |
| 200 | 100 | 8.00 | 8.00 | 1.15 | 3.02 | 2.13 | 1.28 | 1.03 | 3.24 | 6.33 | 2.04 | 2.92 | 4.46 | 1.77 | 2.29 | 2.42 | 2.90 |
| 200 | 200 | 8.00 | 8.00 | 1.86 | 3.14 | 2.98 | 2.35 | 1.43 | 3.28 | 7.98 | 2.70 | 3.00 | 7.53 | 2.41 | 3.02 | 3.08 | 3.46 |
| | $r_{mean}$ | 7.98 | 7.70 | 1.08 | 2.61 | 1.37 | 1.12 | 1.03 | 2.39 | 6.08 | 2.17 | 1.83 | 4.91 | 1.63 | 1.99 | 2.13 | 1.82 |
| | $RMSE$ | 4.98 | 4.77 | 1.93 | 0.91 | 1.72 | 1.90 | 1.97 | 0.82 | 3.63 | 1.27 | 1.36 | 2.96 | 1.55 | 1.08 | 0.94 | 1.40 |
| Part VII.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 3, 0.85, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.98 | 1.01 | 2.51 | 1.04 | 1.00 | 1.00 | 2.14 | 8.00 | 5.05 | 1.13 | 8.00 | 3.92 | 2.29 | 2.34 | 1.01 |
| 20 | 60 | 7.96 | 8.00 | 1.02 | 2.81 | 1.07 | 1.01 | 1.00 | 1.82 | 7.89 | 2.78 | 1.12 | 6.58 | 1.72 | 1.97 | 2.06 | 0.92 |
| 20 | 100 | 7.95 | 8.00 | 1.01 | 3.24 | 1.07 | 1.01 | 1.00 | 1.47 | 4.69 | 1.61 | 1.03 | 1.63 | 1.05 | 2.01 | 2.13 | 0.68 |
| 20 | 200 | 7.97 | 8.00 | 1.03 | 3.28 | 1.13 | 1.02 | 1.01 | 1.35 | 1.83 | 1.13 | 1.04 | 1.15 | 1.00 | 1.59 | 1.70 | 0.99 |
| 30 | 30 | 7.97 | 7.96 | 1.01 | 1.49 | 1.02 | 1.00 | 1.00 | 1.68 | 8.00 | 3.76 | 1.11 | 8.00 | 2.65 | 2.00 | 2.22 | 1.18 |
| 30 | 60 | 7.98 | 8.00 | 1.01 | 1.57 | 1.04 | 1.01 | 1.00 | 1.73 | 7.89 | 2.05 | 1.17 | 6.83 | 1.30 | 1.91 | 2.10 | 0.99 |
| 30 | 100 | 7.97 | 8.00 | 1.01 | 1.67 | 1.04 | 1.01 | 1.00 | 1.74 | 5.01 | 1.40 | 1.17 | 2.94 | 1.03 | 2.05 | 2.27 | 0.90 |
| 30 | 200 | 7.97 | 8.00 | 1.01 | 1.87 | 1.03 | 1.01 | 1.00 | 1.62 | 3.04 | 1.05 | 1.14 | 1.94 | 1.00 | 2.30 | 2.57 | 1.00 |
| 100 | 30 | 8.00 | 6.08 | 1.00 | 1.40 | 1.00 | 1.00 | 1.00 | 2.34 | 6.91 | 1.90 | 1.61 | 4.82 | 1.32 | 1.78 | 2.00 | 1.06 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 2.06 | 1.04 | 1.00 | 1.00 | 2.35 | 7.90 | 1.56 | 1.80 | 7.19 | 1.18 | 1.85 | 2.02 | 1.19 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 2.53 | 1.06 | 1.00 | 1.00 | 2.22 | 8.00 | 1.30 | 1.75 | 7.99 | 1.06 | 1.99 | 2.24 | 1.09 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 3.72 | 1.22 | 1.01 | 1.00 | 3.43 | 6.00 | 1.29 | 2.96 | 4.95 | 1.06 | 2.75 | 3.10 | 2.17 |
| 200 | 30 | 8.00 | 5.46 | 1.00 | 1.65 | 1.01 | 1.00 | 1.00 | 2.88 | 4.76 | 1.66 | 2.27 | 3.59 | 1.33 | 1.75 | 1.92 | 1.25 |
| 200 | 60 | 8.00 | 7.99 | 1.00 | 2.58 | 1.14 | 1.00 | 1.00 | 3.09 | 6.03 | 1.49 | 2.61 | 4.78 | 1.23 | 1.79 | 1.98 | 1.27 |
| 200 | 100 | 8.00 | 8.00 | 1.01 | 3.61 | 1.51 | 1.02 | 1.00 | 3.61 | 7.44 | 1.59 | 3.20 | 6.07 | 1.31 | 2.15 | 2.38 | 1.48 |
| 200 | 200 | 8.00 | 8.00 | 1.04 | 4.80 | 2.50 | 1.15 | 1.01 | 4.29 | 8.00 | 1.83 | 3.94 | 7.93 | 1.45 | 3.53 | 3.83 | 3.40 |
| | $r_{mean}$ | 7.98 | 7.72 | 1.01 | 2.55 | 1.18 | 1.02 | 1.00 | 2.36 | 6.34 | 1.97 | 1.82 | 5.27 | 1.48 | 2.11 | 2.30 | 1.29 |
| | $RMSE$ | 2.98 | 2.82 | 3.99 | 2.63 | 3.83 | 3.98 | 4.00 | 2.77 | 2.33 | 3.20 | 3.31 | 2.36 | 3.60 | 2.93 | 2.74 | 3.77 |

Part VIII.1 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (1, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.95 | 7.99 | 1.03 | 3.40 | 1.09 | 1.01 | 1.00 | 3.42 | 8.00 | 5.43 | 1.92 | 8.00 | 5.09 | 1.62 | 1.79 | 1.85 |
| 20 | 60 | 7.96 | 8.00 | 1.05 | 3.60 | 1.17 | 1.02 | 1.00 | 2.76 | 7.96 | 3.10 | 1.83 | 7.91 | 2.54 | 1.45 | 1.61 | 2.02 |
| 20 | 100 | 7.95 | 8.00 | 1.06 | 3.64 | 1.25 | 1.03 | 1.01 | 2.42 | 4.90 | 2.29 | 1.82 | 5.09 | 1.94 | 1.29 | 1.43 | 2.14 |
| 20 | 200 | 7.95 | 8.00 | 1.07 | 4.35 | 1.41 | 1.05 | 1.01 | 2.27 | 2.44 | 2.09 | 1.97 | 2.00 | 1.50 | 1.88 | 1.98 | 2.26 |
| 30 | 30 | 7.99 | 7.98 | 1.06 | 2.59 | 1.23 | 1.01 | 1.00 | 3.02 | 8.00 | 4.44 | 2.00 | 8.00 | 3.83 | 1.74 | 2.05 | 2.12 |
| 30 | 60 | 7.99 | 8.00 | 1.15 | 2.86 | 1.66 | 1.08 | 1.01 | 3.17 | 7.99 | 3.07 | 2.38 | 8.00 | 2.47 | 1.83 | 2.04 | 2.38 |
| 30 | 100 | 7.98 | 8.00 | 1.22 | 3.10 | 1.88 | 1.13 | 1.01 | 3.21 | 6.32 | 2.61 | 2.48 | 6.46 | 2.13 | 1.98 | 2.16 | 2.66 |
| 30 | 200 | 7.97 | 8.00 | 1.21 | 3.21 | 2.06 | 1.22 | 1.00 | 3.13 | 3.54 | 2.21 | 2.40 | 2.96 | 2.00 | 1.95 | 2.10 | 2.98 |
| 100 | 30 | 8.00 | 7.73 | 1.01 | 3.85 | 1.10 | 1.01 | 1.00 | 6.50 | 8.00 | 3.93 | 5.84 | 8.00 | 3.51 | 1.86 | 2.12 | 1.38 |
| 100 | 60 | 8.00 | 8.00 | 1.05 | 6.18 | 1.82 | 1.05 | 1.01 | 6.64 | 8.00 | 4.06 | 6.26 | 8.00 | 3.81 | 1.49 | 1.72 | 1.57 |
| 100 | 100 | 8.00 | 8.00 | 1.08 | 7.19 | 3.30 | 1.13 | 1.02 | 6.94 | 8.00 | 4.42 | 6.54 | 8.00 | 4.13 | 1.16 | 1.35 | 1.70 |
| 100 | 200 | 8.00 | 8.00 | 1.23 | 7.76 | 5.34 | 1.64 | 1.03 | 7.78 | 8.00 | 4.92 | 7.58 | 8.00 | 4.68 | 1.03 | 1.04 | 1.94 |
| 200 | 30 | 8.00 | 7.17 | 1.00 | 2.14 | 1.02 | 1.00 | 1.00 | 5.85 | 7.89 | 2.51 | 4.62 | 7.75 | 1.89 | 1.39 | 1.50 | 1.28 |
| 200 | 60 | 8.00 | 8.00 | 1.01 | 5.43 | 1.06 | 1.01 | 1.01 | 6.80 | 8.00 | 2.26 | 6.04 | 8.00 | 1.66 | 1.09 | 1.11 | 1.28 |
| 200 | 100 | 8.00 | 8.00 | 1.02 | 7.84 | 1.38 | 1.03 | 1.02 | 7.79 | 8.00 | 2.41 | 7.54 | 8.00 | 1.71 | 1.09 | 1.10 | 1.23 |
| 200 | 200 | 8.00 | 8.00 | 1.02 | 8.00 | 4.22 | 1.06 | 1.01 | 8.00 | 8.00 | 2.93 | 8.00 | 8.00 | 2.31 | 1.03 | 1.04 | 1.19 |
| | $r_{mean}$ | 7.98 | 7.93 | 1.08 | 4.70 | 1.94 | 1.09 | 1.01 | 4.98 | 7.07 | 3.29 | 4.33 | 7.01 | 2.83 | 1.49 | 1.63 | 1.87 |
| | $RMSE$ | 6.98 | 6.93 | 0.11 | 4.20 | 1.54 | 0.18 | 0.01 | 4.52 | 6.31 | 2.52 | 4.07 | 6.30 | 2.15 | 0.60 | 0.75 | 1.02 |
| Part VIII.2 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (3, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 7.99 | 1.01 | 3.75 | 1.08 | 1.00 | 1.00 | 4.52 | 8.00 | 5.67 | 3.05 | 8.00 | 5.33 | 2.12 | 2.41 | 1.73 |
| 20 | 60 | 7.97 | 8.00 | 1.01 | 4.01 | 1.09 | 1.00 | 1.00 | 4.04 | 7.99 | 3.83 | 2.96 | 7.98 | 3.27 | 2.16 | 2.39 | 1.94 |
| 20 | 100 | 7.98 | 8.00 | 1.01 | 3.81 | 1.08 | 1.00 | 1.00 | 3.76 | 7.11 | 3.12 | 2.76 | 6.27 | 2.55 | 2.16 | 2.48 | 1.75 |
| 20 | 200 | 7.95 | 8.00 | 1.00 | 3.69 | 1.07 | 1.00 | 1.00 | 3.17 | 4.01 | 2.08 | 2.16 | 3.08 | 1.29 | 2.49 | 2.73 | 1.75 |
| 30 | 30 | 7.99 | 7.98 | 1.02 | 3.02 | 1.13 | 1.01 | 1.00 | 3.66 | 8.00 | 4.86 | 2.67 | 8.00 | 4.40 | 1.89 | 2.21 | 1.89 |
| 30 | 60 | 8.00 | 8.00 | 1.02 | 3.15 | 1.23 | 1.01 | 1.00 | 4.02 | 8.00 | 3.62 | 2.94 | 8.00 | 2.98 | 1.93 | 2.22 | 1.96 |
| 30 | 100 | 7.99 | 8.00 | 1.03 | 3.65 | 1.32 | 1.01 | 1.00 | 4.33 | 7.46 | 3.26 | 3.58 | 7.33 | 2.75 | 2.04 | 2.33 | 2.27 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 4.23 | 1.14 | 1.00 | 1.00 | 4.86 | 5.58 | 3.00 | 3.81 | 4.76 | 2.37 | 2.51 | 3.08 | 2.91 |
| 100 | 30 | 8.00 | 7.83 | 1.00 | 4.16 | 1.11 | 1.00 | 1.00 | 6.99 | 8.00 | 4.10 | 6.40 | 8.00 | 3.69 | 2.17 | 2.54 | 1.16 |
| 100 | 60 | 8.00 | 8.00 | 1.03 | 7.06 | 1.78 | 1.03 | 1.00 | 7.40 | 8.00 | 4.27 | 7.04 | 8.00 | 3.84 | 2.31 | 2.73 | 0.94 |
| 100 | 100 | 8.00 | 8.00 | 1.14 | 7.95 | 3.85 | 1.20 | 1.01 | 7.90 | 8.00 | 5.00 | 7.81 | 8.00 | 4.67 | 2.88 | 3.48 | 0.73 |
| 100 | 200 | 8.00 | 8.00 | 1.14 | 8.00 | 6.67 | 1.50 | 1.02 | 8.00 | 8.00 | 5.57 | 8.00 | 8.00 | 4.93 | 3.45 | 4.00 | 0.49 |
| 200 | 30 | 8.00 | 7.30 | 1.00 | 2.66 | 1.08 | 1.00 | 1.00 | 6.43 | 7.96 | 2.77 | 5.61 | 7.92 | 2.26 | 1.84 | 2.07 | 1.09 |
| 200 | 60 | 8.00 | 8.00 | 1.02 | 6.33 | 1.46 | 1.04 | 1.00 | 7.41 | 8.00 | 2.77 | 6.85 | 8.00 | 2.17 | 1.76 | 1.94 | 1.10 |
| 200 | 100 | 8.00 | 8.00 | 1.15 | 7.97 | 2.49 | 1.29 | 1.03 | 7.95 | 8.00 | 3.14 | 7.90 | 8.00 | 2.59 | 1.76 | 1.93 | 1.18 |
| 200 | 200 | 8.00 | 8.00 | 2.06 | 8.00 | 5.61 | 2.64 | 1.55 | 8.00 | 8.00 | 3.82 | 8.00 | 8.00 | 3.32 | 2.13 | 2.27 | 1.73 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.10 | 5.09 | 2.07 | 1.17 | 1.04 | 5.78 | 7.51 | 3.81 | 5.10 | 7.33 | 3.28 | 2.23 | 2.55 | 1.54 |
| | $RMSE$ | 4.99 | 4.95 | 1.91 | 2.88 | 1.94 | 1.87 | 1.97 | 3.31 | 4.64 | 1.29 | 3.05 | 4.55 | 1.13 | 0.89 | 0.70 | 1.58 |
| Part VIII.3 :$(r^o, \sigma_\lambda^2, \alpha, \beta) = (5, 3, 0.85, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.02 | 3.01 | 1.03 | 1.00 | 1.00 | 3.92 | 8.00 | 5.62 | 1.99 | 8.00 | 5.07 | 2.29 | 2.51 | 1.36 |
| 20 | 60 | 7.98 | 8.00 | 1.01 | 3.42 | 1.05 | 1.00 | 1.00 | 4.45 | 8.00 | 4.03 | 2.60 | 7.99 | 3.40 | 2.27 | 2.54 | 1.24 |
| 20 | 100 | 7.97 | 8.00 | 1.01 | 3.55 | 1.08 | 1.01 | 1.00 | 3.92 | 7.47 | 3.12 | 2.38 | 7.09 | 2.40 | 2.19 | 2.50 | 1.40 |
| 20 | 200 | 7.97 | 8.00 | 1.01 | 2.89 | 1.07 | 1.01 | 1.00 | 2.46 | 3.76 | 1.80 | 1.45 | 2.10 | 1.14 | 1.91 | 2.08 | 1.24 |
| 30 | 30 | 7.99 | 7.99 | 1.01 | 2.75 | 1.08 | 1.00 | 1.00 | 3.43 | 8.00 | 4.85 | 2.37 | 8.00 | 4.35 | 1.98 | 2.31 | 1.65 |
| 30 | 60 | 8.00 | 8.00 | 1.01 | 2.77 | 1.07 | 1.01 | 1.00 | 3.88 | 8.00 | 3.54 | 2.61 | 7.99 | 2.84 | 1.62 | 1.87 | 1.63 |
| 30 | 100 | 8.00 | 8.00 | 1.01 | 3.75 | 1.11 | 1.01 | 1.00 | 5.14 | 7.89 | 3.44 | 4.19 | 7.83 | 2.91 | 2.13 | 2.50 | 1.53 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 3.69 | 1.07 | 1.00 | 1.00 | 5.26 | 6.78 | 2.96 | 4.28 | 6.10 | 2.26 | 2.18 | 2.64 | 1.47 |
| 100 | 30 | 8.00 | 7.82 | 1.00 | 3.96 | 1.07 | 1.00 | 1.00 | 6.92 | 8.00 | 3.99 | 6.32 | 8.00 | 3.56 | 2.27 | 2.64 | 1.01 |
| 100 | 60 | 8.00 | 8.00 | 1.01 | 7.05 | 1.51 | 1.01 | 1.00 | 7.42 | 8.00 | 4.04 | 7.05 | 8.00 | 3.56 | 2.45 | 2.82 | 0.73 |
| 100 | 100 | 8.00 | 8.00 | 1.01 | 7.91 | 2.44 | 1.02 | 1.00 | 7.83 | 8.00 | 4.37 | 7.60 | 8.00 | 3.84 | 3.01 | 3.46 | 0.41 |
| 100 | 200 | 8.00 | 8.00 | 1.00 | 8.00 | 4.84 | 1.04 | 1.00 | 8.00 | 8.00 | 4.66 | 8.00 | 8.00 | 4.07 | 3.35 | 3.82 | 0.21 |
| 200 | 30 | 8.00 | 7.23 | 1.00 | 2.50 | 1.02 | 1.00 | 1.00 | 6.27 | 7.93 | 2.68 | 5.09 | 7.86 | 2.12 | 2.01 | 2.26 | 0.89 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 6.34 | 1.30 | 1.01 | 1.00 | 7.38 | 8.00 | 2.69 | 6.86 | 8.00 | 2.11 | 2.04 | 2.25 | 0.83 |
| 200 | 100 | 8.00 | 8.00 | 1.03 | 7.97 | 2.42 | 1.10 | 1.00 | 7.98 | 8.00 | 3.15 | 7.92 | 8.00 | 2.58 | 1.94 | 2.17 | 0.73 |
| 200 | 200 | 8.00 | 8.00 | 1.27 | 8.00 | 6.07 | 1.86 | 1.05 | 8.00 | 8.00 | 3.98 | 8.00 | 8.00 | 3.50 | 2.09 | 2.27 | 0.54 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.03 | 4.85 | 1.83 | 1.07 | 1.00 | 5.77 | 7.61 | 3.68 | 4.92 | 7.44 | 3.11 | 2.23 | 2.54 | 1.05 |
| | $RMSE$ | 2.99 | 2.95 | 3.98 | 2.16 | 3.49 | 3.94 | 4.00 | 2.01 | 2.81 | 1.61 | 2.38 | 2.84 | 2.13 | 2.80 | 2.51 | 3.97 |

Table 4: Dynamic factors and heteroskedastic idiosyncratic errors

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 :$(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (1, 1, 1, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.96 | 7.99 | 1.02 | 2.05 | 1.13 | 1.00 | 1.00 | 2.29 | 8.00 | 4.48 | 2.36 | 8.00 | 5.01 | 1.82 | 1.92 | 2.13 |
| 20 | 60 | 7.94 | 8.00 | 1.00 | 1.83 | 1.01 | 1.00 | 1.00 | 2.06 | 6.59 | 2.16 | 2.00 | 7.85 | 2.36 | 1.87 | 1.94 | 2.08 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 2.01 | 1.12 | 1.00 | 1.00 | 2.00 | 2.19 | 2.00 | 2.02 | 4.04 | 2.01 | 1.82 | 1.96 | 2.06 |
| 20 | 200 | 7.91 | 8.00 | 1.03 | 2.01 | 1.95 | 1.02 | 1.00 | 2.00 | 2.00 | 2.00 | 2.01 | 2.14 | 2.00 | 2.00 | 2.00 | 2.04 |
| 30 | 30 | 7.97 | 7.96 | 1.12 | 2.04 | 1.41 | 1.03 | 1.00 | 2.07 | 8.00 | 3.09 | 2.02 | 8.00 | 3.24 | 1.89 | 1.95 | 2.11 |
| 30 | 60 | 7.95 | 8.00 | 1.53 | 2.02 | 1.92 | 1.34 | 1.02 | 2.03 | 6.36 | 2.02 | 2.02 | 7.63 | 2.02 | 2.00 | 2.00 | 2.11 |
| 30 | 100 | 7.92 | 8.00 | 1.57 | 2.00 | 1.98 | 1.44 | 1.01 | 2.00 | 2.18 | 2.00 | 2.00 | 2.94 | 2.00 | 2.00 | 2.00 | 2.08 |
| 30 | 200 | 7.79 | 8.00 | 1.87 | 2.00 | 2.00 | 1.88 | 1.09 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.05 |
| 100 | 30 | 8.00 | 6.41 | 1.51 | 2.06 | 1.88 | 1.43 | 1.07 | 2.31 | 5.96 | 2.04 | 2.19 | 6.09 | 2.02 | 1.98 | 1.99 | 2.16 |
| 100 | 60 | 8.00 | 8.00 | 1.99 | 2.01 | 2.00 | 1.99 | 1.83 | 2.01 | 6.65 | 2.00 | 2.01 | 7.20 | 2.00 | 2.00 | 2.00 | 2.12 |
| 100 | 100 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 7.92 | 2.00 | 2.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.07 |
| 100 | 200 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.05 | 2.00 | 2.00 | 2.14 | 2.00 | 2.00 | 2.00 | 2.06 |
| 200 | 30 | 8.00 | 5.83 | 1.78 | 2.02 | 1.98 | 1.79 | 1.37 | 2.46 | 4.03 | 2.00 | 2.29 | 3.92 | 2.00 | 2.00 | 2.00 | 2.11 |
| 200 | 60 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.98 | 2.02 | 3.65 | 2.00 | 2.01 | 3.65 | 2.00 | 2.00 | 2.00 | 2.04 |
| 200 | 100 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 3.97 | 2.00 | 2.00 | 4.40 | 2.00 | 2.00 | 2.00 | 2.06 |
| 200 | 200 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 7.14 | 2.00 | 2.00 | 7.86 | 2.00 | 2.00 | 2.00 | 2.03 |
| | $r_{mean}$ | 7.96 | 7.76 | 1.59 | 2.00 | 1.77 | 1.56 | 1.40 | 2.08 | 4.92 | 2.24 | 2.06 | 5.37 | 2.29 | 1.96 | 1.99 | 2.08 |
| | $RMSE$ | 5.96 | 5.80 | 0.58 | 0.05 | 0.42 | 0.61 | 0.75 | 0.16 | 3.72 | 0.68 | 0.13 | 4.10 | 0.82 | 0.08 | 0.03 | 0.09 |
| Part I.2 : $(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (3, 1, 1, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.98 | 8.00 | 1.00 | 1.39 | 1.00 | 1.00 | 1.00 | 2.53 | 8.00 | 5.18 | 1.59 | 8.00 | 5.06 | 2.67 | 2.89 | 0.95 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 1.47 | 1.00 | 1.00 | 1.00 | 3.38 | 7.94 | 3.60 | 2.30 | 7.97 | 3.20 | 2.81 | 3.15 | 1.07 |
| 20 | 100 | 7.96 | 8.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 3.07 | 6.46 | 2.73 | 1.81 | 6.77 | 2.19 | 3.38 | 3.71 | 0.74 |
| 20 | 200 | 7.98 | 8.00 | 1.00 | 1.35 | 1.00 | 1.00 | 1.00 | 4.32 | 5.58 | 2.91 | 3.33 | 4.80 | 2.82 | 3.17 | 3.81 | 2.89 |
| 30 | 30 | 8.00 | 8.00 | 1.00 | 1.64 | 1.00 | 1.00 | 1.00 | 2.57 | 8.00 | 4.70 | 1.43 | 8.00 | 4.40 | 3.14 | 3.53 | 0.89 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 1.68 | 1.00 | 1.00 | 1.00 | 2.75 | 7.93 | 2.81 | 2.11 | 7.96 | 2.41 | 2.23 | 2.59 | 1.19 |
| 30 | 100 | 7.99 | 8.00 | 1.00 | 2.95 | 1.00 | 1.00 | 1.00 | 5.46 | 6.62 | 3.46 | 5.06 | 6.88 | 3.14 | 4.68 | 5.19 | 3.23 |
| 30 | 200 | 7.99 | 8.00 | 1.00 | 3.05 | 1.00 | 1.00 | 1.00 | 4.71 | 5.74 | 2.71 | 4.69 | 5.79 | 2.56 | 4.51 | 5.18 | 4.53 |
| 100 | 30 | 8.00 | 7.61 | 1.00 | 2.87 | 1.00 | 1.00 | 1.00 | 5.50 | 7.95 | 3.54 | 4.86 | 7.95 | 3.02 | 3.34 | 3.83 | 1.04 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 5.39 | 1.11 | 1.00 | 1.00 | 5.75 | 7.99 | 3.74 | 5.28 | 7.99 | 2.90 | 5.21 | 5.55 | 3.14 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 5.95 | 3.29 | 1.01 | 1.00 | 5.93 | 8.00 | 4.36 | 5.88 | 8.00 | 3.85 | 5.86 | 5.93 | 5.74 |
| 100 | 200 | 8.00 | 8.00 | 1.02 | 6.00 | 5.39 | 1.18 | 1.00 | 6.00 | 6.09 | 4.98 | 6.00 | 6.13 | 4.05 | 6.00 | 6.00 | 6.01 |
| 200 | 30 | 8.00 | 7.35 | 1.00 | 3.25 | 1.00 | 1.00 | 1.00 | 6.31 | 7.78 | 3.24 | 5.78 | 7.69 | 2.58 | 3.66 | 4.13 | 0.94 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 5.94 | 2.73 | 1.00 | 1.00 | 6.02 | 7.56 | 4.34 | 5.98 | 7.47 | 3.78 | 5.69 | 5.81 | 4.86 |
| 200 | 100 | 8.00 | 8.00 | 1.24 | 6.00 | 5.82 | 2.28 | 1.00 | 6.00 | 7.60 | 5.45 | 6.00 | 7.62 | 5.02 | 6.00 | 6.00 | 6.02 |
| 200 | 200 | 8.00 | 8.00 | 5.86 | 6.00 | 6.00 | 5.99 | 4.58 | 6.00 | 8.00 | 6.00 | 6.00 | 8.00 | 6.00 | 6.00 | 6.00 | 6.01 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.32 | 3.50 | 2.15 | 1.40 | 1.22 | 4.77 | 7.33 | 3.98 | 4.26 | 7.31 | 3.56 | 4.27 | 4.58 | 3.08 |
| | $RMSE$ | 1.99 | 1.94 | 4.82 | 3.18 | 4.27 | 4.76 | 4.85 | 1.85 | 1.59 | 2.26 | 2.48 | 1.62 | 2.66 | 2.18 | 1.86 | 3.59 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part II.1 :$(r^o, \sigma^2_\lambda, \sigma^2_\delta, \beta) = (1, 1, 1, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.31 | 2.85 | 1.60 | 1.09 | 1.00 | 3.36 | 8.00 | 4.96 | 4.33 | 8.00 | 5.77 | 2.13 | 2.26 | 2.63 |
| 20 | 60 | 7.93 | 8.00 | 1.16 | 2.79 | 1.61 | 1.04 | 1.00 | 3.09 | 7.82 | 3.14 | 4.63 | 8.00 | 4.03 | 1.74 | 2.09 | 2.85 |
| 20 | 100 | 7.95 | 8.00 | 1.73 | 2.59 | 2.00 | 1.49 | 1.02 | 3.01 | 3.85 | 2.96 | 3.29 | 7.70 | 3.03 | 2.13 | 2.30 | 2.99 |
| 20 | 200 | 7.94 | 8.00 | 1.00 | 2.75 | 1.00 | 1.00 | 1.00 | 3.00 | 3.00 | 2.86 | 2.99 | 3.12 | 2.48 | 2.69 | 2.96 | 3.04 |
| 30 | 30 | 7.99 | 7.98 | 1.60 | 3.26 | 2.09 | 1.29 | 1.02 | 3.42 | 8.00 | 4.31 | 3.41 | 8.00 | 4.76 | 2.37 | 2.59 | 3.00 |
| 30 | 60 | 7.99 | 8.00 | 2.15 | 3.30 | 2.76 | 1.94 | 1.23 | 3.52 | 7.94 | 3.29 | 3.65 | 8.00 | 3.37 | 2.63 | 2.79 | 3.33 |
| 30 | 100 | 8.00 | 8.00 | 2.33 | 3.14 | 2.94 | 2.23 | 1.58 | 3.52 | 5.40 | 3.05 | 3.66 | 7.68 | 3.09 | 2.80 | 2.95 | 3.45 |
| 30 | 200 | 7.99 | 8.00 | 1.80 | 3.35 | 3.00 | 1.85 | 1.00 | 3.90 | 4.04 | 3.00 | 3.94 | 6.33 | 3.02 | 2.95 | 3.09 | 3.94 |
| 100 | 30 | 8.00 | 7.89 | 1.80 | 4.68 | 2.08 | 1.73 | 1.29 | 6.94 | 8.00 | 4.25 | 7.02 | 8.00 | 4.17 | 2.04 | 2.17 | 2.47 |
| 100 | 60 | 8.00 | 8.00 | 2.00 | 6.87 | 2.52 | 2.00 | 1.87 | 7.22 | 8.00 | 4.39 | 7.28 | 8.00 | 4.34 | 1.99 | 2.02 | 2.35 |
| 100 | 100 | 8.00 | 8.00 | 2.04 | 7.77 | 3.86 | 2.06 | 2.00 | 7.55 | 8.00 | 4.86 | 7.37 | 8.00 | 4.43 | 2.00 | 2.00 | 2.42 |
| 100 | 200 | 8.00 | 8.00 | 2.09 | 8.00 | 5.88 | 2.37 | 2.00 | 8.00 | 8.00 | 5.31 | 7.99 | 8.00 | 5.12 | 2.00 | 2.00 | 2.23 |
| 200 | 30 | 8.00 | 7.38 | 1.68 | 3.23 | 1.96 | 1.69 | 1.23 | 6.43 | 7.94 | 2.91 | 6.00 | 7.92 | 2.63 | 1.98 | 2.02 | 2.25 |
| 200 | 60 | 8.00 | 8.00 | 2.00 | 5.86 | 2.03 | 2.00 | 1.99 | 7.04 | 8.00 | 2.58 | 6.90 | 8.00 | 2.37 | 2.00 | 2.00 | 2.12 |
| 200 | 100 | 8.00 | 8.00 | 2.00 | 7.92 | 2.24 | 2.00 | 2.00 | 7.88 | 8.00 | 2.61 | 7.84 | 8.00 | 2.43 | 2.00 | 2.00 | 2.06 |
| 200 | 200 | 8.00 | 8.00 | 2.00 | 8.00 | 4.57 | 2.00 | 2.00 | 8.00 | 8.00 | 3.06 | 8.00 | 8.00 | 2.80 | 2.00 | 2.00 | 2.02 |
| | $r_{mean}$ | 7.99 | 7.95 | 1.79 | 4.77 | 2.63 | 1.74 | 1.45 | 5.37 | 7.00 | 3.60 | 5.52 | 7.55 | 3.62 | 2.22 | 2.33 | 2.70 |
| | $RMSE$ | 5.99 | 5.96 | 0.41 | 3.50 | 1.35 | 0.49 | 0.70 | 3.95 | 5.29 | 1.83 | 3.99 | 5.68 | 1.91 | 0.41 | 0.51 | 0.88 |
| Part II.2 : $(r^o, \sigma^2_\lambda, \sigma^2_\delta, \beta) = (3, 1, 1, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.99 | 8.00 | 1.00 | 3.81 | 1.01 | 1.00 | 1.00 | 6.67 | 8.00 | 6.49 | 6.25 | 8.00 | 6.51 | 3.47 | 3.70 | 1.23 |
| 20 | 60 | 7.98 | 8.00 | 1.00 | 3.93 | 1.00 | 1.00 | 1.00 | 6.43 | 8.00 | 5.00 | 6.19 | 8.00 | 4.63 | 2.78 | 3.31 | 1.56 |
| 20 | 100 | 8.00 | 8.00 | 1.00 | 5.61 | 1.00 | 1.00 | 1.00 | 7.22 | 7.91 | 5.67 | 7.47 | 7.98 | 5.64 | 6.42 | 6.62 | 4.24 |
| 20 | 200 | 8.00 | 8.00 | 1.00 | 2.68 | 1.00 | 1.00 | 1.00 | 6.46 | 7.05 | 3.81 | 6.72 | 7.80 | 3.22 | 3.05 | 4.13 | 3.34 |
| 30 | 30 | 8.00 | 8.00 | 1.00 | 3.78 | 1.01 | 1.00 | 1.00 | 5.36 | 8.00 | 5.75 | 3.84 | 8.00 | 5.55 | 3.71 | 4.12 | 1.10 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 6.06 | 1.02 | 1.00 | 1.00 | 7.21 | 8.00 | 5.58 | 7.09 | 8.00 | 5.25 | 4.73 | 5.51 | 1.72 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 6.60 | 1.05 | 1.00 | 1.00 | 7.36 | 7.98 | 5.49 | 7.64 | 8.00 | 4.87 | 4.96 | 5.90 | 3.75 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 7.15 | 1.01 | 1.00 | 1.00 | 7.94 | 8.00 | 5.77 | 7.85 | 7.99 | 4.87 | 7.38 | 7.60 | 5.85 |
| 100 | 30 | 8.00 | 8.00 | 1.00 | 6.46 | 1.02 | 1.00 | 1.00 | 7.92 | 8.00 | 5.04 | 7.86 | 8.00 | 4.61 | 3.80 | 4.13 | 0.56 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.99 | 2.01 | 1.00 | 1.00 | 8.00 | 8.00 | 5.63 | 8.00 | 8.00 | 5.17 | 4.48 | 4.82 | 0.18 |
| 100 | 100 | 8.00 | 8.00 | 1.00 | 8.00 | 5.98 | 1.01 | 1.00 | 8.00 | 8.00 | 6.18 | 8.00 | 8.00 | 5.76 | 4.58 | 4.86 | 0.15 |
| 100 | 200 | 8.00 | 8.00 | 2.93 | 8.00 | 7.95 | 5.37 | 1.01 | 8.00 | 8.00 | 6.59 | 8.00 | 8.00 | 6.27 | 5.67 | 5.86 | 0.33 |
| 200 | 30 | 8.00 | 7.94 | 1.00 | 5.73 | 1.02 | 1.00 | 1.00 | 7.93 | 8.00 | 4.20 | 7.89 | 8.00 | 3.69 | 3.93 | 4.33 | 0.54 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 7.92 | 3.61 | 1.01 | 1.00 | 7.99 | 8.00 | 5.07 | 7.98 | 8.00 | 4.58 | 4.77 | 5.25 | 0.74 |
| 200 | 100 | 8.00 | 8.00 | 1.34 | 8.00 | 6.13 | 2.50 | 1.00 | 8.00 | 8.00 | 5.89 | 8.00 | 8.00 | 5.63 | 5.64 | 5.80 | 2.35 |
| 200 | 200 | 8.00 | 8.00 | 5.94 | 8.00 | 7.58 | 6.00 | 5.29 | 8.00 | 8.00 | 6.01 | 8.00 | 8.00 | 6.00 | 5.98 | 5.99 | 5.64 |
| | $r_{mean}$ | 8.00 | 8.00 | 1.45 | 6.23 | 2.71 | 1.68 | 1.27 | 7.41 | 7.93 | 5.51 | 7.30 | 7.99 | 5.14 | 4.71 | 5.12 | 2.08 |
| | $RMSE$ | 2.00 | 2.00 | 4.72 | 1.78 | 4.16 | 4.59 | 4.84 | 1.60 | 1.95 | 0.88 | 1.69 | 1.99 | 1.21 | 1.78 | 1.42 | 4.34 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part III.1 :$(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (1, 3, 3, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.17 | 2.17 | 1.44 | 1.04 | 1.00 | 2.33 | 8.00 | 4.53 | 2.67 | 8.00 | 5.26 | 1.94 | 1.98 | 2.16 |
| 20 | 60 | 7.95 | 8.00 | 1.07 | 2.03 | 1.47 | 1.02 | 1.00 | 2.05 | 6.75 | 2.18 | 2.05 | 7.83 | 2.34 | 1.96 | 1.98 | 2.11 |
| 20 | 100 | 7.91 | 8.00 | 1.00 | 1.89 | 1.02 | 1.00 | 1.00 | 2.00 | 2.22 | 2.00 | 2.01 | 3.12 | 2.01 | 1.91 | 1.96 | 2.07 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 1.94 | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.97 | 1.99 | 2.00 | 2.04 |
| 30 | 30 | 7.97 | 7.96 | 1.43 | 2.08 | 1.77 | 1.18 | 1.01 | 2.10 | 8.00 | 3.02 | 2.13 | 8.00 | 3.59 | 1.95 | 1.98 | 2.17 |
| 30 | 60 | 7.96 | 8.00 | 1.29 | 2.01 | 1.81 | 1.15 | 1.00 | 2.02 | 6.45 | 2.02 | 2.01 | 7.40 | 2.01 | 1.99 | 2.00 | 2.09 |
| 30 | 100 | 7.92 | 8.00 | 1.02 | 2.00 | 1.48 | 1.01 | 1.00 | 2.00 | 2.21 | 2.00 | 2.00 | 2.29 | 1.99 | 2.00 | 2.00 | 2.09 |
| 30 | 200 | 7.75 | 8.00 | 1.01 | 2.00 | 1.87 | 1.01 | 1.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.98 | 2.00 | 2.00 | 2.04 |
| 100 | 30 | 8.00 | 6.45 | 1.41 | 2.04 | 1.85 | 1.33 | 1.04 | 2.31 | 5.89 | 2.04 | 2.19 | 6.13 | 2.02 | 1.96 | 1.98 | 2.13 |
| 100 | 60 | 8.00 | 8.00 | 1.98 | 2.02 | 2.00 | 1.99 | 1.79 | 2.02 | 6.59 | 2.00 | 2.00 | 7.04 | 2.00 | 2.00 | 2.00 | 2.11 |
| 100 | 100 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.97 | 2.00 | 7.92 | 2.00 | 2.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.07 |
| 100 | 200 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.05 | 2.00 | 2.00 | 2.15 | 2.00 | 2.00 | 2.00 | 2.04 |
| 200 | 30 | 8.00 | 5.86 | 1.66 | 2.02 | 1.97 | 1.67 | 1.20 | 2.47 | 4.03 | 2.00 | 2.29 | 3.84 | 2.00 | 1.99 | 2.00 | 2.15 |
| 200 | 60 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.97 | 2.02 | 3.63 | 2.00 | 2.01 | 3.63 | 2.00 | 2.00 | 2.00 | 2.06 |
| 200 | 100 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 4.04 | 2.00 | 2.00 | 4.42 | 2.00 | 2.00 | 2.00 | 2.05 |
| 200 | 200 | 8.00 | 8.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 7.13 | 2.00 | 2.00 | 7.85 | 2.00 | 2.00 | 2.00 | 2.03 |
| | $r_{mean}$ | 7.96 | 7.77 | 1.50 | 2.01 | 1.73 | 1.46 | 1.37 | 2.08 | 4.93 | 2.24 | 2.09 | 5.23 | 2.32 | 1.98 | 1.99 | 2.09 |
| | $RMSE$ | 5.96 | 5.80 | 0.65 | 0.06 | 0.43 | 0.70 | 0.77 | 0.17 | 3.73 | 0.68 | 0.19 | 4.04 | 0.91 | 0.03 | 0.01 | 0.10 |
| Part III.2 : $(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (3, 3, 3, 0)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.97 | 8.00 | 1.00 | 1.36 | 1.00 | 1.00 | 1.00 | 2.93 | 8.00 | 5.42 | 1.64 | 8.00 | 5.23 | 3.29 | 3.46 | 0.83 |
| 20 | 60 | 7.97 | 8.00 | 1.00 | 1.16 | 1.00 | 1.00 | 1.00 | 2.69 | 7.96 | 3.40 | 1.63 | 7.96 | 3.12 | 2.71 | 2.98 | 0.72 |
| 20 | 100 | 7.99 | 8.00 | 1.00 | 1.14 | 1.00 | 1.00 | 1.00 | 4.83 | 7.08 | 3.36 | 4.05 | 7.72 | 3.09 | 2.38 | 3.01 | 1.69 |
| 20 | 200 | 7.96 | 8.00 | 1.00 | 1.11 | 1.00 | 1.00 | 1.00 | 4.75 | 5.28 | 2.73 | 4.04 | 5.20 | 2.10 | 2.47 | 3.07 | 2.79 |
| 30 | 30 | 7.99 | 7.99 | 1.00 | 1.38 | 1.00 | 1.00 | 1.00 | 1.86 | 8.00 | 3.97 | 1.27 | 8.00 | 3.71 | 2.31 | 2.58 | 1.03 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 2.73 | 1.00 | 1.00 | 1.00 | 4.72 | 7.95 | 3.91 | 3.88 | 7.98 | 3.52 | 3.19 | 3.85 | 1.65 |
| 30 | 100 | 7.99 | 8.00 | 1.00 | 2.21 | 1.00 | 1.00 | 1.00 | 4.50 | 6.31 | 2.94 | 3.69 | 6.60 | 2.37 | 3.19 | 3.82 | 1.97 |
| 30 | 200 | 7.97 | 8.00 | 1.00 | 2.08 | 1.00 | 1.00 | 1.00 | 5.12 | 5.82 | 2.12 | 4.08 | 5.46 | 1.94 | 3.98 | 4.66 | 4.70 |
| 100 | 30 | 8.00 | 7.68 | 1.00 | 3.13 | 1.00 | 1.00 | 1.00 | 5.65 | 7.97 | 3.72 | 4.99 | 7.95 | 3.17 | 3.29 | 3.79 | 0.96 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 5.59 | 1.23 | 1.00 | 1.00 | 5.81 | 7.99 | 4.08 | 5.57 | 7.99 | 3.28 | 5.44 | 5.64 | 3.66 |
| 100 | 100 | 8.00 | 8.00 | 1.02 | 5.99 | 4.28 | 1.05 | 1.00 | 5.98 | 8.00 | 4.83 | 5.97 | 8.00 | 4.41 | 5.94 | 5.97 | 5.92 |
| 100 | 200 | 8.00 | 8.00 | 1.09 | 6.00 | 5.81 | 1.45 | 1.00 | 6.00 | 6.07 | 5.16 | 6.00 | 6.13 | 4.42 | 6.00 | 6.00 | 6.02 |
| 200 | 30 | 8.00 | 7.32 | 1.00 | 3.49 | 1.00 | 1.00 | 1.00 | 6.43 | 7.78 | 3.35 | 6.01 | 7.72 | 2.71 | 3.87 | 4.38 | 0.86 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 5.93 | 2.28 | 1.00 | 1.00 | 6.01 | 7.54 | 4.17 | 5.98 | 7.47 | 3.57 | 5.69 | 5.83 | 4.90 |
| 200 | 100 | 8.00 | 8.00 | 1.06 | 6.00 | 5.85 | 1.48 | 1.00 | 6.00 | 7.69 | 5.53 | 6.00 | 7.70 | 4.86 | 6.00 | 6.00 | 6.01 |
| 200 | 200 | 8.00 | 8.00 | 5.87 | 6.00 | 6.00 | 6.00 | 4.67 | 6.00 | 7.99 | 6.00 | 6.00 | 8.00 | 5.99 | 6.00 | 6.00 | 6.01 |
| | $r_{mean}$ | 7.99 | 7.94 | 1.32 | 3.46 | 2.22 | 1.37 | 1.23 | 4.96 | 7.34 | 4.04 | 4.43 | 7.37 | 3.59 | 4.11 | 4.44 | 3.11 |
| | $RMSE$ | 1.99 | 1.94 | 4.83 | 3.25 | 4.25 | 4.78 | 4.85 | 1.69 | 1.61 | 2.23 | 2.28 | 1.65 | 2.65 | 2.37 | 1.99 | 3.57 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2 s}$ | $IC_{p^3 s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part IV.1 :$(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (1, 3, 3, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.95 | 7.99 | 1.89 | 2.21 | 1.97 | 1.66 | 1.19 | 2.49 | 8.00 | 4.04 | 4.96 | 8.00 | 6.26 | 1.89 | 1.98 | 2.50 |
| 20 | 60 | 7.93 | 8.00 | 1.82 | 2.93 | 2.00 | 1.60 | 1.11 | 2.97 | 7.20 | 2.99 | 2.70 | 7.99 | 3.17 | 2.14 | 2.31 | 2.97 |
| 20 | 100 | 7.94 | 8.00 | 1.91 | 3.17 | 2.01 | 1.78 | 1.19 | 3.00 | 3.32 | 2.97 | 3.55 | 7.76 | 2.92 | 2.19 | 2.39 | 3.03 |
| 20 | 200 | 7.97 | 8.00 | 1.98 | 3.80 | 2.01 | 1.97 | 1.33 | 3.00 | 3.00 | 2.88 | 4.73 | 7.74 | 2.43 | 2.06 | 2.28 | 3.01 |
| 30 | 30 | 7.95 | 7.93 | 1.85 | 3.13 | 2.39 | 1.49 | 1.09 | 3.22 | 8.00 | 3.65 | 3.20 | 8.00 | 3.98 | 2.48 | 2.75 | 3.23 |
| 30 | 60 | 7.97 | 8.00 | 2.05 | 3.37 | 2.68 | 1.91 | 1.33 | 3.55 | 7.60 | 3.25 | 3.40 | 7.99 | 3.22 | 2.48 | 2.82 | 3.63 |
| 30 | 100 | 7.95 | 8.00 | 2.50 | 3.58 | 2.98 | 2.36 | 1.74 | 3.75 | 4.30 | 3.08 | 3.48 | 7.71 | 3.03 | 2.70 | 2.97 | 3.84 |
| 30 | 200 | 7.94 | 8.00 | 2.00 | 3.74 | 2.89 | 2.00 | 1.41 | 3.98 | 4.00 | 3.02 | 3.65 | 4.57 | 3.00 | 2.78 | 3.30 | 4.02 |
| 100 | 30 | 8.00 | 7.28 | 1.92 | 3.60 | 2.01 | 1.90 | 1.59 | 6.02 | 7.91 | 3.85 | 6.09 | 7.95 | 3.74 | 1.97 | 2.00 | 2.68 |
| 100 | 60 | 8.00 | 8.00 | 1.99 | 6.89 | 2.37 | 2.00 | 1.91 | 7.31 | 8.00 | 4.60 | 6.76 | 8.00 | 4.06 | 1.99 | 2.02 | 2.46 |
| 100 | 100 | 8.00 | 8.00 | 2.01 | 7.90 | 4.60 | 2.02 | 2.00 | 7.76 | 8.00 | 5.31 | 7.63 | 8.00 | 4.82 | 2.00 | 2.00 | 2.21 |
| 100 | 200 | 8.00 | 8.00 | 2.20 | 8.00 | 7.00 | 2.98 | 2.00 | 8.00 | 8.00 | 6.14 | 8.00 | 8.00 | 5.86 | 2.00 | 2.00 | 2.02 |
| 200 | 30 | 8.00 | 6.06 | 1.81 | 2.02 | 1.99 | 1.82 | 1.42 | 3.38 | 6.96 | 2.04 | 2.78 | 6.54 | 2.01 | 1.99 | 2.00 | 2.08 |
| 200 | 60 | 8.00 | 8.00 | 2.00 | 3.57 | 2.00 | 2.00 | 2.00 | 5.82 | 8.00 | 2.04 | 5.50 | 8.00 | 2.01 | 2.00 | 2.00 | 2.03 |
| 200 | 100 | 8.00 | 8.00 | 2.00 | 7.88 | 2.00 | 2.00 | 2.00 | 7.93 | 8.00 | 2.17 | 7.89 | 8.00 | 2.08 | 2.00 | 2.00 | 2.02 |
| 200 | 200 | 8.00 | 8.00 | 2.00 | 8.00 | 5.26 | 2.00 | 2.00 | 8.00 | 8.00 | 3.15 | 8.00 | 8.00 | 2.89 | 2.00 | 2.00 | 2.00 |
| | $r_{mean}$ | 7.98 | 7.83 | 2.00 | 4.61 | 2.89 | 1.97 | 1.58 | 5.01 | 6.77 | 3.45 | 5.15 | 7.64 | 3.47 | 2.17 | 2.30 | 2.73 |
| | $RMSE$ | 5.98 | 5.85 | 0.16 | 3.40 | 1.68 | 0.33 | 0.55 | 3.67 | 5.11 | 1.82 | 3.69 | 5.71 | 1.92 | 0.32 | 0.51 | 0.99 |
| Part IV.2 : $(r^o, \sigma_\lambda^2, \sigma_\delta^2, \beta) = (3, 3, 3, 0.2)$, $\rho_i \sim i.i.d.\ U(0, 0.85)$ | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.99 | 8.00 | 1.00 | 3.50 | 1.00 | 1.00 | 1.00 | 6.09 | 8.00 | 6.20 | 5.08 | 8.00 | 6.13 | 3.19 | 3.42 | 1.24 |
| 20 | 60 | 7.99 | 8.00 | 1.00 | 4.57 | 1.00 | 1.00 | 1.00 | 7.26 | 8.00 | 5.73 | 7.36 | 8.00 | 5.51 | 5.40 | 5.66 | 1.84 |
| 20 | 100 | 7.99 | 8.00 | 1.00 | 5.22 | 1.11 | 1.00 | 1.00 | 7.17 | 7.91 | 4.93 | 7.59 | 7.99 | 4.78 | 4.54 | 5.29 | 3.84 |
| 20 | 200 | 8.00 | 8.00 | 1.00 | 3.96 | 1.00 | 1.00 | 1.00 | 6.66 | 7.12 | 3.96 | 6.64 | 7.75 | 3.12 | 4.26 | 5.01 | 3.58 |
| 30 | 30 | 8.00 | 8.00 | 1.00 | 5.57 | 1.01 | 1.00 | 1.00 | 6.88 | 8.00 | 6.47 | 6.24 | 8.00 | 6.28 | 4.63 | 4.96 | 1.42 |
| 30 | 60 | 8.00 | 8.00 | 1.00 | 6.47 | 1.06 | 1.00 | 1.00 | 7.52 | 8.00 | 5.98 | 7.61 | 8.00 | 6.01 | 5.42 | 6.09 | 2.11 |
| 30 | 100 | 8.00 | 8.00 | 1.00 | 7.01 | 1.04 | 1.00 | 1.00 | 7.76 | 8.00 | 5.46 | 7.57 | 8.00 | 4.77 | 5.89 | 6.59 | 3.25 |
| 30 | 200 | 8.00 | 8.00 | 1.00 | 7.37 | 1.05 | 1.00 | 1.00 | 7.95 | 8.00 | 5.83 | 7.93 | 8.00 | 5.38 | 7.46 | 7.61 | 6.44 |
| 100 | 30 | 8.00 | 8.00 | 1.00 | 6.79 | 1.01 | 1.00 | 1.00 | 7.96 | 8.00 | 5.21 | 7.93 | 8.00 | 4.75 | 3.72 | 4.03 | 0.55 |
| 100 | 60 | 8.00 | 8.00 | 1.00 | 7.98 | 2.11 | 1.00 | 1.00 | 8.00 | 8.00 | 5.79 | 8.00 | 8.00 | 5.25 | 4.66 | 5.07 | 0.26 |
| 100 | 100 | 8.00 | 8.00 | 1.03 | 8.00 | 6.49 | 1.08 | 1.00 | 8.00 | 8.00 | 6.29 | 8.00 | 8.00 | 5.88 | 4.59 | 4.93 | 0.20 |
| 100 | 200 | 8.00 | 8.00 | 2.41 | 8.00 | 7.97 | 4.23 | 1.03 | 8.00 | 8.00 | 6.85 | 8.00 | 8.00 | 6.54 | 4.93 | 5.29 | 0.02 |
| 200 | 30 | 8.00 | 7.91 | 1.00 | 5.51 | 1.02 | 1.00 | 1.00 | 7.92 | 8.00 | 4.13 | 7.85 | 8.00 | 3.51 | 3.95 | 4.31 | 0.52 |
| 200 | 60 | 8.00 | 8.00 | 1.00 | 7.88 | 3.29 | 1.00 | 1.00 | 7.99 | 8.00 | 5.14 | 7.97 | 8.00 | 4.53 | 4.55 | 5.07 | 1.06 |
| 200 | 100 | 8.00 | 8.00 | 2.09 | 8.00 | 6.16 | 3.94 | 1.01 | 8.00 | 8.00 | 5.94 | 8.00 | 8.00 | 5.85 | 5.83 | 5.93 | 3.18 |
| 200 | 200 | 8.00 | 8.00 | 5.28 | 8.00 | 7.59 | 5.89 | 3.38 | 8.00 | 8.00 | 6.01 | 8.00 | 8.00 | 5.97 | 5.76 | 5.85 | 3.89 |
| | $r_{mean}$ | 8.00 | 7.99 | 1.43 | 6.49 | 2.74 | 1.70 | 1.15 | 7.57 | 7.94 | 5.62 | 7.49 | 7.98 | 5.27 | 4.92 | 5.32 | 2.09 |
| | $RMSE$ | 2.00 | 1.99 | 4.70 | 1.59 | 4.15 | 4.55 | 4.88 | 1.67 | 1.95 | 0.85 | 1.69 | 1.98 | 1.19 | 1.46 | 1.18 | 4.28 |

Table 5: Sensitivity to signal-to-noise ratio

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Part I.1 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (1, 1, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.12 | 1.00 | 8.00 | 2.19 | 1.00 | 1.00 | 1.07 |
| 20 | 60 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.49 | 1.00 | 1.00 | 1.00 | 1.03 |
| 20 | 100 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 20 | 200 | 7.85 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 30 | 30 | 7.88 | 7.87 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.00 | 1.00 | 8.00 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 60 | 7.60 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.57 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 100 | 7.06 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 200 | 5.38 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.66 | 1.86 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 60 | 3.33 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 100 | 3.05 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 200 | 1.31 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 60 | 1.06 | 3.74 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.33 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.77 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | $r_{mean}$ | 4.87 | 6.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 1.94 | 1.07 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.79 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.28 | 0.00 | 2.48 | 0.30 | 0.00 | 0.00 | 0.02 |
| | | Part I.2: $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 1, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 7.98 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.13 | 8.00 | 2.96 | 1.02 | 8.00 | 2.58 | 1.88 | 1.97 | 0.92 |
| 20 | 60 | 7.87 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.51 | 3.22 | 1.89 | 1.17 | 3.13 | 1.58 | 2.08 | 2.21 | 1.84 |
| 20 | 100 | 7.87 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.60 | 2.21 | 1.40 | 1.07 | 1.75 | 1.03 | 1.83 | 1.92 | 2.27 |
| 20 | 200 | 7.88 | 8.00 | 1.00 | 1.66 | 1.00 | 1.00 | 1.00 | 3.00 | 3.00 | 2.30 | 2.98 | 3.00 | 2.10 | 2.99 | 3.00 | 3.00 |
| 30 | 30 | 7.91 | 7.90 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.08 | 8.00 | 1.77 | 1.02 | 8.00 | 1.49 | 1.45 | 1.55 | 1.37 |
| 30 | 60 | 7.84 | 8.00 | 1.00 | 1.48 | 1.00 | 1.00 | 1.00 | 2.05 | 3.04 | 1.92 | 1.69 | 3.20 | 1.66 | 2.04 | 2.26 | 2.76 |
| 30 | 100 | 7.67 | 8.00 | 1.00 | 1.54 | 1.00 | 1.00 | 1.00 | 2.86 | 3.00 | 1.83 | 2.17 | 2.99 | 1.15 | 2.95 | 2.98 | 3.00 |
| 30 | 200 | 7.16 | 8.00 | 1.00 | 1.87 | 1.00 | 1.00 | 1.00 | 2.99 | 3.00 | 1.58 | 2.60 | 2.96 | 1.03 | 3.00 | 3.00 | 3.01 |
| 100 | 30 | 7.70 | 3.12 | 1.00 | 1.20 | 1.00 | 1.00 | 1.00 | 2.01 | 2.80 | 1.33 | 1.48 | 2.57 | 1.05 | 2.28 | 2.43 | 2.84 |
| 100 | 60 | 6.52 | 8.00 | 1.00 | 2.66 | 1.09 | 1.00 | 1.00 | 2.84 | 3.00 | 1.82 | 2.65 | 3.00 | 1.28 | 2.94 | 2.97 | 3.00 |
| 100 | 100 | 6.15 | 8.00 | 1.02 | 3.00 | 1.89 | 1.03 | 1.00 | 3.00 | 3.00 | 2.24 | 2.96 | 3.01 | 1.77 | 3.00 | 3.00 | 3.00 |
| 100 | 200 | 3.78 | 8.00 | 1.02 | 3.00 | 2.73 | 1.16 | 1.00 | 3.00 | 3.00 | 2.37 | 3.00 | 3.00 | 1.91 | 3.00 | 3.00 | 3.01 |
| 200 | 30 | 4.06 | 2.98 | 1.00 | 1.42 | 1.00 | 1.00 | 1.00 | 2.47 | 2.78 | 1.29 | 2.01 | 2.56 | 1.04 | 2.63 | 2.73 | 3.01 |
| 200 | 60 | 3.31 | 4.60 | 1.00 | 2.97 | 1.75 | 1.01 | 1.00 | 2.99 | 3.00 | 2.09 | 2.97 | 3.00 | 1.61 | 2.99 | 2.99 | 3.02 |
| 200 | 100 | 3.75 | 8.00 | 1.23 | 3.00 | 2.87 | 1.56 | 1.01 | 3.00 | 3.00 | 2.67 | 3.00 | 3.00 | 2.23 | 3.00 | 3.00 | 3.00 |
| 200 | 200 | 4.39 | 8.00 | 2.79 | 3.00 | 3.00 | 2.97 | 2.06 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.96 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.36 | 7.16 | 1.13 | 1.93 | 1.46 | 1.17 | 1.07 | 2.41 | 3.57 | 2.03 | 2.17 | 3.51 | 1.65 | 2.57 | 2.63 | 2.63 |
| | $RMSE$ | 3.80 | 4.52 | 1.92 | 1.35 | 1.71 | 1.89 | 1.95 | 0.92 | 1.78 | 1.10 | 1.14 | 1.80 | 1.46 | 0.68 | 0.61 | 0.75 |
| | | Part I.3 : $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (5, 1, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | |
| 20 | 30 | 7.89 | 7.99 | 1.00 | 1.03 | 1.00 | 1.00 | 1.00 | 1.16 | 8.00 | 3.19 | 1.02 | 8.00 | 2.87 | 1.98 | 2.07 | 0.97 |
| 20 | 60 | 7.92 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.16 | 4.77 | 1.75 | 1.03 | 4.55 | 1.36 | 1.84 | 1.98 | 1.08 |
| 20 | 100 | 7.88 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.14 | 2.51 | 1.13 | 1.02 | 1.61 | 1.02 | 2.05 | 2.24 | 1.05 |
| 20 | 200 | 7.91 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.17 | 2.04 | 1.00 | 1.00 | 1.05 | 1.00 | 3.25 | 3.47 | 2.00 |
| 30 | 30 | 7.93 | 7.93 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.08 | 8.00 | 2.07 | 1.01 | 8.00 | 1.62 | 1.80 | 1.95 | 1.15 |
| 30 | 60 | 7.86 | 8.00 | 1.00 | 1.03 | 1.00 | 1.00 | 1.00 | 1.44 | 4.48 | 1.48 | 1.15 | 4.12 | 1.20 | 2.23 | 2.42 | 1.42 |
| 30 | 100 | 7.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.10 | 3.21 | 1.01 | 1.01 | 2.35 | 1.00 | 2.45 | 2.72 | 1.62 |
| 30 | 200 | 7.57 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.71 | 2.80 | 1.00 | 1.10 | 2.13 | 1.00 | 3.31 | 3.66 | 3.25 |
| 100 | 30 | 7.96 | 4.21 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.38 | 3.11 | 1.06 | 1.09 | 2.45 | 1.00 | 2.05 | 2.33 | 1.74 |
| 100 | 60 | 7.64 | 8.00 | 1.00 | 1.70 | 1.00 | 1.00 | 1.00 | 2.27 | 4.92 | 1.16 | 1.51 | 4.86 | 1.01 | 4.06 | 4.29 | 4.71 |
| 100 | 100 | 7.55 | 8.00 | 1.00 | 3.21 | 1.00 | 1.00 | 1.00 | 3.16 | 5.00 | 1.26 | 2.44 | 5.01 | 1.03 | 4.93 | 4.98 | 5.01 |
| 100 | 200 | 6.41 | 8.00 | 1.00 | 4.93 | 1.63 | 1.00 | 1.00 | 4.97 | 5.00 | 2.07 | 4.89 | 5.00 | 1.33 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.36 | 3.82 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.93 | 2.87 | 1.04 | 1.35 | 2.24 | 1.00 | 2.59 | 2.95 | 3.45 |
| 200 | 60 | 5.97 | 5.71 | 1.00 | 2.62 | 1.00 | 1.00 | 1.00 | 3.67 | 4.86 | 1.18 | 2.97 | 4.75 | 1.02 | 4.86 | 4.93 | 4.99 |
| 200 | 100 | 6.37 | 8.00 | 1.00 | 4.59 | 1.30 | 1.00 | 1.00 | 4.74 | 5.00 | 1.70 | 4.36 | 5.00 | 1.14 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.77 | 8.00 | 1.03 | 5.00 | 4.71 | 1.50 | 1.00 | 5.00 | 5.00 | 3.44 | 5.00 | 5.00 | 2.67 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 7.42 | 7.35 | 1.00 | 2.01 | 1.29 | 1.03 | 1.00 | 2.32 | 4.47 | 1.60 | 2.00 | 4.13 | 1.33 | 3.28 | 3.44 | 2.97 |
| | $RMSE$ | 2.51 | 2.73 | 4.00 | 3.34 | 3.82 | 3.97 | 4.00 | 3.04 | 1.76 | 3.48 | 3.33 | 2.17 | 3.71 | 2.14 | 1.98 | 2.64 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Part II.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 3, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.91 | 7.98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.06 | 1.00 | 8.00 | 2.37 | 1.00 | 1.00 | 1.05 |
| 20 | 60 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 1.00 | 1.03 |
| 20 | 100 | 7.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 |
| 20 | 200 | 7.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 30 | 30 | 7.88 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.01 | 1.00 | 1.00 | 1.03 |
| 30 | 60 | 7.69 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 1.00 | 1.01 |
| 30 | 100 | 6.91 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 200 | 5.24 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.62 | 1.87 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 60 | 3.39 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 100 | 2.85 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 200 | 1.34 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 60 | 1.09 | 3.70 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 200 | 100 | 1.31 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.75 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| | $r_{mean}$ | 4.85 | 6.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.00 | 1.89 | 1.09 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.78 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.27 | 0.00 | 2.48 | 0.34 | 0.00 | 0.00 | 0.02 |
| | Part II.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 3, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.91 | 7.98 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.17 | 8.00 | 3.04 | 1.03 | 8.00 | 2.80 | 2.11 | 2.20 | 1.01 |
| 20 | 60 | 7.91 | 8.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.45 | 3.33 | 1.90 | 1.10 | 3.72 | 1.50 | 1.53 | 1.67 | 1.90 |
| 20 | 100 | 7.91 | 8.00 | 1.00 | 1.24 | 1.00 | 1.00 | 1.00 | 2.71 | 2.98 | 2.15 | 1.95 | 2.90 | 1.68 | 2.71 | 2.84 | 2.98 |
| 20 | 200 | 7.87 | 8.00 | 1.00 | 1.24 | 1.00 | 1.00 | 1.00 | 2.81 | 2.97 | 1.67 | 2.33 | 2.78 | 1.22 | 2.94 | 2.97 | 3.00 |
| 30 | 30 | 7.90 | 7.88 | 1.00 | 1.07 | 1.00 | 1.00 | 1.00 | 1.27 | 8.00 | 2.19 | 1.06 | 8.00 | 1.90 | 1.73 | 1.84 | 1.69 |
| 30 | 60 | 7.84 | 8.00 | 1.00 | 1.13 | 1.00 | 1.00 | 1.00 | 1.91 | 3.10 | 1.67 | 1.29 | 3.13 | 1.24 | 2.44 | 2.59 | 2.73 |
| 30 | 100 | 7.62 | 8.00 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.71 | 2.72 | 1.10 | 1.24 | 2.47 | 1.01 | 2.07 | 2.25 | 2.84 |
| 30 | 200 | 7.03 | 8.00 | 1.00 | 1.66 | 1.00 | 1.00 | 1.00 | 2.07 | 2.46 | 1.42 | 1.75 | 2.28 | 1.00 | 2.42 | 2.52 | 3.00 |
| 100 | 30 | 7.70 | 3.18 | 1.00 | 1.38 | 1.00 | 1.00 | 1.00 | 2.22 | 2.85 | 1.54 | 1.82 | 2.75 | 1.20 | 2.36 | 2.52 | 2.88 |
| 100 | 60 | 6.42 | 8.00 | 1.00 | 2.47 | 1.19 | 1.00 | 1.00 | 2.69 | 3.00 | 1.81 | 2.48 | 3.00 | 1.45 | 2.81 | 2.87 | 3.00 |
| 100 | 100 | 6.10 | 8.00 | 1.00 | 2.97 | 1.50 | 1.00 | 1.00 | 2.98 | 3.00 | 1.99 | 2.91 | 3.00 | 1.33 | 3.00 | 3.00 | 3.02 |
| 100 | 200 | 3.74 | 8.00 | 1.06 | 3.00 | 2.95 | 1.35 | 1.00 | 3.00 | 3.00 | 2.77 | 3.00 | 3.00 | 2.30 | 3.00 | 3.00 | 3.00 |
| 200 | 30 | 4.36 | 2.98 | 1.00 | 1.32 | 1.00 | 1.00 | 1.00 | 2.39 | 2.73 | 1.21 | 1.89 | 2.49 | 1.03 | 2.56 | 2.66 | 2.99 |
| 200 | 60 | 3.30 | 4.62 | 1.00 | 2.93 | 1.49 | 1.01 | 1.00 | 2.99 | 3.00 | 1.91 | 2.95 | 3.00 | 1.38 | 3.00 | 3.00 | 3.01 |
| 200 | 100 | 3.80 | 8.00 | 1.66 | 3.00 | 2.98 | 2.14 | 1.09 | 3.00 | 3.00 | 2.90 | 3.00 | 3.00 | 2.60 | 3.00 | 3.00 | 3.00 |
| 200 | 200 | 4.40 | 8.00 | 2.92 | 3.00 | 3.00 | 2.99 | 2.51 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.36 | 7.17 | 1.17 | 1.84 | 1.44 | 1.22 | 1.10 | 2.34 | 3.57 | 2.02 | 2.05 | 3.53 | 1.66 | 2.54 | 2.62 | 2.69 |
| | $RMSE$ | 3.79 | 4.52 | 1.90 | 1.43 | 1.73 | 1.86 | 1.93 | 0.92 | 1.78 | 1.15 | 1.21 | 1.80 | 1.48 | 0.65 | 0.56 | 0.66 |
| | Part II.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 3, 0.5, 0, 0)$, SNR=$\frac{1}{2}$ | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 8.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.08 | 8.00 | 3.22 | 1.01 | 8.00 | 2.87 | 2.18 | 2.23 | 0.77 |
| 20 | 60 | 7.94 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.29 | 4.62 | 1.80 | 1.10 | 4.59 | 1.56 | 1.70 | 1.80 | 1.20 |
| 20 | 100 | 7.89 | 8.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.10 | 1.90 | 1.07 | 1.00 | 1.15 | 1.00 | 1.75 | 1.87 | 1.05 |
| 20 | 200 | 7.90 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.59 | 1.00 | 1.00 | 1.05 | 1.00 | 2.67 | 3.03 | 3.20 |
| 30 | 30 | 7.94 | 7.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 8.00 | 1.63 | 1.01 | 8.00 | 1.33 | 1.86 | 1.97 | 0.69 |
| 30 | 60 | 7.86 | 8.00 | 1.00 | 1.03 | 1.00 | 1.00 | 1.00 | 1.34 | 4.01 | 1.33 | 1.07 | 4.01 | 1.08 | 2.09 | 2.25 | 1.64 |
| 30 | 100 | 7.82 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.23 | 2.86 | 1.01 | 1.03 | 2.42 | 1.00 | 2.35 | 2.53 | 1.80 |
| 30 | 200 | 7.70 | 8.00 | 1.00 | 1.09 | 1.00 | 1.00 | 1.00 | 2.81 | 3.05 | 1.03 | 1.86 | 2.96 | 1.00 | 3.08 | 3.28 | 3.77 |
| 100 | 30 | 7.93 | 4.31 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.39 | 3.23 | 1.10 | 1.12 | 2.66 | 1.01 | 2.17 | 2.38 | 1.69 |
| 100 | 60 | 7.67 | 8.00 | 1.00 | 2.10 | 1.00 | 1.00 | 1.00 | 2.76 | 4.97 | 1.32 | 2.11 | 4.96 | 1.07 | 4.36 | 4.57 | 4.83 |
| 100 | 100 | 7.50 | 8.00 | 1.00 | 3.76 | 1.06 | 1.00 | 1.00 | 3.78 | 5.01 | 1.73 | 2.96 | 5.02 | 1.17 | 4.89 | 4.95 | 5.01 |
| 100 | 200 | 6.35 | 8.00 | 1.00 | 4.75 | 1.83 | 1.00 | 1.00 | 4.86 | 5.00 | 2.04 | 4.58 | 5.00 | 1.56 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.39 | 3.70 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.61 | 2.50 | 1.01 | 1.21 | 1.94 | 1.00 | 2.42 | 2.72 | 2.91 |
| 200 | 60 | 5.91 | 5.67 | 1.00 | 2.63 | 1.01 | 1.00 | 1.00 | 3.63 | 4.83 | 1.16 | 2.97 | 4.70 | 1.01 | 4.85 | 4.92 | 5.00 |
| 200 | 100 | 6.34 | 8.00 | 1.00 | 4.64 | 1.44 | 1.00 | 1.00 | 4.73 | 5.00 | 1.76 | 4.49 | 5.00 | 1.20 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.77 | 8.00 | 1.02 | 5.00 | 4.63 | 1.43 | 1.00 | 5.00 | 5.00 | 3.31 | 5.00 | 5.00 | 2.58 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 7.43 | 7.35 | 1.00 | 2.07 | 1.31 | 1.03 | 1.00 | 2.42 | 4.35 | 1.60 | 2.10 | 4.15 | 1.34 | 3.21 | 3.34 | 3.04 |
| | $RMSE$ | 2.52 | 2.73 | 4.00 | 3.30 | 3.79 | 3.97 | 4.00 | 2.97 | 1.91 | 3.48 | 3.23 | 2.17 | 3.70 | 2.22 | 2.09 | 2.60 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part III.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 1, 0.5, 0, 0)$, SNR=2 | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.91 | 7.98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.14 | 1.01 | 8.00 | 3.72 | 1.00 | 1.00 | 1.08 |
| 20 | 60 | 7.85 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.23 | 8.00 | 2.56 | 1.00 | 1.00 | 1.02 |
| 20 | 100 | 7.87 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.41 | 1.00 | 1.00 | 1.00 | 1.01 |
| 20 | 200 | 7.84 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.06 | 4.96 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 30 | 7.92 | 7.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.34 | 1.00 | 1.00 | 1.03 |
| 30 | 60 | 7.63 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 7.84 | 1.03 | 1.00 | 1.00 | 1.01 |
| 30 | 100 | 7.03 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 1.00 | 1.02 |
| 30 | 200 | 5.38 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 30 | 5.35 | 1.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 60 | 3.31 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 100 | 2.77 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.51 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 200 | 1.36 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.03 |
| 200 | 30 | 1.02 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 60 | 1.07 | 3.70 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 100 | 1.39 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| | $r_{mean}$ | 4.84 | 6.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.07 | 1.02 | 3.06 | 1.29 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.77 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.29 | 0.06 | 3.62 | 0.79 | 0.00 | 0.00 | 0.03 |
| Part III.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 1, 0.5, 0, 0)$, SNR=2 | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.93 | 7.99 | 1.04 | 2.41 | 1.14 | 1.00 | 1.00 | 2.86 | 8.00 | 3.39 | 2.73 | 8.00 | 3.98 | 2.37 | 2.60 | 2.90 |
| 20 | 60 | 7.93 | 8.00 | 1.79 | 3.00 | 2.44 | 1.41 | 1.02 | 3.00 | 3.58 | 3.00 | 3.21 | 8.00 | 3.28 | 2.93 | 2.98 | 3.01 |
| 20 | 100 | 7.90 | 8.00 | 1.24 | 2.99 | 2.08 | 1.09 | 1.00 | 3.00 | 3.00 | 3.00 | 3.01 | 5.04 | 3.00 | 2.96 | 2.99 | 3.00 |
| 20 | 200 | 7.87 | 8.00 | 1.16 | 3.01 | 2.78 | 1.12 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 30 | 30 | 7.94 | 7.91 | 1.05 | 2.80 | 1.35 | 1.00 | 1.00 | 2.96 | 8.00 | 3.00 | 2.86 | 8.00 | 3.02 | 2.78 | 2.89 | 3.01 |
| 30 | 60 | 7.84 | 8.00 | 1.26 | 3.00 | 2.39 | 1.09 | 1.00 | 3.00 | 3.10 | 3.00 | 3.00 | 4.04 | 3.00 | 2.99 | 3.00 | 3.01 |
| 30 | 100 | 7.76 | 8.00 | 1.37 | 3.00 | 2.84 | 1.22 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.05 | 3.00 | 3.00 | 3.00 | 3.01 |
| 30 | 200 | 7.14 | 8.00 | 2.40 | 3.00 | 3.00 | 2.44 | 1.27 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 100 | 30 | 7.61 | 3.12 | 1.94 | 3.00 | 2.76 | 1.75 | 1.09 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.98 | 2.99 | 3.00 | 3.03 |
| 100 | 60 | 6.34 | 8.00 | 2.99 | 3.00 | 3.00 | 2.99 | 2.91 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 | 3.00 | 3.00 | 3.00 | 3.00 |
| 100 | 100 | 5.84 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 3.00 | 3.00 | 3.97 | 3.00 | 3.00 | 3.00 | 3.01 |
| 100 | 200 | 3.85 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 30 | 4.23 | 3.00 | 2.04 | 3.00 | 2.88 | 2.05 | 1.21 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 2.97 | 3.00 | 3.00 | 3.00 |
| 200 | 60 | 3.29 | 4.65 | 2.99 | 3.00 | 3.00 | 3.00 | 2.90 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 100 | 3.76 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 200 | 4.23 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.34 | 7.17 | 2.08 | 2.95 | 2.60 | 2.01 | 1.77 | 2.99 | 3.67 | 3.02 | 2.99 | 4.19 | 3.08 | 2.94 | 2.97 | 3.00 |
| | $RMSE$ | 3.78 | 4.52 | 1.22 | 0.16 | 0.70 | 1.31 | 1.54 | 0.04 | 1.77 | 0.10 | 0.09 | 2.25 | 0.26 | 0.17 | 0.10 | 0.03 |
| Part III.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 1, 0.5, 0, 0)$, SNR=2 | | | | | | | | | | | | | | | | | |
| 20 | 30 | 7.92 | 8.00 | 1.00 | 2.29 | 1.03 | 1.00 | 1.00 | 3.28 | 8.00 | 4.42 | 2.61 | 8.00 | 4.74 | 2.25 | 2.55 | 2.63 |
| 20 | 60 | 7.95 | 8.00 | 1.00 | 3.22 | 1.04 | 1.00 | 1.00 | 4.73 | 6.85 | 4.32 | 4.53 | 7.86 | 4.19 | 3.95 | 4.37 | 4.53 |
| 20 | 100 | 7.93 | 8.00 | 1.00 | 4.51 | 1.00 | 1.00 | 1.00 | 4.99 | 5.01 | 4.36 | 5.00 | 7.35 | 4.05 | 3.04 | 4.00 | 5.00 |
| 20 | 200 | 7.93 | 8.00 | 1.00 | 2.33 | 1.00 | 1.00 | 1.00 | 4.98 | 5.00 | 3.20 | 4.87 | 4.99 | 3.08 | 4.69 | 4.92 | 5.00 |
| 30 | 30 | 7.97 | 7.95 | 1.02 | 3.29 | 1.12 | 1.00 | 1.00 | 4.16 | 8.00 | 4.47 | 3.54 | 8.00 | 4.37 | 2.75 | 3.36 | 3.92 |
| 30 | 60 | 7.92 | 8.00 | 1.01 | 4.64 | 1.62 | 1.00 | 1.00 | 4.96 | 5.57 | 4.58 | 4.94 | 7.67 | 4.63 | 4.71 | 4.87 | 5.00 |
| 30 | 100 | 7.89 | 8.00 | 1.01 | 4.97 | 1.64 | 1.00 | 1.00 | 5.00 | 5.00 | 4.84 | 5.00 | 6.45 | 4.58 | 4.95 | 5.00 | 5.00 |
| 30 | 200 | 7.69 | 8.00 | 1.00 | 5.00 | 1.69 | 1.00 | 1.00 | 5.00 | 5.00 | 4.87 | 5.00 | 5.00 | 4.58 | 5.00 | 5.00 | 5.00 |
| 100 | 30 | 7.98 | 5.01 | 1.03 | 4.58 | 1.74 | 1.02 | 1.00 | 4.93 | 5.00 | 4.33 | 4.85 | 5.01 | 3.90 | 4.72 | 4.87 | 4.99 |
| 100 | 60 | 7.71 | 8.00 | 2.96 | 5.00 | 4.92 | 3.03 | 1.06 | 5.00 | 5.00 | 4.98 | 5.00 | 5.05 | 4.88 | 5.00 | 5.00 | 5.01 |
| 100 | 100 | 7.56 | 8.00 | 4.75 | 5.00 | 5.00 | 4.86 | 2.71 | 5.00 | 5.00 | 5.00 | 5.00 | 5.89 | 4.99 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 6.39 | 8.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| 200 | 30 | 7.55 | 5.00 | 1.02 | 4.68 | 2.14 | 1.02 | 1.00 | 4.96 | 4.99 | 4.03 | 4.94 | 4.99 | 3.64 | 4.92 | 4.96 | 5.00 |
| 200 | 60 | 5.94 | 5.70 | 4.18 | 5.00 | 4.98 | 4.50 | 1.85 | 5.00 | 5.00 | 4.97 | 5.00 | 5.00 | 4.92 | 5.00 | 5.00 | 5.01 |
| 200 | 100 | 6.37 | 8.00 | 4.99 | 5.00 | 5.00 | 4.99 | 4.92 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.74 | 8.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 7.47 | 7.48 | 2.31 | 4.34 | 2.75 | 2.34 | 1.91 | 4.81 | 5.53 | 4.59 | 4.71 | 6.02 | 4.47 | 4.44 | 4.62 | 4.76 |
| | $RMSE$ | 2.55 | 2.71 | 3.20 | 1.15 | 2.86 | 3.20 | 3.45 | 0.48 | 1.17 | 0.63 | 0.71 | 1.61 | 0.76 | 1.06 | 0.80 | 0.66 |

Part IV.1 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (1, 3, 0.5, 0, 0)$, SNR=2

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.90 | 7.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 2.02 | 1.01 | 8.00 | 4.35 | 1.00 | 1.00 | 1.08 |
| 20 | 60 | 7.85 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 2.67 | 1.01 | 1.00 | 1.00 | 1.02 |
| 20 | 100 | 7.79 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 4.71 | 1.03 | 1.00 | 1.00 | 1.02 |
| 20 | 200 | 7.83 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 30 | 30 | 7.89 | 7.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 8.00 | 1.01 | 1.00 | 8.00 | 1.12 | 1.00 | 1.00 | 1.04 |
| 30 | 60 | 7.60 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 5.34 | 1.00 | 1.00 | 1.00 | 1.04 |
| 30 | 100 | 6.97 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.03 |
| 30 | 200 | 5.39 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 100 | 30 | 5.67 | 1.86 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 100 | 60 | 3.41 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 | 1.04 |
| 100 | 100 | 2.80 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.68 | 1.00 | 1.00 | 1.00 | 1.02 |
| 100 | 200 | 1.31 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 30 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 60 | 1.06 | 3.78 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.02 |
| 200 | 100 | 1.33 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 200 | 200 | 1.86 | 8.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | $r_{mean}$ | 4.85 | 6.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.88 | 1.06 | 1.00 | 2.53 | 1.22 | 1.00 | 1.00 | 1.02 |
| | $RMSE$ | 4.78 | 6.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.47 | 0.26 | 0.00 | 2.89 | 0.84 | 0.00 | 0.00 | 0.03 |

Part IV.2 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (3, 3, 0.5, 0, 0)$, SNR=2

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.92 | 7.99 | 1.01 | 2.32 | 1.07 | 1.00 | 1.00 | 2.84 | 8.00 | 3.41 | 2.55 | 8.00 | 4.20 | 2.22 | 2.48 | 2.87 |
| 20 | 60 | 7.94 | 8.00 | 1.01 | 2.83 | 1.22 | 1.00 | 1.00 | 3.00 | 3.54 | 3.00 | 3.00 | 6.98 | 3.00 | 2.70 | 2.91 | 3.01 |
| 20 | 100 | 7.92 | 8.00 | 1.41 | 3.01 | 2.53 | 1.21 | 1.00 | 3.00 | 3.00 | 3.00 | 3.01 | 3.67 | 3.00 | 3.00 | 3.00 | 3.01 |
| 20 | 200 | 7.86 | 8.00 | 1.03 | 3.01 | 2.43 | 1.02 | 1.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 30 | 30 | 7.93 | 7.90 | 1.05 | 2.57 | 1.28 | 1.00 | 1.00 | 2.88 | 8.00 | 2.98 | 2.66 | 8.00 | 2.99 | 2.50 | 2.70 | 3.00 |
| 30 | 60 | 7.84 | 8.00 | 1.65 | 3.00 | 2.71 | 1.31 | 1.00 | 3.00 | 3.10 | 3.00 | 3.00 | 6.33 | 3.00 | 3.00 | 3.00 | 3.01 |
| 30 | 100 | 7.66 | 8.00 | 2.51 | 3.00 | 2.99 | 2.21 | 1.02 | 3.00 | 3.00 | 3.00 | 3.00 | 3.46 | 3.00 | 3.00 | 3.00 | 3.01 |
| 30 | 200 | 7.30 | 8.00 | 1.53 | 3.00 | 2.97 | 1.58 | 1.02 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.02 |
| 100 | 30 | 7.74 | 3.15 | 1.33 | 2.97 | 2.36 | 1.24 | 1.00 | 3.00 | 3.00 | 2.97 | 2.99 | 3.00 | 2.88 | 2.94 | 2.98 | 3.00 |
| 100 | 60 | 6.57 | 8.00 | 2.99 | 3.00 | 3.00 | 2.99 | 2.74 | 3.00 | 3.00 | 3.00 | 3.00 | 3.02 | 3.00 | 3.00 | 3.00 | 3.00 |
| 100 | 100 | 6.01 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.59 | 3.00 | 3.00 | 3.00 | 3.01 |
| 100 | 200 | 3.84 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 30 | 4.07 | 3.00 | 2.00 | 3.00 | 2.85 | 2.03 | 1.25 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 2.96 | 2.99 | 3.00 | 3.00 |
| 200 | 60 | 3.31 | 4.66 | 3.00 | 3.00 | 3.00 | 3.00 | 2.98 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 100 | 3.93 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 200 | 4.28 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 6.38 | 7.17 | 2.03 | 2.92 | 2.53 | 1.97 | 1.75 | 2.98 | 3.67 | 3.02 | 2.95 | 4.19 | 3.06 | 2.90 | 2.94 | 3.00 |
| | $RMSE$ | 3.82 | 4.52 | 1.28 | 0.21 | 0.83 | 1.34 | 1.56 | 0.05 | 1.77 | 0.10 | 0.14 | 2.21 | 0.30 | 0.24 | 0.15 | 0.03 |

Part IV.3 : $(r^o, \sigma^2_\lambda, \alpha, \rho, \beta) = (5, 3, 0.5, 0, 0)$, SNR=2

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $IC_{p^2s}$ | $IC_{p^3s}$ | $BIC_{3s}$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 30 | 7.94 | 8.00 | 1.03 | 3.10 | 1.16 | 1.00 | 1.00 | 4.06 | 8.00 | 4.76 | 4.18 | 8.00 | 5.02 | 2.72 | 3.18 | 3.19 |
| 20 | 60 | 7.95 | 8.00 | 1.01 | 4.86 | 1.20 | 1.00 | 1.00 | 5.00 | 6.88 | 4.96 | 5.03 | 7.91 | 4.93 | 4.79 | 4.91 | 4.99 |
| 20 | 100 | 7.94 | 8.00 | 1.00 | 3.52 | 1.06 | 1.00 | 1.00 | 4.58 | 4.96 | 3.92 | 4.12 | 6.26 | 3.49 | 3.81 | 4.27 | 4.77 |
| 20 | 200 | 7.92 | 8.00 | 1.00 | 4.95 | 1.05 | 1.00 | 1.00 | 5.00 | 5.00 | 4.49 | 5.00 | 5.07 | 4.38 | 5.00 | 5.00 | 5.00 |
| 30 | 30 | 7.95 | 7.93 | 1.01 | 3.01 | 1.07 | 1.00 | 1.00 | 3.73 | 8.00 | 4.14 | 2.97 | 8.00 | 4.03 | 2.99 | 3.35 | 3.49 |
| 30 | 60 | 7.92 | 8.00 | 1.01 | 4.07 | 1.42 | 1.00 | 1.00 | 4.83 | 5.60 | 4.21 | 4.72 | 7.50 | 4.09 | 4.48 | 4.77 | 4.95 |
| 30 | 100 | 7.89 | 8.00 | 1.06 | 4.66 | 2.05 | 1.03 | 1.00 | 4.99 | 5.00 | 4.26 | 4.96 | 5.23 | 4.14 | 4.88 | 4.97 | 5.00 |
| 30 | 200 | 7.72 | 8.00 | 1.02 | 4.92 | 3.63 | 1.02 | 1.00 | 5.00 | 5.00 | 4.20 | 5.00 | 5.00 | 4.19 | 4.99 | 5.00 | 5.01 |
| 100 | 30 | 7.96 | 5.00 | 1.03 | 4.26 | 1.62 | 1.01 | 1.00 | 4.84 | 4.99 | 3.96 | 4.78 | 4.99 | 3.69 | 4.44 | 4.74 | 4.97 |
| 100 | 60 | 7.72 | 8.00 | 2.44 | 5.00 | 4.72 | 2.51 | 1.06 | 5.00 | 5.00 | 4.89 | 5.00 | 5.03 | 4.71 | 4.99 | 5.00 | 5.00 |
| 100 | 100 | 7.54 | 8.00 | 4.81 | 5.00 | 5.00 | 4.89 | 3.01 | 5.00 | 5.01 | 5.00 | 5.00 | 5.77 | 5.00 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 6.35 | 8.00 | 5.00 | 5.00 | 5.00 | 5.00 | 4.98 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.01 |
| 200 | 30 | 7.52 | 5.00 | 1.03 | 4.62 | 1.95 | 1.03 | 1.00 | 4.96 | 4.99 | 3.96 | 4.93 | 4.99 | 3.51 | 4.82 | 4.94 | 5.01 |
| 200 | 60 | 5.98 | 5.69 | 4.10 | 5.00 | 4.99 | 4.44 | 1.75 | 5.00 | 5.00 | 4.98 | 5.00 | 5.00 | 4.93 | 5.00 | 5.00 | 5.01 |
| 200 | 100 | 6.36 | 8.00 | 4.99 | 5.00 | 5.00 | 5.00 | 4.92 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.79 | 8.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 7.47 | 7.48 | 2.28 | 4.50 | 2.87 | 2.31 | 1.92 | 4.81 | 5.53 | 4.55 | 4.73 | 5.86 | 4.44 | 4.56 | 4.70 | 4.78 |
| | $RMSE$ | 2.55 | 2.70 | 3.22 | 0.85 | 2.74 | 3.22 | 3.45 | 0.41 | 1.17 | 0.62 | 0.60 | 1.48 | 0.79 | 0.84 | 0.65 | 0.59 |

Table 6: Large variances for some cross-sectional units (DGP VI)

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 : $(r^o, p) = (3, 2)$ without standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.61 | 8.00 | 1.04 | 3.87 | 1.58 | 1.01 | 1.00 | 3.82 | 7.75 | 3.41 | 3.79 | 3.89 | 4.00 |
| 50 | 100 | 7.08 | 8.00 | 1.21 | 4.00 | 3.86 | 1.18 | 1.00 | 4.00 | 4.00 | 3.58 | 4.00 | 4.00 | 4.01 |
| 50 | 200 | 5.48 | 8.00 | 1.69 | 4.00 | 4.00 | 2.50 | 1.00 | 4.00 | 4.00 | 3.27 | 4.00 | 4.00 | 4.01 |
| 100 | 50 | 7.67 | 7.80 | 1.44 | 3.00 | 2.70 | 1.42 | 1.01 | 3.92 | 4.98 | 2.98 | 3.34 | 3.76 | 4.96 |
| 100 | 100 | 7.29 | 8.00 | 2.91 | 4.84 | 3.00 | 2.95 | 2.19 | 4.61 | 5.00 | 3.02 | 4.14 | 4.48 | 5.00 |
| 100 | 200 | 6.46 | 8.00 | 3.00 | 5.00 | 4.95 | 3.00 | 2.99 | 5.00 | 5.00 | 3.04 | 4.61 | 4.88 | 5.00 |
| 200 | 50 | 4.17 | 3.14 | 1.26 | 2.99 | 2.58 | 1.40 | 1.03 | 3.02 | 3.95 | 2.79 | 2.79 | 2.86 | 5.03 |
| 200 | 100 | 7.79 | 8.00 | 2.99 | 3.00 | 3.00 | 3.00 | 2.88 | 3.27 | 6.82 | 3.00 | 3.00 | 3.00 | 7.01 |
| 200 | 200 | 7.76 | 8.00 | 3.00 | 6.46 | 3.00 | 3.00 | 3.00 | 4.70 | 7.00 | 3.00 | 3.00 | 3.00 | 7.00 |
| | $r_{mean}$ | 6.81 | 7.44 | 2.06 | 4.13 | 3.19 | 2.16 | 1.79 | 4.04 | 5.39 | 3.12 | 3.63 | 3.76 | 5.11 |
| | $RMSE$ | 3.99 | 4.69 | 1.26 | 1.57 | 0.93 | 1.18 | 1.51 | 1.21 | 2.75 | 0.26 | 0.86 | 1.01 | 2.38 |
| Part I.2 : $(r^o, p) = (3, 2)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.36 | 8.00 | 1.04 | 2.73 | 1.53 | 1.02 | 1.00 | 2.64 | 7.96 | 2.34 | 2.62 | 2.79 | 3.00 |
| 50 | 100 | 5.82 | 8.00 | 1.10 | 3.00 | 2.58 | 1.09 | 1.00 | 3.00 | 3.00 | 2.87 | 3.00 | 3.00 | 3.02 |
| 50 | 200 | 5.36 | 8.00 | 1.58 | 3.00 | 3.00 | 1.94 | 1.00 | 3.00 | 3.00 | 2.99 | 3.00 | 3.00 | 3.00 |
| 100 | 50 | 6.13 | 7.59 | 1.37 | 3.00 | 2.58 | 1.34 | 1.01 | 3.00 | 3.00 | 2.76 | 3.00 | 3.00 | 3.00 |
| 100 | 100 | 5.06 | 8.00 | 2.68 | 3.00 | 2.99 | 2.77 | 1.64 | 3.00 | 3.11 | 2.98 | 3.00 | 3.00 | 3.01 |
| 100 | 200 | 3.48 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.97 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 50 | 3.21 | 3.22 | 1.44 | 3.00 | 2.76 | 1.60 | 1.03 | 3.00 | 3.00 | 2.63 | 3.00 | 3.00 | 3.01 |
| 200 | 100 | 3.62 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 2.98 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 200 | 200 | 4.03 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| | $r_{mean}$ | 4.90 | 7.42 | 2.02 | 2.97 | 2.72 | 2.08 | 1.74 | 2.96 | 3.56 | 2.84 | 2.96 | 2.98 | 3.01 |
| | $RMSE$ | 2.32 | 4.67 | 1.28 | 0.09 | 0.53 | 1.22 | 1.55 | 0.12 | 1.65 | 0.27 | 0.13 | 0.07 | 0.01 |
| Part II.1 : $(r^o, p) = (3, 4)$ without standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.69 | 8.00 | 4.13 | 4.99 | 4.85 | 3.37 | 1.17 | 5.00 | 7.86 | 4.90 | 3.79 | 4.70 | 5.01 |
| 50 | 100 | 7.23 | 8.00 | 4.03 | 4.97 | 4.56 | 4.02 | 2.97 | 4.99 | 5.00 | 4.21 | 2.14 | 3.26 | 5.00 |
| 50 | 200 | 6.28 | 8.00 | 4.91 | 5.00 | 5.00 | 4.93 | 4.74 | 5.00 | 5.00 | 5.00 | 4.96 | 5.00 | 5.01 |
| 100 | 50 | 7.91 | 7.91 | 1.18 | 7.00 | 3.47 | 1.18 | 1.01 | 6.99 | 7.00 | 6.53 | 6.95 | 6.98 | 7.00 |
| 100 | 100 | 7.83 | 8.00 | 6.15 | 7.00 | 7.00 | 6.92 | 1.11 | 7.00 | 7.01 | 6.91 | 7.00 | 7.00 | 7.01 |
| 100 | 200 | 7.56 | 8.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 | 7.00 |
| 200 | 50 | 4.76 | 3.70 | 1.31 | 2.99 | 2.56 | 1.40 | 1.02 | 8.00 | 8.00 | 7.71 | 5.11 | 4.76 | 0.03 |
| 200 | 100 | 7.34 | 7.63 | 2.94 | 3.02 | 2.99 | 2.98 | 2.57 | 8.00 | 8.00 | 8.00 | 5.21 | 4.34 | 0.00 |
| 200 | 200 | 7.08 | 7.18 | 3.00 | 3.07 | 3.00 | 3.00 | 3.00 | 8.00 | 8.00 | 8.00 | 6.96 | 5.67 | 0.00 |
| | $r_{mean}$ | 7.08 | 7.38 | 3.85 | 5.00 | 4.49 | 3.87 | 2.73 | 6.66 | 6.99 | 6.47 | 5.46 | 5.41 | 4.01 |
| | $RMSE$ | 4.18 | 4.58 | 2.06 | 2.58 | 2.17 | 2.16 | 1.94 | 3.87 | 4.15 | 3.73 | 2.94 | 2.72 | 3.11 |
| Part II.2 : $(r^o, p) = (3, 4)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.35 | 8.00 | 1.11 | 2.98 | 2.04 | 1.05 | 1.00 | 2.97 | 7.99 | 2.89 | 2.95 | 2.98 | 3.01 |
| 50 | 100 | 5.91 | 8.00 | 1.15 | 3.00 | 2.36 | 1.14 | 1.00 | 3.00 | 3.00 | 2.68 | 3.00 | 3.00 | 3.01 |
| 50 | 200 | 5.44 | 8.00 | 1.91 | 3.00 | 3.00 | 2.30 | 1.02 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| 100 | 50 | 6.14 | 7.54 | 1.11 | 2.99 | 2.23 | 1.10 | 1.00 | 2.99 | 3.00 | 2.50 | 2.99 | 2.99 | 3.01 |
| 100 | 100 | 5.14 | 8.00 | 2.77 | 3.00 | 3.00 | 2.85 | 1.90 | 3.00 | 3.16 | 2.99 | 3.00 | 3.00 | 3.00 |
| 100 | 200 | 3.46 | 8.00 | 2.98 | 3.00 | 3.00 | 3.00 | 2.67 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 50 | 3.25 | 3.20 | 1.48 | 3.00 | 2.81 | 1.62 | 1.03 | 3.00 | 3.00 | 2.65 | 3.00 | 3.00 | 3.00 |
| 200 | 100 | 3.55 | 8.00 | 2.98 | 3.00 | 3.00 | 2.99 | 2.75 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.01 |
| 200 | 200 | 4.06 | 8.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
| | $r_{mean}$ | 4.92 | 7.42 | 2.05 | 3.00 | 2.72 | 2.12 | 1.71 | 3.00 | 3.57 | 2.86 | 2.99 | 3.00 | 3.01 |
| | $RMSE$ | 2.34 | 4.66 | 1.25 | 0.01 | 0.47 | 1.22 | 1.53 | 0.01 | 1.66 | 0.23 | 0.02 | 0.01 | 0.01 |

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part III.1 : $(r^o, p) = (5, 2)$ without standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.92 | 8.00 | 1.00 | 3.78 | 1.01 | 1.00 | 1.00 | 4.01 | 7.97 | 3.38 | 4.68 | 5.18 | 4.96 |
| 50 | 100 | 7.74 | 8.00 | 1.00 | 5.67 | 1.14 | 1.00 | 1.00 | 5.53 | 6.00 | 3.43 | 5.79 | 5.94 | 6.00 |
| 50 | 200 | 7.12 | 8.00 | 1.00 | 5.99 | 5.13 | 1.01 | 1.00 | 6.00 | 6.00 | 3.86 | 6.00 | 6.00 | 6.00 |
| 100 | 50 | 7.97 | 7.97 | 1.00 | 3.36 | 1.07 | 1.00 | 1.00 | 4.59 | 6.92 | 2.68 | 4.89 | 5.64 | 5.90 |
| 100 | 100 | 7.93 | 8.00 | 1.03 | 6.82 | 2.56 | 1.05 | 1.00 | 6.27 | 7.02 | 3.65 | 6.99 | 6.99 | 7.00 |
| 100 | 200 | 7.68 | 8.00 | 1.37 | 7.00 | 6.98 | 2.07 | 1.01 | 7.00 | 7.00 | 4.79 | 7.00 | 7.00 | 7.01 |
| 200 | 50 | 5.38 | 5.01 | 1.00 | 4.63 | 1.77 | 1.00 | 1.00 | 4.90 | 5.94 | 3.09 | 3.88 | 4.42 | 2.30 |
| 200 | 100 | 7.73 | 8.00 | 1.75 | 5.00 | 4.86 | 2.64 | 1.04 | 5.25 | 8.00 | 4.42 | 4.82 | 4.89 | 2.87 |
| 200 | 200 | 7.99 | 8.00 | 4.96 | 5.01 | 5.00 | 5.00 | 4.69 | 6.65 | 8.00 | 5.00 | 5.00 | 5.00 | 4.62 |
| | $r_{mean}$ | 7.50 | 7.66 | 1.57 | 5.25 | 3.28 | 1.75 | 1.42 | 5.58 | 6.98 | 3.81 | 5.45 | 5.67 | 5.18 |
| | $RMSE$ | 2.62 | 2.82 | 3.64 | 1.20 | 2.72 | 3.49 | 3.77 | 1.10 | 2.15 | 1.40 | 1.10 | 1.09 | 1.59 |
| Part III.2 : $(r^o, p) = (5, 2)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.83 | 8.00 | 1.00 | 2.91 | 1.02 | 1.00 | 1.00 | 2.70 | 7.99 | 2.45 | 3.96 | 4.31 | 3.81 |
| 50 | 100 | 7.16 | 8.00 | 1.00 | 4.05 | 1.48 | 1.00 | 1.00 | 4.26 | 5.00 | 2.86 | 4.73 | 4.82 | 4.96 |
| 50 | 200 | 6.49 | 8.00 | 1.00 | 4.86 | 1.52 | 1.00 | 1.00 | 4.96 | 5.00 | 2.40 | 5.00 | 5.00 | 5.00 |
| 100 | 50 | 7.64 | 7.81 | 1.00 | 3.96 | 1.21 | 1.00 | 1.00 | 4.09 | 4.97 | 2.36 | 4.56 | 4.74 | 4.91 |
| 100 | 100 | 7.27 | 8.00 | 1.07 | 5.00 | 3.97 | 1.14 | 1.00 | 4.99 | 5.08 | 3.68 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 5.78 | 8.00 | 1.81 | 5.00 | 4.89 | 3.04 | 1.01 | 5.00 | 5.00 | 4.11 | 5.00 | 5.00 | 5.01 |
| 200 | 50 | 5.92 | 5.03 | 1.00 | 4.61 | 1.69 | 1.00 | 1.00 | 4.82 | 4.99 | 2.37 | 4.98 | 4.99 | 5.00 |
| 200 | 100 | 6.11 | 8.00 | 2.09 | 5.00 | 4.92 | 3.20 | 1.10 | 5.00 | 5.00 | 4.24 | 5.00 | 5.00 | 5.00 |
| 200 | 200 | 6.40 | 8.00 | 4.99 | 5.00 | 5.00 | 5.00 | 4.95 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| | $r_{mean}$ | 6.73 | 7.65 | 1.66 | 4.49 | 2.86 | 1.93 | 1.45 | 4.54 | 5.34 | 3.27 | 4.80 | 4.87 | 4.86 |
| | $RMSE$ | 1.88 | 2.81 | 3.56 | 0.85 | 2.72 | 3.37 | 3.76 | 0.86 | 1.00 | 1.97 | 0.39 | 0.25 | 0.40 |
| Part IV.1 : $(r^o, p) = (5, 4)$ without standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.96 | 8.00 | 1.00 | 6.01 | 1.03 | 1.00 | 1.00 | 5.06 | 8.00 | 4.13 | 5.63 | 6.18 | 5.64 |
| 50 | 100 | 7.86 | 8.00 | 1.00 | 6.98 | 1.63 | 1.00 | 1.00 | 6.76 | 7.00 | 4.31 | 6.96 | 6.97 | 7.00 |
| 50 | 200 | 7.62 | 8.00 | 1.00 | 7.00 | 6.99 | 1.00 | 1.00 | 7.00 | 7.00 | 4.67 | 7.00 | 7.00 | 7.00 |
| 100 | 50 | 7.91 | 7.94 | 1.00 | 3.44 | 1.07 | 1.00 | 1.00 | 4.68 | 7.98 | 2.77 | 2.90 | 3.69 | 1.01 |
| 100 | 100 | 7.98 | 8.00 | 1.19 | 5.11 | 3.89 | 1.34 | 1.00 | 7.08 | 8.00 | 4.32 | 3.33 | 3.97 | 0.35 |
| 100 | 200 | 7.98 | 8.00 | 2.00 | 6.75 | 5.04 | 3.28 | 1.06 | 8.00 | 8.00 | 4.93 | 4.27 | 4.56 | 0.10 |
| 200 | 50 | 5.19 | 5.00 | 1.00 | 4.62 | 1.65 | 1.00 | 1.00 | 4.77 | 6.51 | 2.68 | 3.45 | 3.83 | 1.88 |
| 200 | 100 | 5.47 | 7.92 | 2.10 | 5.00 | 4.89 | 3.09 | 1.11 | 5.21 | 8.00 | 4.21 | 4.75 | 4.86 | 4.31 |
| 200 | 200 | 6.53 | 8.00 | 4.86 | 5.00 | 5.00 | 4.99 | 4.38 | 6.39 | 8.00 | 4.98 | 4.99 | 4.99 | 4.99 |
| | $r_{mean}$ | 7.17 | 7.65 | 1.68 | 5.55 | 3.47 | 1.97 | 1.39 | 6.11 | 7.61 | 4.11 | 4.81 | 5.12 | 3.59 |
| | $RMSE$ | 2.42 | 2.81 | 3.53 | 1.27 | 2.56 | 3.33 | 3.76 | 1.58 | 2.67 | 1.19 | 1.43 | 1.23 | 2.98 |
| Part IV.2 : $(r^o, p) = (5, 4)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 7.81 | 8.00 | 1.00 | 2.98 | 1.02 | 1.00 | 1.00 | 2.68 | 8.00 | 2.47 | 3.88 | 4.16 | 3.48 |
| 50 | 100 | 7.31 | 8.00 | 1.00 | 4.41 | 1.30 | 1.00 | 1.00 | 4.70 | 5.01 | 2.54 | 4.88 | 4.96 | 5.01 |
| 50 | 200 | 6.57 | 8.00 | 1.00 | 4.75 | 2.01 | 1.00 | 1.00 | 4.91 | 5.00 | 3.06 | 4.96 | 4.97 | 5.01 |
| 100 | 50 | 7.60 | 7.79 | 1.00 | 3.67 | 1.11 | 1.00 | 1.00 | 4.01 | 4.99 | 2.13 | 4.62 | 4.79 | 4.95 |
| 100 | 100 | 7.19 | 8.00 | 1.05 | 4.99 | 3.39 | 1.11 | 1.00 | 4.95 | 5.13 | 3.17 | 5.00 | 5.00 | 5.01 |
| 100 | 200 | 5.74 | 8.00 | 1.17 | 5.00 | 4.86 | 1.70 | 1.00 | 5.00 | 5.00 | 3.57 | 5.00 | 5.00 | 5.00 |
| 200 | 50 | 5.80 | 5.01 | 1.00 | 4.59 | 1.79 | 1.01 | 1.00 | 4.78 | 4.99 | 2.39 | 4.95 | 4.98 | 5.00 |
| 200 | 100 | 6.19 | 8.00 | 1.48 | 5.00 | 4.70 | 2.29 | 1.03 | 5.00 | 5.00 | 3.69 | 5.00 | 5.00 | 5.01 |
| 200 | 200 | 6.39 | 8.00 | 4.86 | 5.00 | 5.00 | 4.99 | 4.13 | 5.00 | 5.00 | 4.97 | 5.00 | 5.00 | 5.01 |
| | $r_{mean}$ | 6.73 | 7.64 | 1.51 | 4.49 | 2.80 | 1.68 | 1.35 | 4.56 | 5.35 | 3.11 | 4.81 | 4.87 | 4.83 |
| | $RMSE$ | 1.88 | 2.80 | 3.69 | 0.85 | 2.72 | 3.55 | 3.78 | 0.85 | 1.00 | 2.06 | 0.40 | 0.29 | 0.51 |

Part V.1 : $(r^o, p) = (8, 2)$ without standardization

| $N$ | $T$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 50 | 12.00 | 12.00 | 1.00 | 1.67 | 1.00 | 1.00 | 1.00 | 2.21 | 12.00 | 4.51 | 3.16 | 3.93 | 1.08 |
| 50 | 100 | 11.90 | 12.00 | 1.00 | 6.22 | 1.00 | 1.00 | 1.00 | 5.47 | 8.93 | 4.75 | 7.29 | 7.95 | 6.13 |
| 50 | 200 | 11.80 | 12.00 | 1.00 | 8.15 | 1.05 | 1.00 | 1.00 | 7.34 | 8.41 | 4.18 | 8.41 | 8.70 | 8.82 |
| 100 | 50 | 12.00 | 10.90 | 1.00 | 1.88 | 1.00 | 1.00 | 1.00 | 3.07 | 9.25 | 3.49 | 4.69 | 5.67 | 2.00 |
| 100 | 100 | 12.00 | 12.00 | 1.00 | 7.42 | 1.30 | 1.00 | 1.00 | 6.55 | 10.10 | 4.66 | 9.60 | 9.87 | 9.88 |
| 100 | 200 | 11.60 | 12.00 | 1.00 | 10.00 | 3.23 | 1.00 | 1.00 | 9.93 | 10.00 | 5.40 | 10.00 | 10.00 | 10.00 |
| 200 | 50 | 11.00 | 7.87 | 1.00 | 3.09 | 1.01 | 1.00 | 1.00 | 4.63 | 7.86 | 3.13 | 3.93 | 5.08 | 1.74 |
| 200 | 100 | 12.00 | 12.00 | 1.00 | 7.52 | 2.85 | 1.01 | 1.00 | 7.58 | 11.70 | 4.63 | 11.10 | 11.80 | 11.80 |
| 200 | 200 | 12.00 | 12.00 | 1.14 | 11.30 | 7.85 | 2.95 | 1.00 | 9.22 | 12.00 | 6.85 | 12.00 | 12.00 | 12.00 |
| | $r_{mean}$ | 11.81 | 11.42 | 1.02 | 6.36 | 2.25 | 1.22 | 1.00 | 6.22 | 10.03 | 4.62 | 7.80 | 8.33 | 7.05 |
| | $RMSE$ | 3.82 | 3.66 | 6.98 | 3.66 | 6.13 | 6.81 | 7.00 | 3.05 | 2.51 | 3.53 | 3.05 | 2.77 | 4.28 |
| Part V.2 :$(r^o, p) = (8, 2)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 12.00 | 12.00 | 1.00 | 1.71 | 1.00 | 1.00 | 1.00 | 1.49 | 12.00 | 3.94 | 3.34 | 3.88 | 0.80 |
| 50 | 100 | 11.80 | 12.00 | 1.00 | 2.03 | 1.00 | 1.00 | 1.00 | 2.12 | 7.57 | 2.67 | 4.46 | 5.38 | 1.99 |
| 50 | 200 | 11.70 | 12.00 | 1.00 | 5.02 | 1.00 | 1.00 | 1.00 | 5.49 | 7.27 | 3.75 | 6.76 | 7.14 | 5.04 |
| 100 | 50 | 12.00 | 9.61 | 1.00 | 2.11 | 1.00 | 1.00 | 1.00 | 2.30 | 7.46 | 2.72 | 4.57 | 5.42 | 2.61 |
| 100 | 100 | 11.60 | 12.00 | 1.00 | 5.84 | 1.11 | 1.00 | 1.00 | 4.65 | 8.27 | 3.19 | 7.60 | 7.74 | 7.72 |
| 100 | 200 | 10.10 | 12.00 | 1.00 | 7.61 | 2.57 | 1.01 | 1.00 | 7.54 | 8.00 | 3.93 | 8.00 | 8.00 | 8.01 |
| 200 | 50 | 11.60 | 7.96 | 1.00 | 3.49 | 1.01 | 1.00 | 1.00 | 4.47 | 6.79 | 2.53 | 6.18 | 6.83 | 6.82 |
| 200 | 100 | 10.80 | 12.00 | 1.00 | 7.45 | 2.21 | 1.00 | 1.00 | 7.32 | 8.00 | 3.56 | 8.00 | 8.00 | 8.00 |
| 200 | 200 | 10.40 | 12.00 | 1.56 | 8.00 | 7.94 | 4.42 | 1.02 | 8.00 | 8.00 | 6.60 | 8.00 | 8.00 | 8.01 |
| | $r_{mean}$ | 11.33 | 11.29 | 1.06 | 4.81 | 2.09 | 1.38 | 1.00 | 4.82 | 8.15 | 3.65 | 6.32 | 6.71 | 5.44 |
| | $RMSE$ | 3.40 | 3.57 | 6.94 | 4.00 | 6.28 | 6.71 | 7.00 | 3.95 | 1.44 | 4.50 | 2.38 | 1.91 | 3.76 |
| Part VI.1 : $(r^o, p) = (8, 4)$ without standardization | | | | | | | | | | | | | | |
| 50 | 50 | 12.00 | 12.00 | 1.00 | 1.45 | 1.00 | 1.00 | 1.00 | 2.31 | 12.00 | 5.22 | 4.10 | 4.83 | 0.78 |
| 50 | 100 | 11.90 | 12.00 | 1.00 | 8.28 | 1.00 | 1.00 | 1.00 | 5.09 | 9.79 | 4.64 | 5.96 | 7.25 | 4.33 |
| 50 | 200 | 11.80 | 12.00 | 1.00 | 9.61 | 1.11 | 1.00 | 1.00 | 7.83 | 9.84 | 4.81 | 9.88 | 9.94 | 9.95 |
| 100 | 50 | 12.00 | 11.90 | 1.00 | 2.21 | 1.00 | 1.00 | 1.00 | 3.31 | 11.10 | 4.00 | 4.08 | 5.25 | 1.33 |
| 100 | 100 | 12.00 | 12.00 | 1.00 | 9.86 | 1.16 | 1.00 | 1.00 | 6.19 | 12.00 | 4.81 | 10.60 | 11.40 | 10.70 |
| 100 | 200 | 12.00 | 12.00 | 1.00 | 12.00 | 2.23 | 1.00 | 1.00 | 11.60 | 12.00 | 5.64 | 12.00 | 12.00 | 12.00 |
| 200 | 50 | 8.95 | 7.91 | 1.00 | 3.21 | 1.01 | 1.00 | 1.00 | 3.73 | 8.07 | 2.77 | 2.87 | 3.50 | 0.44 |
| 200 | 100 | 9.39 | 11.80 | 1.00 | 7.56 | 2.61 | 1.00 | 1.00 | 7.11 | 12.00 | 4.12 | 4.15 | 4.87 | 0.20 |
| 200 | 200 | 11.40 | 12.00 | 1.21 | 8.00 | 7.89 | 3.15 | 1.00 | 9.13 | 12.00 | 6.31 | 6.81 | 7.20 | 0.09 |
| | $r_{mean}$ | 11.27 | 11.51 | 1.02 | 6.91 | 2.11 | 1.24 | 1.00 | 6.26 | 10.98 | 4.70 | 6.72 | 7.36 | 4.42 |
| | $RMSE$ | 3.46 | 3.74 | 6.98 | 3.68 | 6.26 | 6.79 | 7.00 | 3.32 | 3.27 | 3.44 | 3.39 | 2.98 | 5.94 |
| Part VI.2 : $(r^o, p) = (8, 4)$ with standardization | | | | | | | | | | | | | | |
| 50 | 50 | 12.00 | 12.00 | 1.00 | 1.22 | 1.00 | 1.00 | 1.00 | 1.10 | 12.00 | 3.25 | 2.98 | 3.51 | 0.62 |
| 50 | 100 | 11.80 | 12.00 | 1.00 | 2.44 | 1.00 | 1.00 | 1.00 | 2.71 | 7.58 | 2.98 | 4.87 | 5.64 | 1.98 |
| 50 | 200 | 11.40 | 12.00 | 1.00 | 2.06 | 1.00 | 1.00 | 1.00 | 3.21 | 6.78 | 1.67 | 7.23 | 7.52 | 5.93 |
| 100 | 50 | 12.00 | 9.59 | 1.00 | 2.48 | 1.00 | 1.00 | 1.00 | 2.49 | 7.46 | 2.86 | 4.47 | 5.26 | 2.25 |
| 100 | 100 | 11.60 | 12.00 | 1.00 | 6.10 | 1.10 | 1.00 | 1.00 | 4.94 | 8.24 | 3.36 | 7.87 | 7.94 | 7.95 |
| 100 | 200 | 10.00 | 12.00 | 1.00 | 7.61 | 3.11 | 1.00 | 1.00 | 7.54 | 8.00 | 4.20 | 8.00 | 8.00 | 8.00 |
| 200 | 50 | 11.60 | 7.97 | 1.00 | 3.03 | 1.00 | 1.00 | 1.00 | 3.77 | 6.46 | 2.15 | 5.96 | 6.76 | 6.15 |
| 200 | 100 | 10.60 | 12.00 | 1.00 | 7.49 | 2.29 | 1.00 | 1.00 | 7.19 | 8.00 | 3.54 | 8.00 | 8.00 | 8.00 |
| 200 | 200 | 10.30 | 12.00 | 1.80 | 8.00 | 7.81 | 4.83 | 1.05 | 8.00 | 8.00 | 6.50 | 8.00 | 8.00 | 8.00 |
| | $r_{mean}$ | 11.26 | 11.28 | 1.09 | 4.49 | 2.15 | 1.43 | 1.01 | 4.55 | 8.06 | 3.39 | 6.38 | 6.74 | 5.43 |
| | $RMSE$ | 3.33 | 3.57 | 6.92 | 4.36 | 6.23 | 6.68 | 6.99 | 4.18 | 1.51 | 4.79 | 2.41 | 1.97 | 3.82 |

Table 7: Estimated number of factors for Stock and Watson's (2002, 2005) data sets

| $r_{max}$ | $AIC$ | $CAIC$ | $BIC$ | $HQ_2$ | $HQ_3$ | $HQ_4$ | $HQ_5$ | $IC_{p^2}$ | $IC_{p^3}$ | $BIC_3$ | $ER$ | $GR$ | $ED$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part I.1 : Stock and Watson (2002) without standardization | | | | | | | | | | | | | |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 4 | 4 | 3 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 4 | 4 | 4 |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 4 | 4 | 6 |
| 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 4 | 4 | 6 |
| 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 6 |
| Part I.2 : Stock and Watson (2002) with standardization | | | | | | | | | | | | | |
| 6 | 6 | 6 | 4 | 6 | 6 | 4 | 4 | 6 | 6 | 2 | 1 | 2 | 2 |
| 8 | 8 | 8 | 4 | 8 | 8 | 7 | 4 | 8 | 8 | 3 | 1 | 2 | 2 |
| 10 | 10 | 10 | 4 | 10 | 10 | 7 | 4 | 10 | 10 | 4 | 1 | 2 | 2 |
| 12 | 12 | 12 | 4 | 12 | 12 | 12 | 4 | 12 | 12 | 5 | 1 | 2 | 2 |
| 14 | 14 | 14 | 4 | 14 | 14 | 12 | 4 | 14 | 14 | 6 | 1 | 2 | 2 |
| 16 | 16 | 16 | 4 | 16 | 16 | 12 | 4 | 16 | 16 | 7 | 1 | 2 | 2 |
| Part II.1 : Stock and Watson (2005) without standardization | | | | | | | | | | | | | |
| 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 1 | 1 | 3 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 1 | 1 | 4 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 1 | 10 | 4 |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 1 | 10 | 4 |
| 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 1 | 10 | 4 |
| 16 | 16 | 16 | 15 | 16 | 16 | 16 | 15 | 16 | 16 | 16 | 1 | 10 | 4 |
| Part II.2 : Stock and Watson (2005) with standardization | | | | | | | | | | | | | |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 2 | 6 | 6 | 2 | 1 | 1 | 1 |
| 8 | 8 | 8 | 6 | 8 | 7 | 6 | 2 | 7 | 8 | 2 | 1 | 1 | 1 |
| 10 | 10 | 10 | 6 | 10 | 7 | 6 | 2 | 7 | 10 | 3 | 1 | 1 | 1 |
| 12 | 12 | 12 | 6 | 12 | 7 | 6 | 2 | 7 | 12 | 4 | 1 | 1 | 1 |
| 14 | 14 | 14 | 6 | 14 | 7 | 6 | 2 | 7 | 14 | 4 | 1 | 1 | 1 |
| 16 | 16 | 16 | 6 | 16 | 7 | 6 | 2 | 7 | 16 | 5 | 1 | 1 | 1 |

Figure 1: Sensitivity to signal-to-noise ratios
$(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 1, 0.5, 0, 0)$

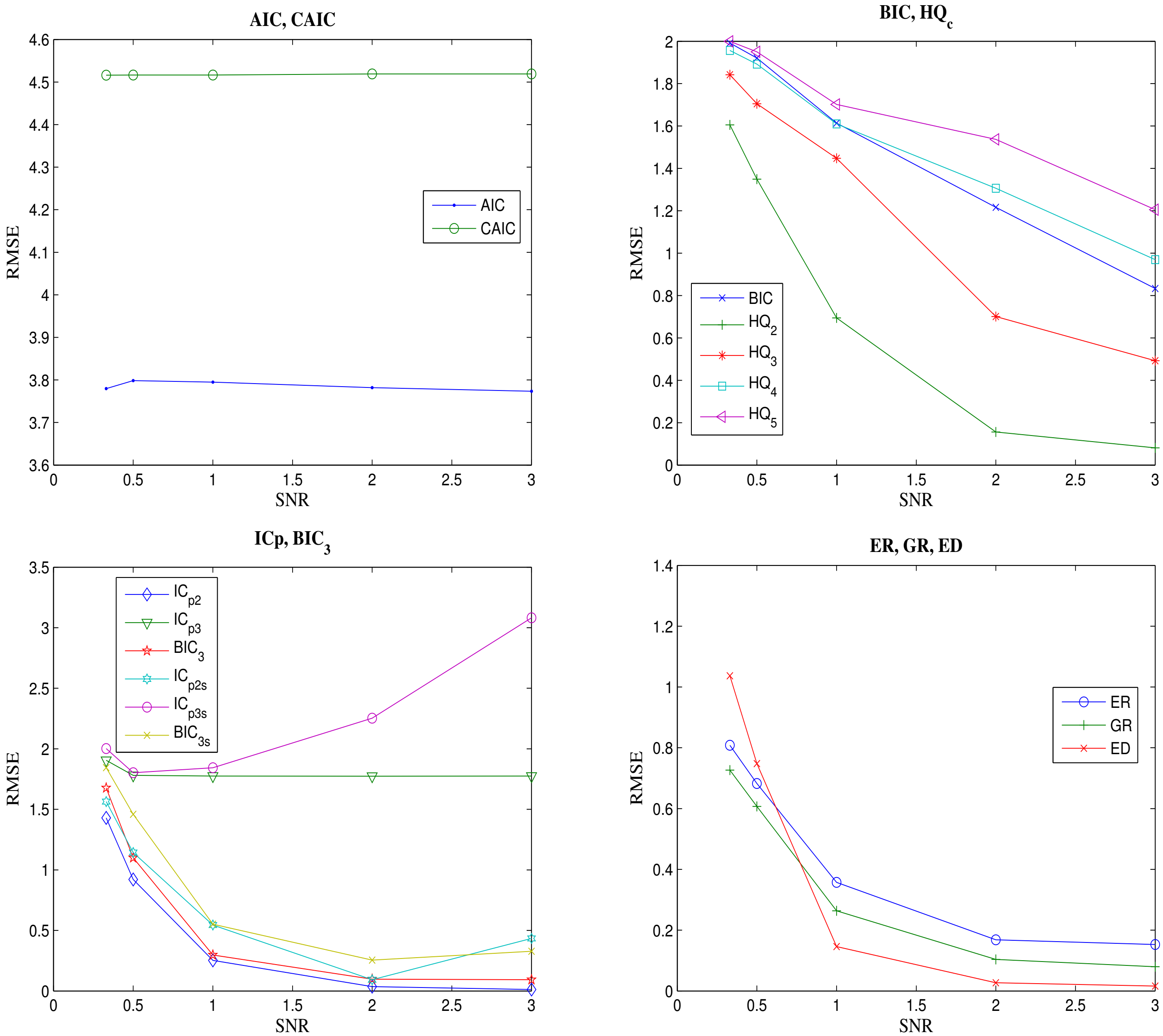
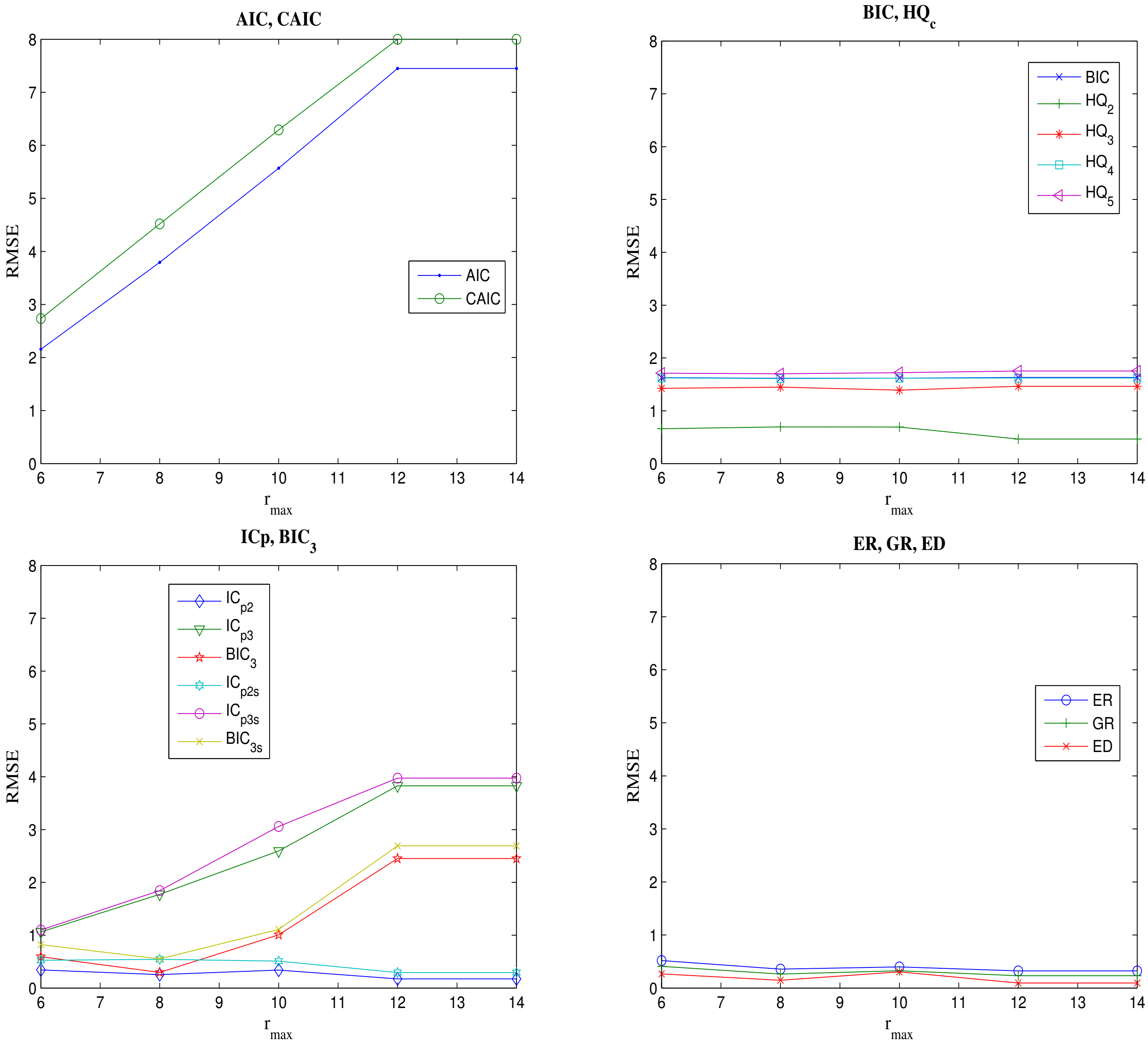


Figure 2: Sensitivity to $r_{max}$

Part I. $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 1, 0.5, 0, 0)$



Part II. $(r^o, \sigma_\lambda^2, \alpha, \rho, \beta) = (3, 1, 0.5, 0.5, 0.2)$

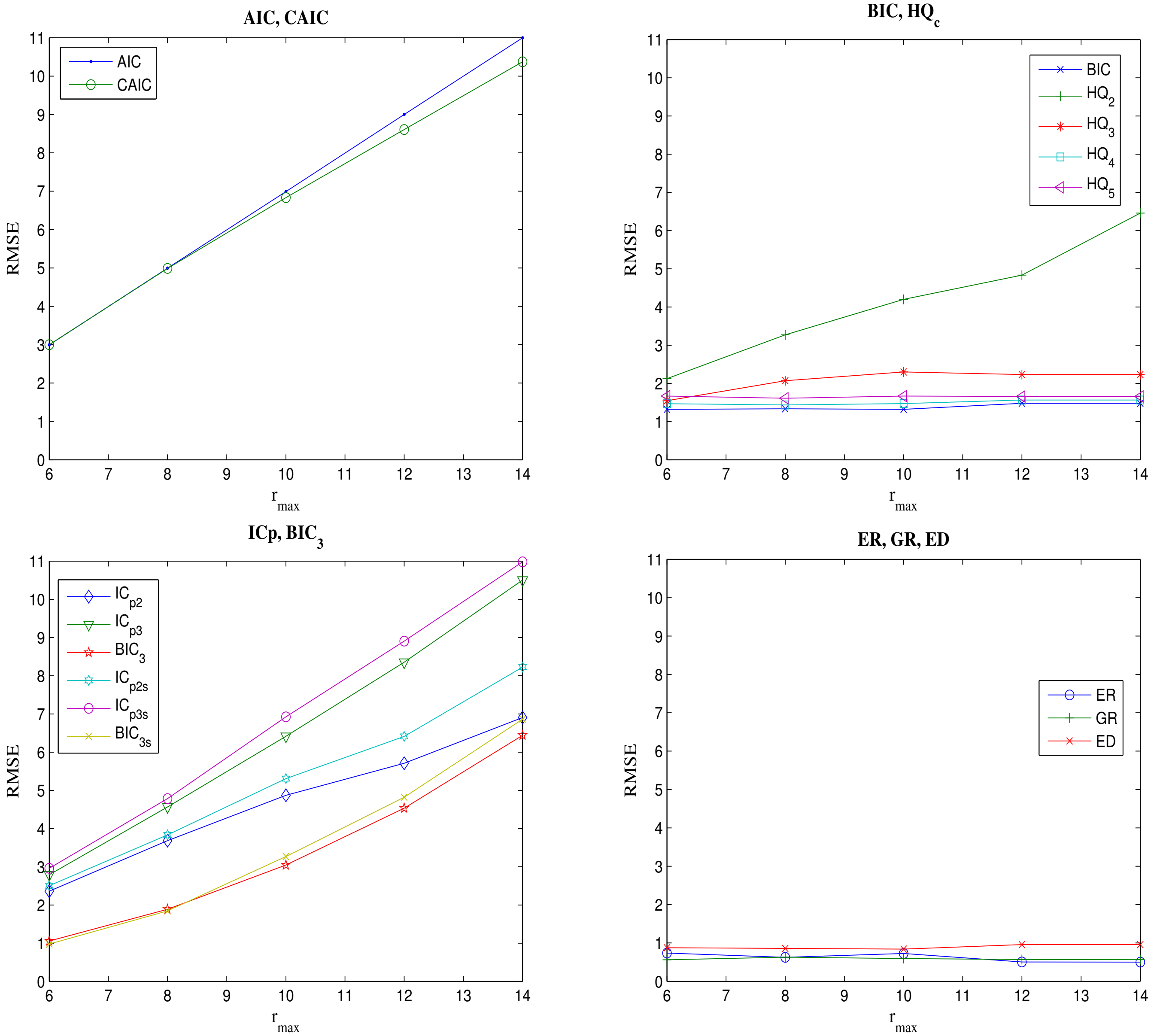


Figure 3: Size of penalty terms
($N = T = 200$)

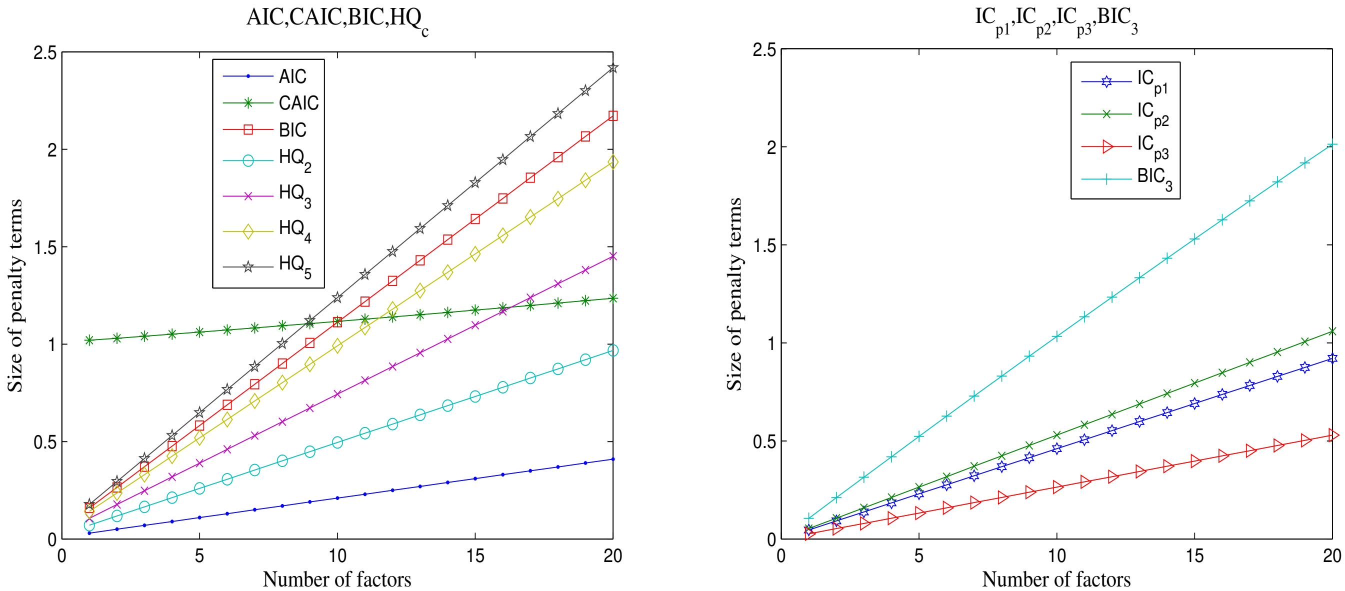



Figure 4: Scree plots

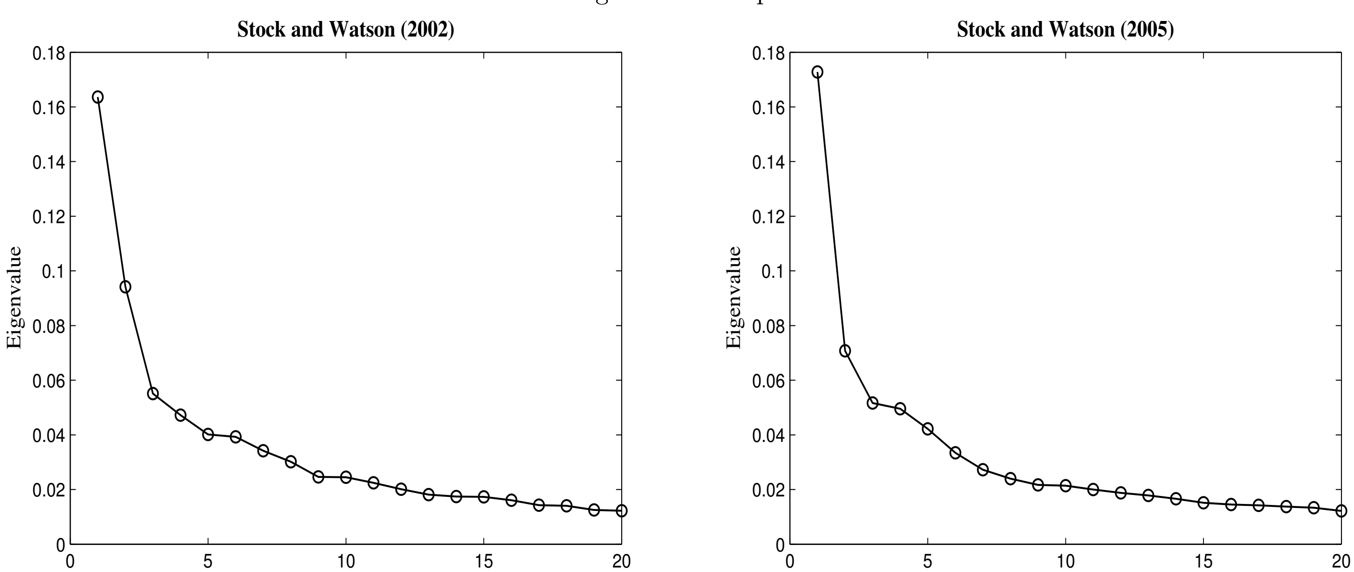


Figure 5: Absolute values of Moscone and Tosetti's (2009) test statistic